

Shear strength estimation of a FRP-strengthened RC beam: A comparison between an artificial neural network and guideline equations

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Abstract

In recent years, several experimental tests have been conducted on the shear strengthening of reinforced concrete (RC) beams strengthened by fiber-reinforced polymer (FRP) systems. In this regard, some equations have also been proposed to estimate the shear strength of beams reinforced with FRP systems. The aim of this study is to investigate the estimation of the shear strength of beams reinforced with FRP systems using an artificial neural network model. For this purpose, a comprehensive and extensive review of forty published articles has been carried out to compile data on 304 RC beams strengthened with externally bonded FRP systems to improve their shear strength. These laboratory results have been used to provide a database for the ANN model to evaluate the shear behavior. The input to the ANN model consists of the 11 variables, including the sectional geometry, reinforcement ratio, FRP ratio, and the characteristics of concrete, steel reinforcement, and composite material, while the output variable is the shear strength of the FRP-strengthened RC beam. In order to evaluate the effectiveness of the neural network model in estimating the shear capacity of RC beams, the results obtained from the neural network model are compared with the equations from the Publication No. 345 and ACI 440.2R guidelines. Specifically, the mean absolute relative error (MARE) criteria for the studied data is 13%, 34% and 39% for the ANN model, ACI 440.2R guideline and the Publication No. 345 guideline, respectively.

Keywords: concrete beam, fiber-reinforced polymers, shear strength, artificial neural network, ACI440.2R

1. Introduction

All over the world, concrete bridges and structures require reconstruction, repair, strengthening or complete replacement as they approach the end of the structure's service life, increase in traffic loads, change of use and reduced structural integrity caused by corrosion to the reinforcing steel (Mabsout et al., 2004; Noël & Soudki, 2011; Saadatmanesh & Ehsani, 1991). The use of polymer-based composites for the improvement of reinforced concrete structures has grown significantly in recent years. Fiber-reinforced polymers are among the most desirable materials for repair due to their high strength-to-weight ratio and anti-corrosion properties and can lead to an increase in the service life of the structure (Saadatmanesh & Ehsani, 1991; Shahawy et al., 1996; Sobuz et al., 2011). In general, the strengthening of existing concrete structures or their restoration due to the lack of proper design and construction, lack of maintenance and repair, structural events such as earthquakes, bearing double design loads, improving deficiencies caused by erosion, increasing the structure's

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ductility, or other cases by using suitable materials, and the correct implementation methods are done conventionally (Gamino et al., 2010; Sobuz et al., 2011). The use of composite materials made of fibers embedded in a polymer resin medium as polymers reinforced with FRP fibers has been introduced as a necessity in replacing traditional materials and existing practices (Beber & Campos-Filho, 2005). An FRP system is defined as the fibers and resins used to create the composite laminate, all applicable resins used to bond it to the concrete substrate, and all applied coatings used to protect the constituent materials. Coatings used exclusively for aesthetic reasons are not considered part of an FRP system. FRP materials are lightweight, noncorrosive, and exhibit high tensile strength. These materials are readily available in several forms, ranging from factory-made laminates to dry fiber sheets that can be wrapped to conform to the geometry of a structure before adding the polymer resin (American Concrete Institute, 2008).

Following the expansion of the need and attention to strengthening using composite materials and in order to apply technical knowledge, design methods have also been developed. Explaining the analysis methods and considering the safety coefficients in the design with economic considerations led to the formulation of the calculation and implementation guidelines and regulations, including the American Concrete Institute (2008) 440.2R regulations, Management and Planning Organization of Iran (MPO) (2006) Publication No. 345, International Federation for Structural Concrete (fib) (2001), National Cooperative Highway Research Program (NCHRP) Report 514 (Mirmiran et al., 2004), and standard Japan Society of Civil Engineers (Maruyama, 2001) pointed out. The International Federation for Structural Concrete (2001) recently published a publication on the guidelines for designing externally bonded FRP reinforcement for RC structures. The CSA-S806 by the Canadian Standard Association (CSA) (2002) is also active in developing and formulating guidelines for FRP systems. In the United States, ACI Guide 440.2R has been published as a guide for the design and implementation of strengthening concrete buildings with FRP systems.

The design equations presented in the guidelines are based on research results on conventional dimensions and proportional prismatic members, while FRP systems have effective performance on other non-prismatic members as well. These guidelines only apply to FRP strengthening systems used as additional tensile reinforcement. It is not recommended to use these systems as compressive reinforcement. While FRP materials can support compressive stresses, there are numerous issues surrounding the use of FRP for compression. For example, the microbuckling of fibers can occur if any resin voids are present in the laminate; laminates themselves can buckle if not properly adhered or anchored to the substrate, and highly unreliable compressive strengths result from misaligning fibers in the field. Therefore, the compressive strength of FRP materials is ignored (American Concrete Institute, 2008).

The cause of shear failure, even for simple reinforced concrete elements, is a complex mechanism, and this will take a more complicated form with the use of polymer fibers in reinforced concrete elements (Täljsten, 2003). Therefore, estimating the final shear strength of reinforced concrete beams is very important, especially in design cases. In most of the existing design models, the evaluation of the design shear strength of a reinforced concrete beam reinforced with polymer fibers is obtained from the sum of the contributions of concrete, reinforcing steel (stirrups, ties, or spirals), and polymer fibers (Khalifa & Nanni, 2000; Triantafillou & Antonopoulos, 2000). The contribution of the first two cases can be calculated according to the provisions of the existing design regulations. Therefore, the main difference between the existing design patterns lies in how to evaluate the contribution of polymer fibers. In this regard, experimental relationships (Chaallal et al., 1988; Khalifa et al., 1999; Norris et al., 1997; Täljsten, 2003; Triantafillou & Antonopoulos, 2000; Triantafillou & Plevris, 1992; Zhang et al., 2004) and several analytical equations (Chaallal et al., 1988; Ianniruberto & Imbimbo, 2004; Pellegrino & Modena, 2008; Täljsten, 2003) have been introduced using regression analysis of laboratory data. To develop such models, it is necessary to assume a combination and a template for empirical relationships and then obtain unknown parameters. Several studies have been done on beams from different viewpoints (Alambeigi et al., 2020; Areias et al., 2019; Firouzi & Kazemi, 2023; Dadgar-Rad & Firouzi, 2021; Karami et al., 2020; Żur et al., 2023). The success of such a process in the mentioned studies is difficult due to the large number of parameters affecting the strength of the beam.

In contrast, the use of artificial neural networks provides an alternative method that overcomes these problems. An artificial neural network consists of a network of simple processing elements (neurons), which can display the overall complex behavior determined by the relationship between processing elements and system parameters. This unique feature enables the neural network to solve complex problems that cannot be solved with existing analytical methods. This is true even for problems whose mathematical and physical models are not well known. For this reason, the neural network can be used to estimate and evaluate the shear strength of beams reinforced with appropriate polymer fibers.

The purpose of this study is to investigate the possibility of using a multi-layer feed-forward network to estimate the ultimate shear strength of concrete beams reinforced with polymer fibers. For this purpose, a database has been compiled from the report analyzing the results of the existing articles. Then, in order to evaluate the efficiency and performance of the neural network model in estimating the shear capacity of reinforced beams, the results obtained from the neural network model are compared with the relationship values of regulations from Iran and America.

2. Materials and methods

2.1. FRP materials

The FRP reinforcement system is one of the composite materials consisting of two parts of fiber or reinforcement fibers surrounded by a polymer resin matrix, as shown in Figure 1 (American Concrete Institute, 1996). FRP fibers, which have elastic technical characteristics and are very resistant, are considered the main bearing component in the FRP material (they make up between 40% and 70% of the volume). The FRP resin also basically acts as an adhesive medium that holds the fibers together (Norris et al., 1997). Among the most common resins, e.g., epoxies, polyesters, are often used in a wide range of environmental conditions. The most widely used resins are epoxy resins, which are used to impregnate dry FRP sheets and bond them to reinforced concrete members. The main role of the resin matrix is to transfer shear from the reinforcing fiber to the adjacent material, protect the fiber in environmental conditions, prevent mechanical damage to the fibers, and finally control the local buckling of the fibers under pressure (Shahawy et al., 1996).



Fig. 1. Components of the FRP system (American Concrete Institute, 1996)

FRP composite fibers (FRP Sheets) are sheets of FRP with a thickness of several millimeters. FRP sheets are attached to the concrete surface with strong and suitable adhesives. The fibers form the stiffness and strength of the FRP system. In general, 4 types of woven FRP fibers are used for strengthening with FRP, which include carbon fibers, glass, aramid and basalt (International Federation for Structural Concrete, 2001). In FRP reinforcements, one should not rely on their strength to compressive loads. However, their tolerance against the pressure caused by the application of alternating bending anchors or changes in the way of loading can be considered. However, in any case, the compressive strength of FRP reinforcing components is ignored (American Concrete Institute, 2008). Design recommendations are based on limit state design principles. Reinforced or improved concrete buildings with FRP materials are designed based on the recommendations for resistance and serviceability, and from the load factors provided in the Iranian concrete regulations and ACI318 and ACI 440 regulations for American calculations. Additional reduction factors applied to the contribution of the FRP reinforcement are recommended to reflect uncertainties inherent in FRP systems compared with steel reinforced and prestressed concrete. The reduction factors associated with FRP are adjusted to achieve a confidence index above 3.5. Reliability indexes between 3.0 and 3.5 can be encountered in cases where relatively low ratios of steel reinforcement combined with high ratios of FRP reinforcement are used (American Concrete Institute, 2008).

The existing concrete substrate strength is an important parameter for bond-critical applications, including flexure or shear strengthening. It should possess the necessary strength to develop the design stresses of the FRP system through bond. The working concrete, including all repaired surfaces, as well as the main concrete, must have sufficient direct tensile and shear strength to transfer force to the FRP system. The minimum tensile strength of concrete is 1.4 MPa, which is measured by tensile testing according to ACI 503R or ASTM D4541. FRP systems should not be used when the concrete substrate has a compressive strength of less than 17 MPa (American Concrete Institute, 2008; Mirmiran et al., 2004).

When increasing flexure capacity, the structure will be loaded closer to its maximum shear capacity. Täljsten (2003), even shows in a full-scale test that flexural strengthening can induce shear failure. On the other hand, a structure with brittle failure in shear can be strengthened so that the failure mode will change to a more ductile and friendly mode (Carolin & Täljsten, 2005; Collins & Roper, 1990). A beam must have a certain safety margin against shear failure since shear failure is more dangerous and less predictable than flexural failure (Al-Sulaimani et al., 1994; Carolin & Täljsten, 2005).

There are several methods for shear strengthening, such as additional reinforcements covered with concrete and bracing with steel and CFRP external bonding. For optimal use of the capacity of FRP materials during the shear strengthening of reinforced concrete members by FRP, external reinforcements are attached in the main direction of the fibers, parallel to the maximum of the main tensile stresses. FRPs used for shear reinforcement and repair can be used in three configurations: Complete wrapping, U-Wrap, and bonding to two opposite sides of the beam. In all wrapping schemes, the FRP system can be installed continuously along the span of a member or placed as intermittent (discrete) strips. For the complete wrapping repair technique, the FRP sheet fully wraps the beam cross-section with the fibers fixed in the transverse direction along the beam, as shown in Figure 2 (Chen & Teng, 2003). When the beam is completely wrapped (the most effective case), the probability of debonding is slim, and the full capacity of the FRP sheet can be utilized (American Concrete Institute, 2008). In the U-Wrapping repair technique, the FRP sheet is applied to three sides of the beam's cross-section because the top face is not accessible. For side bonding, the FRP sheets are applied on the two side faces of the beam when the bottom face of the beam is not accessible.

Both ends of the FRP sheet tend to separate from the concrete surface before the FRP sheet reaches its ultimate tensile capacity (Chen & Teng, 2003).

A rupture of an FRP occurs when the materials have reached their ultimate tensile strength, causing the fibers to fracture.

2.2. Neural Network

The most important application of Artificial Neural Networks (ANN) in civil engineering is the estimation of nonlinear functions with appropriate accuracy.

The most common type of neural networks is multi-layer Feed-Forward networks, which consist of an input layer, one or more hidden layers, and an output layer. A weight is considered for each connection. The input layer receives the input data and transmits it to the neurons of the hidden layer. Then, after processing, the data is entered into the output layer. The output of the neurons of the output layer of the prediction neural network is a function of the input data.

There is no universally accepted method for determining the optimal number of neurons and layers for each problem, and for that, it is necessary to act based on experience and trial and error. An example of a feed-forward multilayer neural network is shown in Figure 3.



Fig. 2. Schematics of shear reinforcement by adding externally bonded FRP laminates: a) side bonding; b) U-Wrap; c) complete wrapping. Each applied as intermittent strips (I) or a continuous sheet (II) along the length of the member (Chen & Teng, 2003)

FRP debonding is the process where an FRP sheet peels off the concrete surface to which it is bonded.



Fig. 3. Schematic diagram of 3-layer network

According to Figure 3, the output in the last layer is obtained from Equation (1):

$$O = f\left(\sum_{q} g\left(\sum_{p} x_{i} w_{ij}^{l}\right) w_{j}^{H}\right)$$
(1)

In this case, I and H represent the input and the hidden layer, respectively, and w represents the

weights of the layers. p and q are the number of input and hidden layer neurons respectively, f is the transfer function of the output layer, and g is the transfer function of the hidden layer. The error backpropagation algorithm is one of the most widely used algorithms for training multi-layer feed-forward artificial neural networks. In this method, the technique of gradient descent is used to minimize the error function, during which the errors are propagated backward from the output layer to the input layer and the weights are modified to minimize the error. Therefore, the training process includes the gradual modification of the weights in order to minimize the error function. This process continues until one of the stopping criteria is satisfied.

Before starting the simulation, the input data should be divided into three groups (Kasabov, 1996):

- Training data: These data are used among the labeled data to guide the training process, and these data are also used to update the weights of the neural network during training. Typically, 60% to 70% of the total data are randomly selected as training data. After the network has been trained by these data, the weights have found their final value so that the network obtains the least error for the training data.
- 2. Validation data: This is used to monitor the quality of the neural network model of the system during the learning process and to determine the learning stop condition for the training process (20% of the total data).
- 3. Test data: after the network was trained by the training data until reaching the minimum error, the rest of the data (remaining 20%) that did not play a role in training was given as input to the network and the network's response with the desired response (their label) is compared and thus the efficiency of the trained network is tested.

3. Methodologies for estimating the shear strength of FRP-strengthened RC beams

Today, due to the increase in the use of FRP materials, numerous regulations and recommendations such as ACI Committee 440.2R (American Concrete Institute, 2008), Management and Planning Organization of Iran (MPO) (2006), International Federation for Structural Concrete (fib) (2001), National Cooperative Highway Research Program (Mirmiran et al., 2004), Japan Society of Civil Engineers (Maruyama, 2001), and Canadian Standard Association (CSA) (2002) have been published in different countries of the world for the design of concrete structures reinforced or reinforced with polymer fibers. It seems that flexural reinforcement is well documented in these regulations, although some unclear points remain. This is despite the fact that the behavioral understanding of reinforced concrete structures that are designed for shear strengthening is still at a stage where similar design rules either do not exist or have been addressed briefly (Adhikary & Mutsuyoshi, 2004; Diagana et al., 2003; Pellegrino & Modena, 2002; Täljsten, 2003; Triantafillou & Antonopoulos, 2000). For example, Carolin & Taljsten (2005) have shown that the composite is not uniformly stressed when bonded to the sides of a beam and that the strain field must be studied further to understand the behavior of a member strengthened in shear.

According to Figure 2, the use of externally bonded FRP laminates in typical wrapping schemes (Completely wrapped, U-wraps or side bonding of the web) and the placement of its fibers along the cross-section or perpendicular to the possible shear cracks, effectively increases the shear strength of the elements. The additional shear strength created by this method depends on various parameters such as: sectional geometry, the strengthening pattern, the compressive strength of the concrete, etc. (Diagana et al., 2003; Pellegrino & Modena, 2002; Ozden et al., 2014). Therefore, the amount of this increase in strength is always limited depending on the operation criteria and in accordance with the load conditions of the member. In recent years, several guidelines and design recommendations have been proposed to estimate the shear strength of reinforced concrete beams when they are reinforced with polymer fibers. In all the proposed designs, the design shear strength, V_{d} , of a concrete beam reinforced with polymer fibers is calculated from Equation (2):

$$V_d = V_c + V_s + V_f \tag{2}$$

where V_c is the contribution of concrete, V_s is the contribution of transverse steel and V_f is the contribution of FRP in the shear strength of the beam. In this regard, V_c and V_s can be obtained by using the relations of the existing regulations, so the main difference between the proposed plans is the way to calculate V_f . In the following, each of these parameters will be explained in the ACI Committee 440.2R (American Concrete Institute, 2008) and the Publication No. 345 (Management and Planning Organization of Iran, 2006). In addition, the method of simulating the shear behavior of reinforced concrete beams reinforced with polymer fibers is explained with the help of a neural network.

3.1. Shear strength of FRP-strengthened RC beams – ACI 440.2R

According to chapter 11 of this regulation, the shear strengthening of members is important because the increase in shear strength leads to flexural failure of the member, which in this case, has a more ductile behavior than shear failure. This regulation stipulates that the design shear strength of concrete member reinforced with FRP system, $(\emptyset V_n)$, should be higher than the required shear strength (factored shear), (V_u) , in the section under consideration. The design shear strength is calculated by multiplying the nominal shear strength (V_n) by the strength reduction factor according to Equation (3):

$$\emptyset V_n \ge V_n \tag{3}$$

According to American concrete regulations, the strength reduction factor \emptyset is considered equal to 0.85 in shear or torsion mode. The nominal shear strength of a concrete member reinforced with the FRP system is obtained from the total contribution of concrete, shear steel and polymer fibers. Also, an additional reduction factor (Ψ_{f}) is applied to the contribution of FRP, which is in accordance with Equation (4):

$$V_n = \emptyset(V_c + V_s + \Psi_f V_f) \tag{4}$$

where Ψ_f is equal to 0.95 for the case where the polymer fibers have completely wrapped members, and 0.85 for the three-sided FRP U-wrap or two-opposite-sides strengthening schemes. V_c , which is equal to the shear strength provided by concrete for non-prestressed members in a state that is only under the effect of shear and bending, using Equations (5) and (6), and V_s , which is equal to the nominal shear strength provided by shear bars, according to Equation (7) is derived according to the ACI Committee 440.2R (American Concrete Institute, 2008).

$$V_c = 0.17 \sqrt{f_c} b_w d \tag{5}$$

$$V_c = \left(0.16\sqrt{f_c} + 17\rho_w \frac{V_u d}{M_u}\right) b_w \tag{6}$$

$$V_s = \frac{A_V f_{yt} (\sin \alpha + \cos \alpha) d}{s}$$
(7)

where f_c is specified compressive strength of concrete [MPa], b_w is the width of the web [mm], d is the distance from extreme compression fiber to the centroid of tension reinforcement [mm], M_u is the factored moment at a section [N-mm], V_u is the factored shear force at the section [N], and finally, the ratio $\rho_w = A_s$ / $(b_w d)$ where A_s is the area of nonprestressed steel reinforcement [mm²]. In Equation (7), A_{ν} is the area of shear reinforcement spacing *s*. Angle α is the angle defining the orientation of reinforcement. f_{yt} is the specified yield strength f_y of transverse reinforcement [MPa].

The contribution of the FRP system to the shear strength of a member is based on the fiber orientation and an assumed crack pattern (Khalifa et al., 1999). The shear strength provided by the FRP reinforcement can be determined by calculating the force resulting from the tensile stress in the FRP across the assumed crack. The shear contribution of the FRP shear reinforcement is then given by Equation (8):

$$V_f = \frac{A_{fv} f_{fe} (\sin \alpha + \cos \alpha) d_{fv}}{s_f}$$
(8)

where A_{fv} is the area of FRP shear reinforcement with spacing s_f , whose value is equal to $2nt_fw_f$ [mm²], where *n* is the number of plies of FRP reinforcement, t_f is the nominal thickness of one ply of FRP reinforcement, and w_f is the width of FRP reinforcing plies [mm]. d_{fv} is the effective depth of FRP shear reinforcement in millimeters, and α is the angle of the FRP shear reinforcement strip with the longitudinal axis of the member. f_{fe} is the effective stress in the FRP, stress level attained at section failure and is obtained from Equation (9):

$$f_{fe} = \varepsilon_{fe} E_f \tag{9}$$

where ε_{f_e} is equal to the effective strain and it is calculated according to Equations (10) and (11) according to the different arrangements of FRP sheets.

For reinforced concrete column and beam members completely wrapped by FRP, the loss of aggregate interlock of the concrete has been observed to occur at fiber strains less than the ultimate fiber strain. To preclude this mode of failure, the maximum strain used for design should be limited to the smallest value of 0.004 and 0.75 of the design failure strain (ε_{fu}). In this case, the effective strain is calculated from Equation (10).

$$\varepsilon_{fu} = 0.75\varepsilon_{fu} \le 0.004 \tag{10}$$

Bonded U-wraps or bonded face plies, FRP systems do not enclose the entire section (two- and three-sided wraps), have been observed to delaminate from the concrete before the loss of aggregate interlock of the section. For this reason, bond stresses have been analyzed to determine the usefulness of these systems (Triantafillou & Plevris, 1992), and the effective strain is obtained according to Equation (11):

$$\varepsilon_{fe} = K_v \varepsilon_{fu} \le 0.004 \tag{11}$$

It should be noted that in Equations (10) and (11), ε_{fit} is obtained from the product of the reduction factor of the environmental conditions and the ultimate failure strain reported by the factory. This coefficient is considered equal to 0.95 and 0.75 for mild environmental conditions and the type of carbon and glass fibers, respectively. The bond-reduction coefficient is a function of the concrete strength, the type of wrapping scheme used, and the stiffness of the laminate. It is obtained from Equation (12):

$$K_{v} = \frac{k_{1}k_{2}L_{e}}{11900\varepsilon_{fu}} \le 0.75$$
(12)

The active bond length (L_e) is the length over which the majority of the bond stress is maintained. This length is calculated by Equation (13):

$$L_e = \frac{23300}{(nt_f E_f)^{0.58}}$$
(13)

The bond-reduction coefficient also relies on two modification factors, k_1 and k_2 , that account for the concrete strength and the type of wrapping scheme used, respectively. Calculate k_1 from Equation (14) and calculate k_2 , in case a U-wraps stirrup is used, from Equation (15). Equation (16) is used for two sides bonded scheme.

$$k_1 = \left(\frac{f'_c}{27}\right)^{\frac{2}{3}}$$
(14)

$$k_2 = \frac{d_{fv} - L_e}{d_{fv}}$$
(15)

$$k_2 = \frac{d_{fv} - 2L_e}{d_{fv}}$$
(16)

It should be noted that the total shear strength provided by the steel shear reinforcement and the FRP shear reinforcement must follow Equation (17):

$$V_s + V_f \le 0.66 \sqrt{f_c'} b_w d \tag{17}$$

3.2. Shear strength of FRP-strengthened RC beams – Publication No. 345

Chapter 9 of this guide contains the general criteria for using FRP materials as external stirrups, in order to increase the shear and torsion strength of reinforced concrete sections. Based on this, the final shear strength of the section V_r is calculated using Equation (18):

$$V_r = \varphi_c V_c + \varphi_s V_s + \varphi_{frp} V_{frp}$$
(18)

where φ_{c} , φ_{s} and φ_{frp} are the partial safety factor of concrete (equal to 0.6), steel equal to (0.85) and FRP material equal to 0.85, respectively. The ultimate shear strength provided by concrete, V_{c} , and the ultimate shear

strength provided by shear reinforcement, V_s , are obtained from the Iranian concrete code "ABA". The shear strength provided by FRP materials (V_{frp}) is added to include the contribution of the FRP shear reinforcement. V_c and V_s are calculated from Equations (19) and (20):

$$V_c = 0.2\sqrt{f_c'}b_w d \tag{19}$$

$$V_s = A_V f_y \frac{d}{s} \tag{20}$$

The contribution of FRP material from the shear is determined according to Equations (21) and (22):

$$V_{f} = \frac{E_{frp} \varepsilon_{frpe} A_{frp} d_{frp} (\sin\beta + \cos\beta)}{s_{frp}}$$
(21)

$$A_{frp} = 2t_{frp} w_{frp} \tag{22}$$

The effective depth of FRP stirrups, d_{fip} , is considered as the distance from extreme compression fiber to centroid of tension reinforcement, and in the case where the section is completely wrapped around, it is assumed to be equal to h (beam height). The effective strain of FRP materials, ε_{frpe} , is obtained through testing and applying Equations (23) and (26), and in any case, the lowest value obtained from the above two methods is considered. But the effective strain ε_{frpe} should be limited to 0.004, because in the higher range of strain, the loss of the aggregate interlock of the concrete has been observed due to the opening of cracks.

$$\varepsilon_{frpe} = R\varepsilon_{frpu} \tag{23}$$

R, the ratio of effective strain to ultimate strain in FRP stirrups is obtained from Equation (24), and the ratio of FRP shear reinforcement, ρ_{frp} , is obtained from Equation (25):

$$R = 0.8\lambda_1 \left(\frac{f_c^{2/3}}{\rho_{frp} E_{frp}}\right)^{\lambda_2}$$
(24)

$$\rho_{frp} = \frac{2t_{frp} w_{frp}}{b_w s_{frp}}$$
(25)

In Equation (24), coefficients λ_1 and λ_2 for carbon fibers are: $\lambda_1 = 1.35$ and $\lambda_2 = 0.3$, aramid fibers and glass: $\lambda_1 = 1.23$ and $\lambda_2 = 0.47$.

$$\varepsilon_{frpu} = \frac{\varphi_{frp} k_1 k_2 L_e}{9525} \tag{26}$$

In order to consider the possibility of debonding of FRP sheets, the effective strain is considered equal to the lowest three values A, B and C: effective strain limit ε_{frpe} , the value obtained from Equation (10) and the value presented in Equation (11), respectively.

$$k_1 = \left[\frac{f'_c}{27.65}\right]^{\frac{2}{3}}$$
(27)

$$k_2 = \frac{d_{frp} - n_e L_e}{d_{frp}} \tag{28}$$

where k_1 is an index of concrete shear strength and k_2 is an index for the type of wrapping scheme. In Equation (28), n_e is the number of free ends of FRP stirrups on one side of the beam. If we have FRP beams on only 2 side faces, $n_e = 2$, and if thP strup is U-wraps, then $n_e = 1$. When $k_2 \le 0$, the FRP system is ineffective in sheanless FRP restraint is provided in a suitable manner. The effective restraint length, L_e , is calculated ung Equation (29), which is proposed based on experimental data:

$$L_e = \frac{253350}{\left(t_{frp} E_{frp}\right)^{0.58}}$$
(29)

It should be noted that the ultimate shear strength of the section is limited to the Equation (30):

$$V_{r} = V_{c} + V_{s} + V_{frp} \le V_{c} + 0.8\varphi_{c}\sqrt{f_{c}}b_{w}d$$
(30)

3.3. Shear strength of FRP-strengthened RC beams using a neural network

In Figure 4, the neural network architecture used in this research is presented. This feed-forward neural network consists of three layers. Nodes represent neurons, and arrows represent weighted connections. This network includes 11 neurons in the input layer, 10 in the hidden layer, and 1 in the output layer. The number of input and output neurons depends on the problem to be solved, while the number of hidden layers and the number of neurons may be checked by different situations, and the most optimal one is selected. The network training is done by updating the weights related to the connections between neurons based on the known input and the output patterns or learning patterns using aeraverocess. These weights indicate the influence and strength of a connection among neurons.

Considering the many parameters that influence the beam shear failure mode, creating an efficient neural network requires an appropriate selection of input variables. However, it should be kept in mind that increasing the input neurons to a neural network may reduce the accuracy and efficiency of the training process (Kasabov, 1996). In this article, the selection of input parameters is based on the shear capacity relationships that were examined earlier. As a basic idea, the ACI Committee 318M (American Concrete Institute, 2005) regulations specify that three variables have the greatest contribution to the shear strength of concrete, these variables include: $1 - f_c'$, $2 - A_s / b_w d$, $3 - V_u d / M_u$. After counting trial and error, 11 main variables were considered as input to the network (Tab. 1).

In this research, a comprehensive and extensive review of forty articles¹, has been carried out to compile data on 304 reinforced concrete beams reinforced in shear using FRP sheets (Tab. 6²). These laboratory results have been used to provide a database for the ANN model. The 304 analyzed beam samples are randomly divided into three groups: a) training data set comprising 60% of the total data set (182 data); b) validation data set comprising 20% of the total data set (61 data); c) the test data set consists of 20% of the total data set (61 data).

All beams were simply supported with various span lengths, and they were subjected to two concentrated loads symmetrically placed about the midspan. In the data collection, different schemes of FRP shear strengthening for reinforced concrete beams are considered (Fig. 2) (Completely wrapped, U-wraps or side bonding of the web). The FRP sheets can be applied as intermittent strips (like stirrups) or a continuous sheet along the length of the member. In addition, the parameters available in the laboratory database have a wide range of loading arrangement, sectional geometry, material properties, reinforcement ratio, as well as geometry, arrangement and mechanical properties of FRP external reinforcement. Therefore, we will have a wide range of failure loads.

¹ Abdel-Jaber et al., 2003; Adhikary & Mutsuyoshi, 2004; Alzate et al., 2013; Barros & Dias, 2006; Beber & Campos-Filho, 2005; Bousselham & Chaallal, 1988; Bukhari et al., 2010; 2013; Cao et al., 2005; Carolin & Täljsten, 2005; Chaallal et al., 1988; Deniaud & Cheng, 2001; Diagana et al., 2003; Dias & Barros, 2012; Grace et al., 2003; Grande et al., 2009; Islam et al., 2005; Kamiharako et al., 1997; Khalifa et al., 1999; Khalifa & Nanni, 2000, 2002; Kim et al., 2008; Leung et al., 2007; Mofidi & Chaallal, 2014; Mofidi et al., 2014; Monti & Liotta, 2007; Norris et al., 1997; Ozden et al., 2014; Panda et al., 2011; Pellegrino & Modena, 2002, 2008; Saadatmanesh & Ehsani, 1991; Singh, 2013; Sundarraja & Rajamohan, 2009; Täljsten, 2003; Tanarslan et al., 2008; Teng et al., 2004.

² The table has been placed at the page 21 and is available in the online version of the article: https://doi.org/10.7494/ cmms.2024.3.0830.



Fig. 4. Schematic diagram of the neural network architecture

In the neural network structure, according to Table 1, all the significant variables in the data are considered input variables for the model. The number of hidden layers was equal to one, and the number of neurons in the hidden layer was chosen equal to 10. Here, the number of neurons in the hidden layer was optionally considered to be 10, it should be noted that more neurons in the hidden layer require more calculations, and if the number is too high, it will cause overfitting of the data. However, a network with more neurons will solve more complex problems (Alwosheel et al., 2018). Also, in neural network training, the Levenberg– Marquardt backpropagation algorithm is used with a tangent sigmoid transfer function in the hidden layer and a linear transfer function in the output layer. It should be noted that the neural network's performance is based on the Mean Squared Error (MSE) using the MATLAB program. The optimal network was selected based on the lowest error rate and the highest correlation coefficient between the data. The network specifications are given in Table 2.

No.	Variable	Symbol	Minimum	Maximum	Average
	Input element				
1	beam/slab width [mm]	b	75.00	600.00	172.09
2	effective depth of cross-section [mm]	d	124.60	720.00	304.10
3	cylindrical compressive strength of concrete [MPa]	f_c'	10.60	71.40	34.80
4	area of transverse reinforcement per spacing s [mm ² /mm]	A_{sy}/s	0.00	2.01	0.24
5	yield stress of transverse steel [MPa]	$f_{_{Y90}}$	0.00	650.00	225.03
6	number of free ends of an FRP stirrup*	n _e	-	—	_
7	tensile modulus of elasticity of FRP [MPa]	E_{f}	5.30	640.00	201.23
8	design stress of externally bonded FRP reinforcement [MPa]	$f_{_{fd}}$	112.00	4500.00	3032.89
9	effective depth of FRP shear reinforcement [mm]**	d_{fy}	80.00	800.00	306.97
10	the effective width of FRP shear reinforcement strip in the longitudinal direction of the beam to the center-to-enter distance of FRP stirrups	w_f / s_f	0.11	1.50	0.73
11	number of plies of FRP reinforcement in the thickness of each layer [mm]	$n_f \times t_f$	0.04	2.10	0.43
	Output element				
1	laboratory ultimate shear strength [N]	V_{U-exp}	45.90	881.20	173.80

Table 1. Range of input and output variables

* n_e is the number of free ends of the FRP stirrup on the side of the beam ($n_e = 2$ for side plates, $n_e = 1$ for a U-wrap, and $n_e = 0$ for completely wrapped).

^{**} The effective depth of FRP shear reinforcement d_{j_0} , is taken as the distance from the free end of the FRP shear reinforcement underneath the slab to the bottom of the internal steel stirrups. For the rare case of completely wrapped member, d_{j_0} is taken as the total height of the section (Canadian Standard Association, 2002).

No.	Parameter	Specifications		
1	Network function	feed-forward backprop with Trainlm (Levenberg– Marquardt backpropagation) as a training function	R	MSE
2	Network structure	11-10-1		
3	Number of training data	182	0.991	296.29
4	Number of validation data	61	0.969	700.13
5	Number of test data	61	0.946	964.38
6	Number of all data	304	0.981	511.38

Table 2. Network specifications

3.3.1. Efficiency criteria of models

There are various indicators to evaluate the performance of estimation and forecasting models. In the following section, the criteria for measuring the number of prediction errors will be discussed.

The correlation coefficient R expresses the degree of correlation between the estimated results of the model and the real data, which is calculated based on Equation (31). In fact, the correlation coefficient measures the linear relationship between two variables, obviously, the closer its value is to one, the closer the estimated values are to the actual values.

$$R = \frac{\sum_{i=1}^{n} (V_{e,i} - \overline{V_e})(V_{p,i} - \overline{V_e})}{\left[\sum_{i=1}^{n} (V_{e,i} - \overline{V_e})^2 \sum_{i=1}^{n} (V_{p,i} - \overline{V_e})^2\right]^{0.5}}$$
(31)

The performance indicators used are the Mean Absolute Relative Error (MARE), Mean Absolute Error (MAE), Mean Squared Error (MSE), Root Mean Squared Error (RMSE), and Root Mean Squared Relative Error (RRMSE). These are calculated using Equations (32) to (36), and can range from zero (indicating excellent performance) to infinity (indicating very poor performance).

$$MARE = \frac{1}{n} \sum_{i=1}^{n} \frac{\left| V_{e,i} - V_{p,i} \right|}{V_{e,i}} \times 100$$
(32)

$$MAE = \frac{\sum_{i=1}^{n} |V_{e,i} - V_{p,i}|}{n}$$
(33)

$$MSE = \frac{\sum_{i=1}^{n} (V_{e,i} - V_{p,i})^2}{n}$$
(34)

$$RMSE = \sqrt{MSE} \tag{35}$$

$$RRMSE = \frac{RMSE}{\overline{V_e}}$$
(36)

In the above relationships, $V_{e,i}$ is the laboratory shear values, \overline{V}_e is the average laboratory shear values, V_{ni} is the predicted shear values by regulation relations or neural network model and \overline{V}_p is the average predicted shear values. *n* is the number of exaned samples.

3.3.2. Comparison of neural network architectures

The environment used to build neural networks is MATLAB R2013b. It will be possible to compare the architecture of neural networks by running nntool toolbox and addressing input and output data (an 11×304 input data matrix that represents 304 samples of 11 elements and an 1×304 output data matrix that represents 304 samples of 1 elements are listed in Table 1).

By considering feed-forward backprop network type, there will be a wide range of learning functions. A comparison of all learning functions and corresponding performance results are listed in Table 3. Note that as a primary solution, the default values for the nntool toolbox were selected as follows: MSE as a performance function, tansig (tangent sigmoid transfer function) as a transfer function, two-layer network and ten hidden neurons. The MATLAB toolbox does not refer to the inputs as a layer. Therefore, a two-layer network means a network that consists of an output layer, a hidden layer plus an input layer, as shown in Figure 3. In this section, learning and transfer functions are discussed, and in the next section, the number of layers and the number of hidden neurons are discussed in ANN complexity adjustment.

As can be seen in Table 3, the best performance results are obtained by Trainbr (Bayesian regularization backpropagation) and Trainlm (Levenberg–Marquardt backpropagation) learning functions. Both of these learning functions update the weight and bias values according to Levenberg–Marquardt optimization, but the Trainlm function is faster than the Trainbr function and is often even the fastest backpropagation algorithm in the toolbox, although it does require more memory than other algorithms. Therefore, by choosing Trainlm as the training function, other effective parameters will be investigated.

Training function	MSE	R	Training function	MSE	R
Trainbfg: BFGS quasi-Newton back- propagation	2041.51	0.933	Traingda: Gradient descent with adap- tive learning rate backpropagation	26520.96	1.853e ⁻²⁶
Trainbr ¹ : Bayesian regularization backpropagation	435.59	0.975	Traingdx: Gradient descent with momentum and adaptive learning rate backpropagation	22076.16	1.853e ⁻²⁶
Traincgb: Conjugate gradient backprop- agation with Powell–Beale restarts	1118.55	0.949	Trainlm ² : Levenberg–Marquardt backpropagation	511.38	0.981
Traincgf: Conjugate gradient backprop- agation with Fletcher–Reeves updates	1772.57	0.912	Trainoss: One-step secant backprop- agation	1005.10	0.954
Traincgp: Conjugate gradient backprop- agation with Polak–Ribiére updates	1610.26	0.930	Trainr ³ : Random order incremental training with learning functions	take a long time	-
Traingd: Gradient descent backprop- agation	77763.62	-0.117	Trainrp: Resilient backpropagation	1225.23	0.920
Traingdm: Gradient descent with momentum backpropagation	253100.80	0.338	Trainscg: Scaled conjugate gradient backpropagation	1244.57	0.950

Table 3. Comparison of all learning functions and corresponding performance results using nntool – MATLAB R2013b toolbox

¹Trainbr: is a network training function that updates the weight and bias values according to Levenberg–Marquardt optimization. It minimizes a combination of squared errors and weights, and then determines the correct combination so as to produce a network that generalizes well. The process is called Bayesian regularization.

² Trainlm: is a network training function that updates weight and bias values according to Levenberg–Marquardt optimization.
³ Trainr: trains a network with weight and bias learning rules with incremental updates after each presentation of an input. Inputs are presented in random order.

Three transfer functions can be selected for each layer (tan-sigmoid, log-sigmoid and linear transfer function – Fig. 5). Therefore, in a two-layer network, there will be a total of 9 choices for the transfer functions. A network of two layers, where the first layer is tangent sigmoid and the second layer is linear, can be shown to have the best performance results.



Fig. 5. Transfer Function Diagrams: a) tan-sigmoid transfer function; b) log-sigmoid transfer function; c) linear transfer function

3.3.3. ANN complexity adjustment

For the number of hidden layers and the number of hidden neurons, different choices lead to ANN with different levels of complexity. For example, adding more neurons to a particular hidden layer increases the network's capacity because it has more degrees of freedom.

An ANN's training process aims to produce a model that approximates the underlying data generating process (DGP) based on previous observations (so-called training data). A successful approximation of the underlying process implies that the trained network is generalizable, meaning that it maintains a consistent performance in the available data used for training and on future data generated by the same DGP. Importantly, an ANN may fail to deliver such performance consistency if the network is excessively complex compared to the underlying data generating process. In this case, ANN performs very well the training data, but fails to maintain a similarly strong performance on different data generated by the same DGP, which are used for validation purposes (socalled validation data). This issue is known as overfitting. Another issue that may impact the extent to which a trained ANN's is generalizable is known as underfitting, which means that the ANN is too sple compared to the underlying DGP. As a result, it performs poorly on both training and validation data. In this case, the ANN cannot accurately capture the relation (embodied in the DGP) between input and observed choices. In sum, it is essential for the analyst to consider the relation between

complexity and performance. The above-described concepts of under- and overfitting a learning machine are shown in Figure 6 (Alwosheel et al., 2018).

Low model complexity (compared to the underlying DGP) is represented in Figure 6 on the left-hand side: here, models perform poorly on both training and future data, as they impose assumptions that are too simplistic on the DGP. In contrast, very complex models are represented on the right hand side. These models perform well on the available data, but fail to obtain a similarly strong performance on validation data generated by the same DGP. The ideal level of complexity is found in the range where the validation error is low, and the divergence between training and validation error (thus the vertical distance between the red and green lines) is small (Alwosheel et al., 2018).

This section adjusts the ANN complexity by adding/ removing hidden neurons and the number of hidden layers. For example, if the underlying DGP is complex, an ANN with very few hidden neurons (in the extreme case: only one hidden neuron) will underfit this DGP. In contrast, using a large number of hidden neurons will lead to overfitting. Various ANNs with different levels of complexity (i.e., different number of hidden neurons) are estimated using the training set. Then, the performance of each of the estimated ANNs is evaluated on the validation set. The network that has the best performance with respect to the validation set is selected, as its complexity falls in the ideal level of complexity range shown in Figure 6. Subsequently, to provide an unbiased evaluation of the selected network, ANN performance is further evaluated on the testing set. If the ANN also performs well on the testing data, the analyst can be confident that the network has successfully learned the underlying DGP³ (Alwosheel et al., 2018).

Figure 7 shows the relationship between ANN complexity (i.e., the number of hidden neurons) and the correlation coefficient R obtained on training, validation, and test datasets. The network that provides the best performance on the validation data is then selected. Figure 7 shows that ten hidden neurons provide the best performance (on the validation set). Using more than ten hidden neurons does not affect the resulting correlation coefficient R, implying that ANN has learned the input/output relationship with ten neurons. According to Occam's razor principle, an explanation of a set of data should be limited to the bare minimum that is consistent with the data (Alwosheel et al., 2018). In Figure 7, increasing the complexity does not result in better performance. Therefore, the simplest model that describes data is preferred, which in this case is an ANN with ten neurons.



Fig. 6. A conceptual representation of the relationship between model complexity and performance





Note that a different number of hidden layers have also been implemented for this study. By choosing the 3-layer network (Fig. 3), it can be shown in a similar way that adding more hidden layers does not improve the prediction performance.

It is expected that the more complex the ANN is, the more data will be needed for training the network, leading to a larger sample size (Alwosheel et al., 2018). Therefore, according to what was ateempted by choosing a network with 3-layers and 10 hidden neurons, the complexity of the network falls in the ideal level of complexity range shown in Figure 6, where the validation error is low, and the divergence between training and validation error is small.

³ In some cases, training the ANN and adjusting its complexity may not result in a low generalization error, which means that the ANN has failed to approximate the underlying DGP to a sufficient extent. One possible reason of this outcome is that the used data are insufficient in size; i.e., when trained on a very small dataset – relative to the number of nodes in the network – the ANN may end up memorizing observations rather than learning the underlying DGP. In this case, it is recommended to use larger datasets (Alwosheel et al., 2018).

4. Presentation of results

Table 4 shows the neural network model's performance criteria and two Iranian and American regulations for all data.

Table 4 calculated from a comparison of design shear strength values obtained by ACI 440.2R-08 and ACI 318-08, design shear strength values obtained by Publication No. 345 – ABA and shear strength values obtained by neural network model with actual experimental shear strength values measured in the laboratory, as it is shown in Table 6.

As it is clear from Table 4, all efficiency criteria for the neural network model are better compared to

both regulations, so that, for example, the correlation coefficient of the neural network model is close to 1, while this criterion does not have suitable values for the two regulations. Also, in the comparison of the two regulations, considering all efficiency criteria, the results of the American regulations are more accurate than the Iranian regulations.

Figure 8a–c shows the predicted values of the neural network model compared to its actual (laboratory) values, respectively, for three categories of training, validation, and testing datasets. Figure 8d shows the predicted values of the neural network model compared to its actual values for all data.

Relationship/model	Equation	MARE [%]	MAE	MSE	RMSE	RRMSE	R
ACI 318-05 and ACI 440.2 (2008)	(3)	34.24	62.13	7261.52	85.21	0.49	0.8419
Publication No. 345 – ABA	(18)	38.94	69.93	8268.94	94.70	0.55	0.8418
ANN MODEL	model	13.46	16.97	511.38	22.61	0.13	0.9811

Table 4. Performance criteria of shear strength estimation models for all data



Fig. 8. Comparison of laboratory shear strength V_{U-exp} (KN) with shear strength predicted by ANN model for: a) training data; b) validation data; c) test data; d) all data

In Figures 8a–c, the diagonal axis shown is located where the predicted and experimental values have the same value. Therefore, the density of points around the diagonal axis shows more accurate predicted values. As expected, a better prediction has been obtained for the training data set compared to the other two sets, and the dispersion of the data around the diagonal axis is less in this set.

Figure 9 presents the graph of the actual shear strength values compared to the output of the regulations and the neural network model for all 304 data is presented. As it is clear from the diagram, the neural network model benefits from more agreement with the real values compared to the output of the regulations.

Table 5 has been prepared to evaluate and compare the relationship between Iranian and American regulations, as well as the neural network prediction model in the use of all types of FRP wrapping schemes (completely wrapped, U-wraps or side bonding of the web, each applied as intermittent strips or a continuous sheet along the length of the member). All the data are placed in their specific wrapping schemes and using performance indicators, the error between the output values of the regulations or the neural network model and its actual values has been estimated.



Fig. 9. Experimental shear strength compared to ANN model and regulations codes for all data

Relationship/model	MARE [%]	MAE	MSE	RMSE	RRMSE	R
	ACI 318-05 and A	ACI 440.2 ((2008)		1	
Side bonding – discrete strips	27.1	29.6	1541.3	39.3	0.39	0.410
Side bonding – continuous sheet	32.2	50.2	3725.1	61.0	0.38	0.820
U-wrap – discrete strips	33.9	50.7	4286.8	65.5	0.44	0.890
U-wrap – continuous sheet	32.5	62.1	6498.8	80.6	0.43	0.830
Completely wrapped – discrete strips	40.1	91.8	14565.6	120.7	0.53	0.900
Completely wrapped – continuous sheet	46.9	130.9	24436.7	156.3	0.56	0.660
	Publication No	o. 345 – AI	BA			
Side bonding – discrete strips	35.2	38.4	2302.2	48.0	0.48	0.440
Side bonding – continuous sheet	35.2	54.5	4342.2	65.9	0.41	0.840
U-wrap – discrete strips	39.1	56.8	5149.3	71.8	0.48	0.890
U-wrap – continuous sheet	36.5	71.6	8739.2	93.5	0.50	0.790
Completely wrapped – discrete strips	44.9	102.1	17256.3	131.4	0.58	0.910
Completely wrapped – continuous sheet	50.5	142.8	30185.0	173.7	0.62	0.660
	Neural netw	vork model				
Side bonding – discrete strips	12.8	11.0	210.4	14.5	0.14	0.900
Side bonding – continuous sheet	13.1	17.6	576.1	24.0	0.15	0.960
U-wrap – discrete strips	19.1	17.5	555.1	23.6	0.16	0.970
U-wrap – continuous sheet	10.4	16.1	429.2	20.7	0.11	0.979
Completely wrapped – discrete strips	11.1	18.4	509.6	22.6	0.10	0.990
Completely wrapped – continuous sheet	10.8	23.5	938.2	30.6	0.11	0.978

 Table 5. Comparison of the relations between Iranian and American regulations, as well as the prediction model of ANN in the use of various types of FRP wrapping schemes

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Figure 10 graphically shows the prediction error of the neural network model, the Iranian and American regulations according to Mean Squared Error (MSE) for different types of FRP wrapping schemes. From Table 5 and Figure 10, it can be concluded that in all the data and in all the beam shear strengthening patterns in all cases, the neural network model has the lowest estimation error by a large difference from the others, and it shows that the output of this model is closer to the real values. After the artificial neural network model, the estimated values of the relationship of the American regulation predict a lower estimation error than its corresponding Iranian relationship. In the American and Iranian regulations, as well as in the neural network model, from a general point of view and ignoring the type of applying FRP laminates (intermittent strips or a continuous sheet), the lowest amount of estimation error belongs to the side bonding of the beam, followed by the U-wraps stirrup and at the end the completely wrapped. In the case that, the beam is strengthened with a completely wrapped scheme, the regulations show the highest amount of error, one of the main reasons for that is limiting the total shear capacity of the steel and FRP shear reinforcement to $0.66\sqrt{f_c}b_w d$ for the relationship of the ACI 318-05 and ACI 440.2 and the value of $4 \times 0.2 \times \sqrt{f_c} b_w$ for the relation of Publication No. 345. This is despite the fact that in such cases, the real shear strength is higher than what the regulations predict. It is worth mentioning that in all wrapping schemes, where the FRP system is installed in the form of intermittent strips (discrete strips), the regulatory relations show a lower error value than the similar case of a continuous sheet along the length of the member.



Fig. 10. Comparison of the prediction error of various FRP wrapping schemes in the regulations and the neural network model

5. Conclusion

In this study, the shear strength of RC beams reinforced by FRP with different wrapping schemes has been investigated by artificial neural network. To model the neural network, beam width, beam height, compressive strength of concrete, the ratio of the area of transverse reinforcements to the distances between them, the yield stress of transverse reinforcements, the number of free ends of FRP stirrups, FRP modulus of elasticity, strength tensile FRP, the effective depth of FRP stirrups, the effective width of the FRP shear reinforcement strip in the longitudinal direction of the beam to the center-to-center distance of the FRP stirrups, the number of FRP layers multiplied by the thickness of each layer as 11 input variables and the value of the total shear strength of the beam as a variable output were all considered.

By comparing the results based on the efficiency criteria of the data-based models, it can be concluded that in all types FRP wrapping schemes (completely wrapped, U-wraps or side bonding of the web, each applied as intermittent strips or a continuous sheet along the length of the member), the neural network model has the lowest error by a large difference from the others, and after that, the relationship of the American concrete code ACI 440.2R is in the second place with less error than its corresponding Iranian relationship. Specifically, the values of the correlation coefficient for an artificial neural network, American concrete code and Iranian concrete code are 0.9811, 0.8419 and 0.8418, respectively. In addition, the percentage of the Mean Absolute Relative Error (MARE) in the neural network model is about 13%, which is 21% and 26% less than the results of the American and Iranian regulations. In addition, in all three wrapping schemes, from a general point of view and ignoring the type of applying FRP laminates (intermittent strips or a continuous sheet), the lowest amount of estimation error belongs to the side bonding of the beam, followed by the U-wraps stirrup and finally the completely wrapped. Finally, it can be said that it is difficult to create a general model to estimate the shear strength due to the complexity of the shearing mechanism of concrete beams and the various parameters affecting it. Therefore, the exact values of shear strength are not available. Regulations such as American concrete regulations and Iranian concrete regulations each use empirical formulas provided for a set of specific data, but using a neural network model using a set of laboratory data has greater accuracy in estimating shear strength.

Data availability

Data will be available upon reasonable request.

Conflict of interest

There is no conflict of interest to declare.

4	()	ANN	$V_{\rm ANN}$	142.1	170.9	112.5	113.6	142.1	134.1	235.2	246.7	258.2	125.8	125.1	125.0	125.0	125.2	126.0	129.5	216.8	241.9	243.9	177.7	166.3	177.7	181.6	138.6	147.6	138.6	147.6	115.9	126.1	115.9	126.1	131.4	90.9	91.4	111.9
			$V_{r\text{-ABA}}$	87.8	162.1	80.4	101.1	87.8	73.0	203.5	204.0	241.6	46.5	51.3	54.4	54.6	57.6	67.0	71.1	123.0	129.3	138.7	56.6	49.8	56.6	76.8	100.1	100.1	100.1	100.1	75.6	75.6	75.6	75.6	98.8	95.8	104.5	115.3
			Check Equation (30)	50.1	124.4	42.7	63.4	50.1	40.7	154.2	154.7	192.3	10.9	15.7	18.9	19.0	22.0	31.4	35.6	87.5	93.8	103.2	36.4	29.7	36.4	56.6	103.3	95.0	103.3	95.0	90.8	88.7	90.8	88.7	59.5	56.5	65.2	76.0
			V_{fp}	61.8	153.6	52.7	78.3	61.8	50.2	121.0	121.5	168.0	13.5	19.4	23.3	23.5	27.2	38.8	43.9	19.4	27.2	38.8	24.9	16.6	24.9	49.9	26.0	13.0	26.0	13.0	6.4	3.2	6.4	3.2	28.9	25.2	35.9	49.3
			$\min_{\epsilon_{fipe}}$	0.0032	0.0032	0.0022	0.0040	0.0032	0.0026	0.0031	0.0022	0.0018	0.0035	0.0035	0.0035	0.0035	0.0035	0.0035	0.0020	0.0035	0.0035	0.0035	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0011	0.0010	0.0010	0.0010
		ABA	ε _{firpe-2}	0.0032	0.0032	0.0022	0.0040	0.0032	0.0026	0.0031	0.0022	0.0018	0.0035	0.0035	0.0035	0.0035	0.0035	0.0035	0.0020	0.0035	0.0035	0.0035	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0011	0.0010	0.0010	0.0010
		0. 345 –	K_2	0.761	0.761	0.523	111	0.761	0.761	0.843	0.895	0.917	0.676	0.676	0.676	0.676	0.676	0.676	0.855	0.676	0.676	0.676	111	111	111	111	111		111	111	111	111	111	111	0.886	0.771	0.773	0.767
	о ;	cation N	$K_{_{l}}$	1.680	1.680 0	1.680 (///	1.680	1.365	0.998	9 866.0	9 866.0	080.1	080.1	080.1	080.1	080.1	080.1	080.1	080.1	080.1	080.1	///	///	///	///	///	///	///	///	///	///	///	///	0.527 0	0.527 (0.527	0.527 0
	;	Public	L_e	36.4	36.4	36.4	///	36.4	36.4	53.8 (36.0 (28.5 (70.8	70.8	70.8	70.8	70.8	70.8	31.7	70.8	70.8	70.8	///	///	///	///	///		///	///	///	///	///	///	34.9 (34.9 (34.9 (34.9 (
0			Efipe-I	.0059	.0045	0.0045	///	.0059	0.0055	0.0065	0.0053	0.0047	.0078	0200.0	.0066	.0066	0.0063	0.0057	0.0037	0.0070	.0063	0.0057	///	///	///	///	///		///	///	///	///	///	///	0.0024	0.0024	0.0024	0.0020
			R	.401 0	.305 0	.305 0	///	.401 0	.377 0	.361 0	.293 0	.260 0	.518 0	.464 0	.440 0	.438 0	.420 0	.377 0	.249 0	.464 (.420 0	.377 0	///	///	///	///	///	///	///	///	///	///	///	///	.301 0	.301 0	.301 0	.245 0
			β _{frp}	.0018 0	.0045 0	.0045 0	.0018	.0018 0	.0018 0	.0016 0	.0031 0	.0047 0	.0005 0	.0007 0	0 6000.	0 6000.	.0010 0	.0014 0	.0058 0	0007 0	.0010 0	.0014 0	///	///	///	///	///	///	///	///	///	///	///	///	0 6000.	0 6000.	0 6000.	.0018 0
			<i>V</i> ^c	2.9 0	2.9 0	2.9 0	2.9 0	2.9 0	3.8 0	2.1 0	2.1 0	2.1 0	9.2 0	9.2 0	9.2 0	9.2 0	9.2 0	9.2 0	9.2 0	9.2 0	9.2 0	9.2 0	3.6	3.6	3.6	3.6	3.4	3.4	3.4	3.4	5.2	5.2	5.2	5.2	5.6 0	5.6 0	5.6 0	5.6 0
	╎		7 _{n-ACI}	5.6 6	70.1 6	2.0 6	08.6 6	15.7 6	6.0 5	17.2 8	18.2 8	57.1 8	3.9 5	8.8 5	2.0 5	2.2 5	5.2 5	4.8 5	3.1 5	30.6 5	37.0 5	46.5 5	0.6 3	3.9 3	0.6 3	0.8 3	17.7 3	21.3 3	17.7 3	21.3 3	8.9 2	8.9 2	8.9 2	8.9 2	9 6.70	05.0 6	14.2 6	25.7 6
			ck tion øl	4	.6 1	5 5	3 1	2 1	0 0	.9 2	.1 2	.0 2	4 5	.1 5	.6 6	8	0	C.	9	.6 1	.4 1	.7 1.	0	7 5	0	8	.0 1	.0	.0 1	.0 1	4.	.2 8	.4 8	.2	2 1	3 1	0	1 6.
			Che Equa	69	5 172	64	78.	97.	79.	7 206	208	3 262	15.	22	26.	26	31	44	69	106	115	128	44	35.	44.	69	128	115	128	115	108	105	108	105	76.	72.	85.	100
		80	V_f	69.4	172.6	64.5	78.3	97.2	79.0	140.7	142.0	195.8	15.4	22.1	26.6	26.8	31.0	44.3	69.69	22.1	31.0	44.3	24.9	16.6	24.9	(49.9	26.0	13.0	26.0	13.0	6.4	3.2	6.4	3.2	33.8	29.8	42.5	58.4
		ACI318-0	ε_{fe}	0.0035	0.0035	0.0027	0.0040	0.0035	0.0029	0.0036	0.0025	0.0021	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0031	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0013	0.0011	0.0011	0.0011
-	(q	08 and ∕	K,	0.255	0.255	0.183	1111	0.255	0.207	0.201	0.142	0.114	0.281	0.281	0.281	0.281	0.281	0.281	0.214	0.281	0.281	0.281	///	///	///	111	111	11	///	///	111	///	///	///	0.161	0.143	0.143	0.142
o		440.2R-	K_2	0.781	0.781	0.561	1111	0.781	0.781	0.856	0.903	0.924	0.702	0.702	0.702	0.702	0.702	0.702	0.801	0.702	0.702	0.702	///	///	///	111	111	111	///	///	111	///	///	///	0.895	0.790	0.791	0.786
		ACL	$K_{_{I}}$	1.707	1.707	1.707	///	1.707	1.387	1.014	1.014	1.014	1.097	1.097	1.097	1.097	1.097	1.097	1.097	1.097	1.097	1.097	///	///	///	111	111	111	///	///	111	///	///	///	0.536	0.536	0.536	0.536
o			L_e	33.5	33.5	33.5	///	33.5	33.5	49.5	33.1	26.2	65.1	65.1	65.1	65.1	65.1	65.1	43.5	65.1	65.1	65.1	///	///	///				///	///		///	///	///	32.1	32.1	32.1	32.1
			V_s	0.0	0.0	0.0	0.0	0.0	0.0	66.2	66.2	66.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	84.4	84.4	84.4	19.1	19.1	19.1	19.1	102.0	102.0	102.0	102.0	102.0	102.0	102.0	102.0	42.4	42.4	42.4	42.4
			V_c	53.4	53.4	53.4	53.4	53.4	45.8	69.8	69.8	69.8	50.4	50.4	50.4	50.4	50.4	50.4	50.4	50.4	50.4	50.4	28.6	28.6	28.6	28.6	28.4	28.4	28.4	28.4	21.4	21.4	21.4	21.4	55.7	55.7	55.7	55.7
	a)	imental	$V_{U\text{-exp}}$	115.4	127.0	115.4	135.6	144.3	138.5	209.5	200.0	234.8	120.9	134.5	131.1	135.7	150.6	120.0	121.7	246.7	253.9	250.6	185.0	187.0	227.0	217.0	136.0	121.0	178.0	161.0	104.0	99.0	162.0	150.0	95.0	100.0	101.0	112.5
		Exper	No.	1	2	3	4	5	9	7	8	6	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35

Table 6. Ultimate experimental shear strength measured in the laboratory (a), design shear strength values obtained by ACI 440.2R-08 and ACI 318-08 (b),design shear strength values obtained by Publication No. 345 – ABA (c) and shear strength values obtained by neural network model (d)

$V_{ m ANN}$	133.2	144.9	137.4	145.5	145.5	168.1	168.1	231.4	243.8	215.6	200.6	192.2	207.4	313.4	185.4	192.3	209.6	219.8	219.8	185.4	218.1	304.8	304.5	272.6	292.9	351.8	309.9	100.7	100.7	72.0	72.0	265.3	275.5	286.1	286.1	116.4	133.8	130.4	128.3
$V_{r\text{-ABA}}$	108.6	112.5	105.3	140.4	121.4	196.7	196.7	196.7	243.8	211.0	160.5	154.8	258.5	268.4	154.9	162.1	179.8	155.7	155.7	184.5	226.8	247.5	247.4	141.8	144.8	153.7	166.1	83.6	83.6	81.3	81.3	126.4	134.6	142.8	142.8	45.0	52.4	52.4	62.4
Check Equation (30)	69.2	73.2	65.9	101.1	82.0	210.7	212.4	388.7	177.9	153.5	106.9	103.2	198.4	320.3	105.7	110.5	122.4	96.0	96.0	135.4	166.7	195.8	195.8	104.7	107.7	116.7	129.1	60.7	60.7	58.4	58.4	93.8	102.0	110.2	110.2	17.4	24.8	24.8	34.8
^{dy}A	41.0	45.8	36.9	80.3	56.8	216.2	218.4	436.8	177.4	147.4	89.7	85.2	202.8	354.4	88.4	94.3	108.9	76.4	76.4	124.9	163.7	199.6	199.6	40.7	44.4	55.4	70.7	49.5	49.5	46.6	46.6	12.8	25.6	38.4	38.4	21.5	30.6	30.6	43.0
$\min_{\mathfrak{S}_{fipe}}$	0.0011	0.0008	0.0008	0.0011	0.0011	0.0040	0.0040	0.0040	0.0028	0.0023	0.0021	0.0020	0.0024	0.0040	0.0025	0.0026	0.0031	0.0032	0.0032	0.0025	0.0032	0.0040	0.0040	0.0016	0.0016	0.0017	0.0040	0.0015	0.0015	0.0015	0.0015	0.0040	0.0040	0.0040	0.0040	0.0028	0.0040	0.0040	0.0028
€ _{firpe-2}	0.0011	0.0008	0.0008	0.0011	0.0011	0.0040	0.0040	0.0040	0.0028	0.0023	0.0021	0.0020	0.0024	0.0040	0.0025	0.0026	0.0031	0.0032	0.0032	0.0025	0.0032	0.0040	0.0040	0.0016	0.0016	0.0017	0.0040	0.0015	0.0015	0.0015	0.0015	0.0040	0.0040	0.0040	0.0040	0.0028	0.0040	0.0040	0.0028
K_2	0.889	0.925	0.919	0.884	0.884	///	///	///	0.870	0.870	0.870	0.870	0.859	///	0.818	0.818	0.818	0.818	0.818	0.818	0.818	///	///	0.925	0.931	0.944	0.787	0.765	0.765	0.765	0.765	///	///	///	///	0.530	///	0.765	0.530
K_l	0.527	0.527	0.527	0.527	0.527	///	///	///	1.214	1.009	0.921	0.875	1.072	///	0.818	0.873	1.009	1.061	1.061	0.818	1.072	///	///	1.142	1.142	1.142	1.142	1.171	1.171	1.171	1.171	///	///	///	///	1.107	///	1.107	1.107
L_e	34.9	23.3	23.3	34.9	34.9	111	///	///	39.1	39.1	39.1	39.1	39.1	111	54.6	54.6	54.6	54.6	54.6	54.6	54.6	///	///	22.7	22.7	22.7	64.9	25.2	25.2	25.2	25.2	///	///	///	///	70.5	111	70.5	70.5
Efipe-1	0.0024	0.0017	0.0017	0.0020	0.0020	///	///	///	0.0044	0.0042	0.0046	0.0045	0.0043	111	0.0048	0.0049	0.0051	0.0059	0.0059	0.0048	0.0052	111	///	0.0043	0.0043	0.0043	0.0055	0.0039	0.0039	0.0044	0.0044	///	///	///	111	0.0069	111	0.0069	0.0056
R	0.301	0.217	0.217	0.245	0.245	111	///	///	0.296	0.280	0.307	0.302	0.285	111	0.312	0.318	0.332	0.381	0.381	0.312	0.338	111	////	0.331	0.331	0.331	0.366	0.278	0.278	0.314	0.314	///	///	111	111	0.465	111	0.465	0.377
ρ_{frp}	0.0009	0.0026	0.0026	0.0018	0.0018	///	///	///	0.0035	0.0035	0.0023	0.0023	0.0035	///	0.0020	0.0020	0.0020	0.0013	0.0013	0.0020	0.0020	///	///	0.0060	0.0060	0.0060	0.0017	0.0067	0.0067	0.0044	0.0044	0.0008	0.0016	0.0024	0.0024	0.0007	0.0007	0.0007	0.0015
V_c	65.6	65.6	65.6	65.6	65.6	65.6	65.6	65.6	109.9	95.7	89.5	86.1	100.2	89.5	81.9	85.9	95.7	99.4	99.4	81.9	100.2	86.1	85.9	61.8	61.8	61.8	61.8	38.2	38.2	38.2	38.2	54.4	54.4	54.4	54.4	45.9	45.9	45.9	45.9
$\omega V_{\rm n-ACI}$	118.0	122.1	114.7	151.2	131.4	231.3	231.3	231.3	263.7	228.0	174.7	168.4	278.3	315.6	168.5	176.5	196.0	171.0	171.0	199.7	245.8	257.5	257.4	150.6	153.7	163.0	168.1	90.6	90.6	88.2	88.2	135.2	145.6	155.9	155.9	52.4	57.9	55.3	71.7
Check Equation (17)	90.3	96.0	85.6	136.3	108.8	260.8	260.8	479.2	248.0	212.8	145.3	140.0	277.9	394.6	144.3	151.2	168.5	130.2	130.2	187.4	233.0	239.8	239.8	131.8	136.1	148.9	156.0	83.0	83.0	79.6	79.6	113.5	126.3	139.1	139.1	26.6	30.6	30.6	53.3
h_{f}	47.9	53.5	43.2	93.8	66.4	218.4	218.4	436.8	207.8	172.6	105.1	99.8	237.7	354.4	104.1	111.1	128.3	90.06	90.06	147.2	192.8	199.6	199.6	47.4	51.6	64.4	71.6	58.6	58.6	55.3	55.3	12.8	25.6	38.4	38.4	26.6	30.6	30.6	53.3
ϵ_{fe}	0.0013	0.0009	0.0009	0.0013	0.0013	0.0040	0.0040	0.0040	0.0033	0.0027	0.0025	0.0024	0.0029	0.0040	0.0029	0.0031	0.0036	0.0038	0.0038	0.0029	0.0038	0.0040	0.0040	0.0019	0.0019	0.0019	0.0040	0.0018	0.0018	0.0018	0.0018	0.0040	0.0040	0.0040	0.0040	0.0035	0.0040	0.0040	0.0035
$K_{ m v}$	0.162	0.112	0.112	0.161	0.161	///	///	///	0.219	0.182	0.166	0.158	0.191	111	0.188	0.201	0.232	0.244	0.244	0.188	0.247	///	///	0.146	0.147	0.148	0.311	0.130	0.130	0.130	0.130	///	///	///	///	0.235	///	0.324	0.235
$K_{_2}$	0.898	0.931	0.925	0.893	0.893	///	///	///	0.880	0.880	0.880	0.880	0.871	111	0.833	0.833	0.833	0.833	0.833	0.833	0.833	///	///	0.931	0.937	0.949	0.804	0.784	0.784	0.784	0.784	///	///	///	///	0.568	111	0.784	0.568
$K_{_{I}}$	0.536	0.536	0.536	0.536	0.536	///	///	///	1.234	1.025	0.936	0.889	1.089	111	0.831	0.887	1.025	1.078	1.078	0.831	1.089	///	///	1.160	1.160	1.160	1.160	1.190	1.190	1.190	1.190	///	///	///	///	1.124	111	1.124	1.124
L_e	32.1	21.4	21.4	32.1	32.1	///	///	///	36.0	36.0	36.0	36.0	36.0	111	50.2	50.2	50.2	50.2	50.2	50.2	50.2	///	///	20.9	20.9	20.9	59.6	23.2	23.2	23.2	23.2	///	///	///	///	64.8	///	64.8	64.8
⊿~	42.4	42.4	42.4	42.4	42.4	42.4	42.4	42.4	40.2	40.2	40.2	40.2	40.2	40.2	40.2	40.2	40.2	40.2	40.2	40.2	40.2	40.2	40.2	84.4	84.4	84.4	84.4	24.3	24.3	24.3	24.3	100.7	100.7	100.7	100.7	0.0	0.0	0.0	0.0
h^{c}	55.7	55.7	55.7	55.7	55.7	55.7	55.7	55.7	93.4	81.4	76.0	73.1	85.1	76.0	69.69	73.0	81.4	84.5	84.5	69.69	85.1	73.1	73.0	52.5	52.5	52.5	52.5	32.4	32.4	32.4	32.4	46.2	46.2	46.2	46.2	39.0	39.0	39.0	39.0
V_{U-exp}	111.0	126.0	133.5	167.0	125.0	177.3	158.5	186.1	232.4	230.8	205.8	232.6	213.8	325.3	201.5	213.7	205.5	212.1	202.9	194.2	209.3	316.4	280.5	275.4	300.3	335.7	294.0	87.5	95.0	94.0	99.5	242.0	270.0	318.0	279.0	105.9	127.9	105.4	123.8
No.	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	99	67	68	69	70	71	72	73	74

149.6	158.5	128.3	132.0	125.6	123.2	145.0	135.2	169.9	131.4	135.2	129.7	146.9	118.9	119.7	119.3	151.1	142.4	105.0	110.1	177.4	226.5	226.5	117.9	123.5	143.2	143.2	150.9	117.6	182.7	166.4	107.4	131.7	155.7	150.0	125.9	104.8	134.3	232.1	182.9	126.7	124.1
77.2	77.2	76.8	107.7	107.7	70.9	87.0	52.8	140.2	61.8	52.7	60.5	89.2	44.8	48.1	43.4	99.4	101.3	62.6	50.8	116.1	100.0	100.0	47.0	58.5	81.6	81.7	97.1	47.1	119.4	126.8	39.6	68.0	106.6	106.6	35.9	34.9	43.9	111.5	85.2	59.1	35.1
49.6	49.6	49.2	80.2	80.2	36.2	52.6	18.1	105.7	26.9	18.1	26.2	54.1	9.3	13.4	9.0	61.3	63.2	24.5	12.8	78.1	167.9	167.9	23.1	34.6	57.6	57.6	73.2	23.1	171.4	94.6	18.9	47.3	74.4	74.4	5.6	2.8	11.1	52.7	26.3	26.3	5.6
61.3	61.3	60.7	99.0	99.0	44.7	65.0	22.3	130.5	33.2	22.3	32.3	66.8	11.5	16.5	11.1	75.7	78.1	30.3	15.8	96.4	67.5	67.5	28.5	42.7	71.2	71.2	90.3	28.5	95.1	116.8	23.4	58.4	91.8	91.8	7.7	3.9	15.4	65.0	32.5	32.5	7.7
0.0040	0.0040	0.0028	0.0011	0.0011	0.0019	0.0027	0.0019	0.0027	0.0028	0.0019	0.0027	0.0028	0.0019	0.0028	0.0018	0.0036	0.0025	0.0036	0.0023	0.0037	0.0017	0.0017	0.0031	0.0031	0.0031	0.0031	0.0022	0.0031	0.0019	0.0029	0.0025	0.0025	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040
0.0040	0.0040	0.0028	0.0011	0.0011	0.0019	0.0027	0.0019	0.0027	0.0028	0.0019	0.0027	0.0028	0.0019	0.0028	0.0018	0.0036	0.0025	0.0036	0.0023	0.0037	0.0017	0.0017	0.0031	0.0031	0.0031	0.0031	0.0022	0.0031	0.0019	0.0029	0.0025	0.0025	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040
0.765	#VALUE!	0.530	0.885	0.885	0.515	0.757	0.515	0.757	0.757	0.515	0.757	0.757	0.515	0.757	0.515	0.800	0.836	0.800	0.511	0.836	0.866	0.866	0.816	0.816	0.816	0.816	0.877	0.816	0.753	0.753	0.816	0.816	0.816	///	///	///	///	///	///	///	///
1.107	1.107	1.107	1.107	1.107	1.469	1.453	1.469	1.460	1.483	1.467	1.446	1.494	1.515	1.475	1.455	1.171	1.171	1.171	1.171	1.171	0.786	0.786	0.996	0.996	0.996	0.996	0.996	0.996	0.996	1.487	0.818	0.818	1.487	///	///	///	///	///			///
70.5	70.5	70.5	17.3	17.3	36.4	36.4	36.4	36.4	36.4	36.4	36.4	36.4	36.4	36.4	36.4	56.3	37.6	56.3	56.3	56.3	37.6	37.6	56.3	56.3	56.3	56.3	37.6	56.3	37.6	37.6	56.3	56.3	56.3	///	111	111	111	111	111	///	111
0.0056	0.0056	0.0056	0.0027	0.0030	0.0043	0.0053	0.0053	0.0043	0.0065	0.0053	0.0065	0.0053	0.0066	0.0080	0.0065	0.0057	0.0046	0.0075	0.0075	0.0057	0.0041	0.0041	0.0071	0.0063	0.0054	0.0054	0.0044	0.0071	0.0041	0.0046	0.0062	0.0047	0.0057	///	///	///		111			///
0.377	0.377	0.377	0.225	0.249	0.292	0.358	0.359	0.291	0.444	0.359	0.440	0.361	0.447	0.545	0.441	0.342	0.278	0.450	0.450	0.342	0.246	0.246	0.429	0.379	0.326	0.326	0.264	0.429	0.264	0.298	0.404	0.307	0.367	///	111	///	///	111			///
0.0015	0.0015	0.0015	0.0093	0.0066	0.0045	0.0023	0.0023	0.0045	0.0011	0.0023	0.0011	0.0023	0.0011	0.0006	0.0011	0.0022	0.0044	0.0009	0.0009	0.0022	0.0044	0.0044	0.0009	0.0013	0.0022	0.0022	0.0044	0.0009	0.0044	0.0044	0.0009	0.0022	0.0022	0.0022	0.0006	0.0003	0.0012	0.0007	0.0004	0.0007	0.0006
45.9	45.9	45.9	45.9	45.9	57.8	57.3	57.8	57.5	58.2	57.7	57.1	58.5	59.1	58.0	57.4	63.4	63.4	63.4	63.4	63.4	33.3	33.3	39.9	39.9	39.9	40.1	39.9	40.1	39.8	53.7	34.4	34.4	53.7	53.7	50.6	53.5	54.6	98.1	98.1	54.6	49.1
86.3	82.7	87.6	116.8	116.8	81.9	97.1	61.9	153.5	70.5	61.8	69.0	9.66	53.1	56.0	51.4	107.0	112.8	70.3	60.0	120.4	117.6	117.6	53.0	65.2	89.4	89.6	109.8	53.2	140.4	138.4	44.7	74.6	105.1	112.9	42.8	41.8	51.9	123.4	97.1	65.7	41.7
73.5	61.3	75.3	115.7	115.7	55.6	77.1	27.8	155.0	39.4	27.8	38.4	79.3	14.3	19.6	13.8	84.7	92.7	33.9	19.7	103.2	213.2	213.2	33.5	50.3	83.8	83.8	112.1	33.5	223.3	137.8	27.5	68.9	91.8	91.8	7.7	3.9	15.4	65.0	32.5	32.5	7.7
73.5	61.3	75.3	115.7	115.7	55.6	77.1	27.8	155.0	39.4	27.8	38.4	79.3	14.3	19.6	13.8	84.7	92.7	33.9	19.7	103.2	79.7	79.7	33.5	50.3	83.8	83.8	112.1	33.5	112.1	137.8	27.5	68.9	91.8	91.8	7.7	3.9	15.4	65.0	32.5	32.5	7.7
0.0048	0.0040	0.0035	0.0013	0.0013	0.0023	0.0032	0.0023	0.0032	0.0033	0.0023	0.0032	0.0033	0.0024	0.0033	0.0023	0.0040	0.0029	0.0040	0.0028	0.0040	0.0020	0.0020	0.0037	0.0037	0.0037	0.0037	0.0023	0.0037	0.0023	0.0034	0.0030	0.0030	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040
0.324		0.235	0.110	0.110	0.158	0.219	0.158	0.220	0.224	0.158	0.218	0.226	0.163	0.223	0.157	0.254	0.177	0.254	0.171	0.265	0.123	0.123	0.220	0.220	0.220	0.220	0.137	0.220	0.148	0.221	0.195	0.195	0.354	///	///	///	///	111		///	111
0.784	111	0.568	0.894	0.894	0.554	0.777	0.554	0.777	0.777	0.554	0.777	0.777	0.554	0.777	0.554	0.816	0.850	0.816	0.550	0.849	0.876	0.876	0.830	0.830	0.830	0.830	0.773	0.830	0.773	0.773	0.830	0.830	0.830	///	111	///	///	///	111	111	111
1.124	11	1.124	1.124	1.124	1.493	1.476	1.493	1.483	1.507	1.490	1.469	1.518	1.539	1.499	1.479	1.190	1.190	1.190	1.190	1.190	0.799	0.799	1.012	1.012	1.012	1.012	1.012	1.012	1.012	1.511	0.831	0.831	1.511	///	///	111	///	111			111
64.8]]	64.8	15.9	15.9	33.5	33.5	33.5	33.5	33.5	33.5	33.5	33.5	33.5	33.5	33.5	51.7	34.6	51.7	51.7	51.7	34.6	34.6	51.7	51.7	51.7	51.7	34.6	51.7	34.6	34.6	51.7	51.7	51.7	///	111	111	111	///	111	11	111
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	133.5	133.5	0.0	0.0	0.0	0.0	0.0	0.0	111.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
39.0	39.0	39.0	39.0	39.0	49.1	48.7	49.1	48.9	49.5	49.1	48.5	49.8	50.3	49.3	48.8	53.9	53.9	53.9	53.9	53.9	28.3	28.3	33.9	33.9	33.9	34.1	33.9	34.1	33.8	45.6	29.2	29.2	45.6	45.6	43.0	45.5	46.4	83.4	83.4	46.4	41.8
125.4	193.2	124.5	127.7	130.7	153.9	153.9	134.7	187.6	134.7	139.5	139.5	168.3	129.9	129.9	125.0	155.0	157.5	162.0	121.5	221.0	177.0	180.5	131.0	133.5	144.5	155.0	169.5	127.5	214.0	131.0	88.0	113.0	119.0	140.0	91.2	89.7	114.0	214.0	159.0	116.0	110.0
75	76	LL	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	66	100	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116

$V_{\rm ANN}$	221.7	222.0	222.6	223.8	226.2	228.7	423.3	379.3	658.3	660.8	189.8	160.7	506.1	468.1	73.7	73.7	73.7	73.7	92.6	92.6	107.1	107.1	107.1	107.1	85.9	85.9	93.3	93.3	90.9	90.9	65.7	75.0	67.1	78.7	71.3	87.9	118.5	199.4	221.5	64.4
$V_{r\text{-ABA}}$	68.3	70.1	73.8	77.8	87.2	9.96	138.4	163.5	241.5	276.7	103.3	86.1	247.3	208.8	49.8	49.8	60.7	60.7	45.4	45.4	44.7	44.7	56.5	56.5	37.7	37.7	61.6	61.6	52.0	52.0	46.2	28.1	43.1	59.6	43.1	57.2	53.7	79.7	111.4	60.0
Check Equation (30)	5.6	11.1	11.1	18.2	27.4	36.5	18.2	27.4	33.4	66.9	34.8	17.6	77.9	39.4	26.1	26.1	36.9	36.9	21.8	21.8	28.4	28.4	40.1	40.1	21.5	21.5	45.3	45.3	35.8	35.8	27.9	10.3	25.8	42.4	25.2	39.1	22.7	49.8	83.5	39.7
V_{fip}	7.7	15.4	15.4	25.2	37.8	50.5	25.2	37.8	46.3	92.5	42.9	24.3	96.2	54.5	40.9	40.9	57.9	57.9	26.9	26.9	44.5	44.5	62.9	62.9	26.5	26.5	56.0	56.0	44.2	44.2	34.6	12.8	32.0	52.6	31.2	48.4	28.1	28.1	28.1	49.2
Min. ϵ_{fipe}	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0011	0.0011	0.0011	0.0011	0.0024	0.0024	0.0012	0.0012	0.0012	0.0012	0.0024	0.0024	0.0025	0.0025	0.0040	0.0040	0.0015	0.0011	0.0014	0.0017	0.0027	0.0032	0.0040	0.0040	0.0040	0.0017
Efrpe-2	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0011	0.0011	0.0011	0.0011	0.0024	0.0024	0.0012	0.0012	0.0012	0.0012	0.0024	0.0024	0.0025	0.0025	0.0040	0.0040	0.0015	0.0011	0.0014	0.0017	0.0027	0.0032	0.0040	0.0040	0.0040	0.0017
K_2	111	111	111	///	///	///	///	111	///	///	///	///	///	///	0.835	0.835	0.835	0.835	0.363	0.363	0.812	0.812	0.812	0.812	0.364	0.364	0.575	0.575	0.682	0.682	0.504	0.259	0.504	0.628	0.629	0.722	///	///		0.777
K_{l}			///	///	///	///	///	///	///	///	///	///	///	///	1.362	1.362	1.362	1.362	1.362	1.362	1.334	1.334	1.334	1.334	1.335	1.335	1.335	1.335	1.335	1.335	1.180	1.137	1.091	1.080	1.142	1.158	///	///		1.407
L _e	111	111	111	///	///	///	111	111	///	///	///	///	///	///	18.9	18.9	18.9	18.9	72.8	72.8	21.5	21.5	21.5	21.5	72.8	72.8	48.7	48.7	72.8	72.8	37.2	55.6	37.2	37.2	55.6	55.6	///	///		22.3
E firpe-1				///	///	///			///	///	///	///	///	///	0.0024	0.0024	0.0024	0.0024	0.0055	0.0055	0.0019	0.0019	0.0019	0.0019	0.0048	0.0048	0.0039	0.0039	0.0048	0.0048	0.0041	0.0050	0.0041	0.0040	0.0051	0.0051	///			0.0132
R			///	///	///	///	///	///	///	///	///	///	///	///	0.125 0	0.125 0	0.125 0	0.125 0	0.410 0	0.410	0.102 0	0.102	0.102 0	0.102	0.362 (0.362 0	0.294 0	0.294 0	0.362	0.362 (0.276 0	0.337 0	0.270	0.269 0	0.337	0.338 0	///	///		0.294
ρ _{frp}	0.0003	0.0006	0.0006	0.0010	0.0014	0.0019	0.0005	0.0006	0.0005	0.0010	0.0004	0.0005	0.0004	0.0005	0.0062	0.0062	0.0062	0.0062	0.0043	0.0043	0.0093	0.0093	0.0093	0.0093	0.0065	0.0065	0.0129	0.0129	0.0065	0.0065	0.0045	0.0022	0.0045	0.0045	0.0022	0.0022	///	///		0.0064
$^{\circ}A$	104.5	98.3	104.5	99.2	99.7	100.1	200.2	226.8	346.7	349.7	114.2	114.2	282.3	282.3	39.3	39.3	39.3	39.3	39.3	39.3	27.0	27.0	27.0	27.0	27.0	27.0	27.0	27.0	27.0	27.0	30.3	29.5	28.6	28.4	29.6	29.9	51.6	49.7	46.4	33.6
$\delta V_{ m n-ACI}$	81.7	83.4	87.9	92.0	102.6	113.1	165.0	194.4	287.9	327.4	117.2	102.2	281.7	248.0	72.5	72.5	90.7	90.7	54.1	54.1	67.5	67.5	95.3	95.3	44.8	44.8	69.1	69.1	51.4	51.4	53.1	34.5	49.5	66.6	48.7	63.3	57.6	83.3	114.6	66.3
Check Equation (17)	7.7	15.4	15.4	25.2	37.8	50.5	25.2	37.8	46.3	92.5	42.9	24.3	96.2	54.5	61.0	61.0	86.2	86.2	35.6	35.6	66.4	66.4	94.0	94.0	35.0	35.0	68.7	68.7	44.2	44.2	43.2	18.2	39.9	63.7	37.8	57.8	28.1	60.0	9.66	58.2
V_f	7.7	15.4	15.4	25.2	37.8	50.5	25.2	37.8	46.3	92.5	42.9	24.3	96.2	54.5	61.0	61.0	86.2	86.2	35.6	35.6	66.4	66.4	94.0	94.0	35.0	35.0	68.7	68.7	44.2	44.2	43.2	18.2	39.9	63.7	37.8	57.8	28.1	28.1	28.1	58.2
E _{fe}	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0017	0.0017	0.0017	0.0017	0.0032	0.0032	0.0019	0.0019	0.0019	0.0019	0.0032	0.0032	0.0031	0.0031	0.0040	0.0040	0.0019	0.0016	0.0017	0.0021	0.0033	0.0038	0.0040	0.0040	0.0040	0.0020
<i>K</i>		111	111	///	///	///	///	111	///	///	///	///	///	///	0.090	0.090	0.090	0.090	0.242	0.242	0.098	0.098	0.098	0.098	0.238	0.238	0.234	0.234	0.406	0.406	0.125	0.105	0.116	0.138	0.219	0.251	///	///		0.043
K_2			///	///	///	///	///	///	///	///	111	///	///	///	0.848	0.848	0.848	0.848	0.415	0.415	0.828	0.828	0.828	0.828	0.416	0.416	0.609	0.609	0.708	0.708	0.544	0.319	0.544	0.658	0.659	0.745	///			0.795
$K_{_{l}}$			///	///	///	///	///	///	///	///	111	///	///	///	1.383	1.383	1.383	1.383	1.383	1.383	1.356	1.356	1.356	1.356	1.356	1.356	1.356	1.356	1.356	1.356	1.199	1.155	1.109	1.097	1.160	1.176	///	///		1.430
L_e			///	///	///	///	///	///	///	///	111	///	///	///	17.3	17.3	17.3	17.3	6.99	6.99	19.7	19.7	19.7	19.7	6.99	6.99	44.8	44.8	6.99	6.99	34.2	51.1	34.2	34.2	51.1	51.1	///	///		20.5
s A	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	31.9	71.6	0.0
o	88.9	83.5	88.8	84.3	84.7	85.1	170.2	192.8	294.7	297.3	97.1	97.1	240.0	240.0	33.4	33.4	33.4	33.4	33.4	33.4	23.0	23.0	23.0	23.0	23.0	23.0	23.0	23.0	23.0	23.0	25.8	25.1	24.3	24.1	25.2	25.4	43.9	42.2	39.4	28.5
V_{U-exp}	173.0	209.0	224.0	254.0	247.0	240.0	424.0	379.0	569.0	662.0	209.1	218.6	469.7	464.5	73.7	63.9	82.2	55.7	53.6	51.2	84.6	104.6	97.0	121.5	76.5	128.6	87.7	133.5	96.1	126.8	63.6	58.6	60.3	80.8	68.5	85.8	132.1	180.9	213.8	55.5
No.	117	118	119	120	121	122	123	124	125	126	127	128	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143	144	145	146	147	148	149	150	151	152	153	154	155	156

13.0	62.7	62.6	92.5	92.6	92.3	90.4	60.6	67.8	94.6	92.6	74.2	76.1	278.9	284.0	247.1	359.9	279.2	226.5	263.8	243.9	361.4	359.9	283.9	299.8	299.8	284.2	296.6	322.3	252.7	278.2	241.0	213.2	231.2	206.1	213.2	116.3	124.3	124.3	206.7	207.0	207.0
46.9	71.9	65.9	103.4	110.7	101.1	94.9	89.8	105.7	41.4	48.9	27.9	18.7	151.1	196.2	185.5	168.8	193.9	181.6	154.7	150.4	150.4	137.8	255.1	178.3	178.3	282.5	282.5	282.5	212.1	236.7	199.8	202.4	227.0	190.1	202.4	67.9	113.2	113.2	126.1	126.1	126.1
25.1	50.5	44.5	102.7	112.6	99.6	129.4	68.3	98.7	31.6	40.6	18.0	8.9	74.8	117.6	117.6	105.5	117.6	117.6	83.1	83.1	83.1	74.6	176.3	129.0	129.0	254.5	266.8	291.4	155.3	179.9	143.0	145.6	170.2	133.3	145.6	44.1	89.3	89.3	121.9	142.5	142.5
31.0	62.5	55.1	127.2	139.4	123.4	160.3	84.6	122.2	11.1	22.3	22.3	11.1	92.6	145.6	145.6	130.7	145.6	145.6	103.0	103.0	103.0	92.4	218.4	118.0	118.0	269.5	269.5	269.5	131.4	131.4	131.4	119.4	119.4	119.4	119.4	54.6	110.6	110.6	95.4	120.9	120.9
0.0010	0.0018	0.0018	0.0017	0.0019	0.0017	0.0015	0.0034	0.0025	0.0023	0.0023	0.0023	0.0023	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0038	0.0038	0.0040	0.0040	0.0040	0.0020	0.0020	0.0020	0.0018	0.0018	0.0018	0.0018	0.0024	0.0016	0.0016	0.0021	0.0017	0.0017
0.0010	0.0018	0.0018	0.0017	0.0019	0.0017	0.0015	0.0034	0.0025	0.0023	0.0023	0.0023	0.0023	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0038	0.0038	0.0040	0.0040	0.0040	0.0020	0.0020	0.0020	0.0018	0.0018	0.0018	0.0018	0.0024	0.0016	0.0016	0.0021	0.0017	0.0017
0.443	0.777	0.777	0.777	0.777	0.777	0.777	0.790	0.859	0.395	0.595	0.595	0.395	0.635	0.720	0.720	111	0.720	0.720	0.720	0.720	///	///	0.778	0.723	0.723	111	///		0.916	0.916	0.916	0.832	0.832	0.832	0.832	0.630	0.804	0.804	0.753	0.804	0.804
1.558	1.518	1.515	1.456	1.596	1.413	1.298	1.512	1.500	0.910	0.910	0.910	0.910	1.809	1.881	1.546	111	1.809	1.427	1.662	1.527	///	///	1.881	1.406	1.406	111	///		0.832	0.832	0.832	0.832	0.832	0.832	0.832	0.996	0.996	0.996	1.089	1.089	1.089
22.3	22.3	22.3	22.3	22.3	22.3	22.3	42.1	28.1	95.0	63.6	63.6	95.0	91.1	70.1	70.1	111	70.1	70.1	70.1	70.1	///	///	55.4	55.4	55.4	111	///	111	37.7	37.7	37.7	37.7	37.7	37.7	37.7	55.5	29.3	29.3	37.1	29.3	29.3
0.0104	0.0143	0.0143	0.0102	0.0104	0.0101	0.0098	0.0143	0.0116	0.0057	0.0046	0.0046	0.0057	0.0100	0.0089	0.0084		0.0088	0.0082	0.0085	0.0083	///	///	0.0078	0.0072	0.0072		///		0.0018	0.0018	0.0018	0.0018	0.0018	0.0018	0.0018	0.0048	0.0044	0.0044	0.0051	0.0045	0.0045
0.230	0.318	0.318	0.226	0.232	0.224	0.218	0.317	0.257	0.362	0.294	0.294	0.362	0.528	0.466	0.440	111	0.461	0.429	0.449	0.438	///	///	0.413	0.378	0.378	111	///		0.292	0.292	0.292	0.292	0.292	0.292	0.292	0.323	0.232	0.232	0.270	0.239	0.239
0.0160	0.0053	0.0053	0.0160	0.0160	0.0160	0.0160	0.0036	0.0072	0.0014	0.0028	0.0028	0.0014	0.0008	0.0012	0.0012	///	0.0012	0.0012	0.0012	0.0012	///	///	0.0018	0.0018	0.0018	111	///		0.0015	0.0015	0.0015	0.0015	0.0015	0.0015	0.0015	0.0022	0.0066	0.0066	0.0044	0.0066	0.0066
36.2	35.5	35.5	34.5	36.9	33.7	31.6	35.4	35.2	16.3	16.3	16.3	16.3	126.7	130.4	112.7	105.0	126.7	106.1	118.9	111.6	111.6	105.0	130.4	81.8	81.8	94.2	94.2	94.2	94.2	94.2	94.2	94.2	94.2	94.2	94.2	39.3	39.3	39.3	42.0	42.0	42.0
54.8	79.1	72.8	121.5	130.2	118.8	111.5	97.4	124.2	44.8	57.5	31.5	22.2	158.5	199.4	186.6	181.4	196.7	181.9	160.3	155.0	163.8	150.5	270.6	182.0	182.0	332.2	332.2	332.2	227.9	252.5	215.6	218.7	243.2	206.4	218.7	76.3	138.7	138.7	148.2	148.2	148.2
39.6	74.0	65.2	150.6	165.0	146.1	189.8	99.4	143.3	41.0	53.9	27.3	14.4	92.6	145.6	145.6	130.7	145.6	145.6	103.0	103.0	103.0	92.4	218.4	163.1	163.1	312.9	327.4	356.3	211.1	240.0	196.6	198.3	227.2	183.8	198.3	66.2	130.5	130.5	166.1	195.4	195.4
39.6	74.0	65.2	150.6	165.0	146.1	189.8	99.4	143.3	14.4	27.3	27.3	14.4	92.6	145.6	145.6	130.7	145.6	145.6	103.0	103.0	103.0	92.4	218.4	123.6	123.6	269.5	269.5	269.5	153.2	153.2	153.2	140.4	140.4	140.4	140.4	66.2	130.5	130.5	113.3	142.7	142.7
0.0013	0.0021	0.0021	0.0020	0.0022	0.0020	0.0018	0.0040	0.0029	0.0030	0.0028	0.0028	0.0030	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0023	0.0023	0.0023	0.0021	0.0021	0.0021	0.0021	0.0029	0.0019	0.0019	0.0025	0.0021	0.0021
0.030	0.047	0.047	0.045	0.049	0.044	0.040	0.089	0.064	0.190	0.180	0.180	0.190	0.453	0.404	0.332	///	0.389	0.307	0.357	0.328	///	///	0.343	0.240	0.240	///	///	111	0.379	0.379	0.379	0.347	0.347	0.347	0.347	0.191	0.099	0.099	0.129	0.108	0.108
0.488	0.795	0.795	0.795	0.795	0.795	0.795	0.807	0.871	0.444	0.628	0.628	0.444	0.665	0.742	0.742	///	0.742	0.742	0.742	0.742	///	///	0.796	0.745	0.745	///	///	111	0.923	0.923	0.923	0.846	0.846	0.846	0.846	0.660	0.820	0.820	0.773	0.820	0.820
1.583	1.542	1.539	1.480	1.622	1.435	1.319	1.536	1.524	0.924	0.924	0.924	0.924	1.838	1.911	1.571		1.838	1.450	1.688	1.551	///	///	1.911	1.429	1.429	111	///	111	0.845	0.845	0.845	0.845	0.845	0.845	0.845	1.012	1.012	1.012	1.106	1.106	1.106
20.5	20.5	20.5	20.5	20.5	20.5	20.5	38.7	25.9	87.3	58.4	58.4	87.3	83.8	64.4	64.4)]]	64.4	64.4	64.4	64.4	///	///	50.9	50.9	50.9	///	///	Ì	34.7	34.7	34.7	34.7	34.7	34.7	34.7	51.0	27.0	27.0	34.1	27.0	27.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	26.6	26.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	39.6	39.6	43.4	57.8	86.8	57.8	86.8	43.4	57.8	86.8	43.4	57.8	0.0	0.0	0.0	52.8	52.8	52.8
30.8	30.2	30.2	29.3	31.4	28.6	26.9	30.1	29.9	13.8	13.8	13.8	13.8	107.7	110.9	95.8	89.2	107.7	90.2	101.1	94.9	94.9	89.2	110.9	69.5	69.5	80.0	80.0	80.0	80.0	80.0	80.0	80.0	80.0	80.0	80.0	33.4	33.4	33.4	35.7	35.7	35.7
55.0	58.0	65.0	99.7	85.0	97.5	94.0	86.9	80.0	95.7	105.1	68.5	59.3	247.0	257.0	305.0	338.0	306.0	251.0	256.0	298.0	367.0	388.0	334.0	298.0	298.0	250.0	330.0	300.0	270.0	280.0	225.0	205.0	240.0	180.0	220.0	120.0	112.8	140.2	193.0	213.3	247.5
157	158	159	160	161	162	163	164	165	166	167	168	169	170	171	172	173	174	175	176	177	178	179	180	181	182	183	184	185	186	187	188	189	190	191	192	193	194	195	196	197	198

$V_{\rm ANN}$	206.6	206.7	206.7	256.5	260.3	244.7	249.5	144.8	142.8	141.5	140.6	153.9	149.8	147.3	145.7	127.9	135.6	121.6	120.1	101.8	700.9	85.3	85.6	70.7	71.0	79.3	79.3	68.0	65.1	65.1	54.9	68.1	68.1	55.8	110.3	43.6	111.6	46.5	67.3	78.6
$V_{r m -ABA}$	118.4	126.1	126.1	152.9	152.9	152.9	152.9	82.0	77.1	80.6	77.3	83.5	78.3	82.0	78.5	53.9	62.6	52.9	52.2	89.3	318.7	76.9	76.9	72.6	33.5	34.8	34.8	30.3	30.9	30.9	37.1	39.0	39.0	32.5	35.7	25.0	29.2	23.4	31.8	36.7
Check Equation (30)	93.1	121.9	121.9	137.6	179.4	126.3	168.1	41.1	36.1	39.7	36.3	42.6	37.3	41.1	37.6	24.3	33.3	23.3	23.3	60.8	357.3	74.9	74.2	57.1	16.5	17.7	17.7	13.7	14.3	14.3	20.0	21.9	21.9	25.2	28.3	17.6	21.8	16.1	24.4	31.2
V_{fp}	59.7	95.4	95.4	91.1	142.8	91.1	142.8	30.6	24.5	28.9	24.8	32.5	26.0	30.6	26.3	14.3	25.3	13.0	13.0	59.5	362.5	72.4	71.6	50.5	5.7	7.3	7.3	2.3	3.1	3.1	10.1	12.5	12.5	9.4	14.4	12.6	19.2	25.2	38.4	18.9
$\min_{\mathfrak{E}_{fipe}}$	0.0026	0.0021	0.0021	0.0040	0.0031	0.0040	0.0031	0.0038	0.0038	0.0038	0.0038	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0009	0.0030	0.0026	0.0026	0.0021	0.0019	0.0020	0.0020	0.0021	0.0024	0.0024	0.0012	0.0012	0.0012	0.0009	0.0014	0.0009	0.0014	0.0009	0.0014	0.0009
€ _{firpe-2}	0.0026	0.0021	0.0021	0.0040	0.0031	0.0040	0.0031	0.0038	0.0038	0.0038	0.0038	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0013	0.0030	0.0026	0.0026	0.0021	0.0019	0.0020	0.0020	0.0021	0.0024	0.0024	0.0012	0.0012	0.0012	0.0009	0.0014	0.0009	0.0014	0.0009	0.0014	0.0009
$K_{_2}$	0.630	0.753	0.753	0.813	0.875	0.813	0.875	0.888	0.888	0.888	0.888	///	///	///	///	0.671	0.738	0.682	0.682	0.876	0.953	0.804	0.801	0.483	0.780	0.818	0.818	0.607	0.675	0.675	0.881	0.901	0.901	0.489	0.745	0.489	0.745	0.489	0.745	0.489
K_l	1.089	1.089	1.089	1.411	1.411	1.411	1.411	1.237	1.237	1.237	1.237	///	///	///	///	1.680	1.662	1.680	1.624	1.585	1.233	1.204	1.204	1.204	0.584	0.584	0.584	0.584	0.584	0.584	0.584	0.584	0.584	0.912	0.912	0.912	0.912	0.912	0.912	0.912
L_e	55.5	37.1	37.1	56.0	37.4	56.0	37.4	50.6	50.6	50.6	50.6	///	///	///	///	72.5	57.7	6.69	6.69	13.6	37.4	39.8	40.3	52.5	62.7	62.7	62.7	112.1	112.1	112.1	34.0	34.0	34.0	38.3	38.3	38.3	38.3	38.3	38.3	38.3
\mathcal{E}_{fipe-I}	0.0063	0.0051	0.0051	0.0054	0.0044	0.0054	0.0044	0.0068	0.0073	0.0077	0.0080	///	///	///	///	0.0070	0.0059	0.0087	0.0086	0.0009	0.0039	0.0031	0.0033	0.0027	0.0085	0.0085	0.0085	0.0199	0.0199	0.0199	0.0014	0.0014	0.0014	0.0068	0.0068	0.0060	0.0060	0.0043	0.0043	0.0049
R	0.332	0.270	0.270	0.360	0.293	0.360	0.293	0.511	0.546	0.577	0.604	///	///	///	///	0.565	0.474	0.580	0.574	0.234	0.263	0.274	0.276	0.316	0.470	0.470	0.470	0.428	0.428	0.428	0.342	0.342	0.342	0.146	0.146	0.128	0.128	0.092	0.092	0.106
β _{frp}	0.0022	0.0044	0.0044	0.0022	0.0044	0.0022	0.0044	0.0013	0.0011	0.0009	0.0008	///	///	///	///	0.0006	0.0011	0.0005	0.0005	0.0078	0.0055	0.0315	0.0315	0.0236	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0067	0.0067	0.0089	0.0089	0.0178	0.0178	0.0133
<i>v</i> ²	42.0	42.0	42.0	51.0	51.0	51.0	51.0	68.1	68.1	68.1	68.1	68.1	68.1	68.1	68.1	49.3	48.9	49.3	48.0	47.2	106.2	25.6	25.6	25.6	28.4	28.4	28.4	28.4	28.4	28.4	28.4	28.4	28.4	12.2	12.2	12.2	12.2	12.2	12.2	12.2
ø $V_{ m n-ACI}$	127.5	148.2	148.2	166.7	179.8	155.4	179.8	89.0	84.3	87.7	84.5	91.8	86.6	90.3	86.8	58.7	66.4	57.8	56.9	116.0	374.8	90.4	90.4	80.7	37.3	38.6	38.6	34.9	35.8	35.8	40.9	42.9	42.9	38.9	43.2	32.9	39.3	37.8	43.2	43.2
Check Equation (17)	125.2	166.1	166.1	166.5	242.5	153.2	229.2	51.7	45.2	49.9	45.5	51.7	45.2	49.9	45.5	29.3	40.4	28.1	28.1	110.7	497.2	104.7	103.8	82.7	20.7	22.5	22.5	17.5	18.7	18.7	25.7	28.5	28.5	37.6	44.2	31.3	40.2	40.1	57.8	52.6
V_f	72.4	113.3	113.3	91.1	167.1	91.1	167.1	32.5	26.0	30.6	26.3	32.5	26.0	30.6	26.3	14.3	25.3	13.0	13.0	95.6	421.1	85.5	84.5	63.4	6.8	8.6	8.6	3.6	4.8	4.8	11.8	14.6	14.6	15.0	21.7	20.0	28.9	40.1	57.8	30.0
ϵ_{fe}	0.0031	0.0025	0.0025	0.0040	0.0037	0.0040	0.0037	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0015	0.0035	0.0031	0.0031	0.0026	0.0023	0.0024	0.0024	0.0033	0.0036	0.0036	0.0014	0.0014	0.0014	0.0015	0.0021	0.0015	0.0021	0.0015	0.0021	0.0015
$K_{ m v}$	0.165	0.129	0.129	0.342	0.245	0.342	0.245	0.331	0.331	0.331	0.331	///	111	111	///	0.536	0.461	0.435	0.421	0.375	0.231	0.270	0.263	0.300	0.127	0.133	0.133	0.070	0.077	0.077	0.347	0.355	0.355	0.031	0.045	0.031	0.045	0.031	0.045	0.031
$K_{_2}$	0.660	0.773	0.773	0.828	0.885	0.828	0.885	0.897	0.897	0.897	0.897	///	111	111	///	0.697	0.759	0.708	0.708	0.886	0.957	0.820	0.817	0.525	0.798	0.833	0.833	0.638	0.701	0.701	0.890	0.909	0.909	0.531	0.765	0.531	0.765	0.531	0.765	0.531
K_{l}	1.106	1.106	1.106	1.433	1.433	1.433	1.433	1.257	1.257	1.257	1.257	///	111	111	111	1.707	1.688	1.707	1.650	1.611	1.253	1.224	1.224	1.224	0.594	0.594	0.594	0.594	0.594	0.594	0.594	0.594	0.594	0.927	0.927	0.927	0.927	0.927	0.927	0.927
L_{e}	51.0	34.1	34.1	51.5	34.4	51.5	34.4	46.5	46.5	46.5	46.5	111	111	111	///	66.6	53.1	64.3	64.3	12.5	34.4	36.6	37.1	48.2	57.7	57.7	57.7	103.0	103.0	103.0	31.3	31.3	31.3	35.2	35.2	35.2	35.2	35.2	35.2	35.2
${}^{s}_{A}$	52.8	52.8	52.8	75.4	75.4	62.1	62.1	19.2	19.2	19.2	19.2	19.2	19.2	19.2	19.2	15.1	15.1	15.1	15.1	15.1	76.0	19.3	19.3	19.3	13.9	13.9	13.9	13.9	13.9	13.9	13.9	13.9	13.9	22.6	22.6	11.3	11.3	0.0	0.0	22.6
s	35.7	35.7	35.7	43.3	43.3	43.3	43.3	57.9	57.9	57.9	57.9	57.9	57.9	57.9	57.9	41.9	41.5	41.9	40.8	40.1	90.3	21.8	21.8	21.8	24.1	24.1	24.1	24.1	24.1	24.1	24.1	24.1	24.1	10.4	10.4	10.4	10.4	10.4	10.4	10.4
$V_{u\text{-exp}}$	161.4	208.8	212.0	252.9	264.8	238.9	243.3	142.5	130.0	154.5	150.0	177.5	155.0	145.5	132.0	110.0	152.5	121.5	137.5	103.5	701.0	71.1	71.6	97.0	62.0	82.6	66.6	61.3	77.5	77.3	55.0	61.8	70.3	53.0	55.0	50.0	52.0	48.0	55.0	49.0
No.	199	200	201	202	203	204	205	206	207	208	209	210	211	212	213	214	215	216	217	218	219	220	221	222	223	224	225	226	227	228	229	230	231	232	233	234	235	236	237	238

130.3	170.9	205.4	222.0	213.6	234.4	231.5	268.3	340.4	327.8	301.4	97.9	94.8	92.6	103.3	93.9	112.2	168.2	168.2	204.9	279.7	279.5	284.4	263.0	284.3	96.7	110.2	67.5	52.0	63.3	67.3	78.9	85.0	87.9	95.4	96.3	98.9	106.1	54.1	54.1	89.9	89.9
36.7	57.0	75.3	105.5	93.5	86.3	136.3	127.7	154.4	137.4	156.0	42.7	41.8	47.0	56.6	47.5	63.4	89.0	89.0	115.6	194.2	151.3	196.4	154.6	255.4	48.2	65.3	38.6	60.7	21.2	22.2	17.5	30.4	31.4	26.7	35.0	36.0	31.3	26.1	26.1	28.9	28.9
37.5	24.4	42.6	72.7	60.8	53.6	73.5	65.1	91.7	74.7	93.4	15.4	15.0	20.9	29.8	20.6	36.6	45.1	45.1	71.7	117.6	74.8	117.6	83.1	176.3	17.2	34.4	22.1	44.2	5.2	6.5	0.5	14.3	15.6	9.6	18.9	20.1	14.2	18.8	18.8	21.6	21.6
28.8	38.2	38.2	45.0	38.2	27.0	91.1	45.5	78.5	57.4	45.5	19.0	18.6	25.9	36.9	25.6	45.3	25.9	25.9	58.8	145.6	92.6	145.6	103.0	218.4	21.3	42.6	27.4	54.8	6.4	8.0	0.6	6.5	8.1	0.6	6.5	8.1	0.6	8.9	8.9	12.4	12.4
0.0014	0.0024	0.0024	0.0036	0.0024	0.0032	0.0040	0.0040	0.0027	0.0038	0.0040	0.0027	0.0026	0.0036	0.0037	0.0026	0.0037	0.0034	0.0034	0.0024	0.0040	0.0040	0.0040	0.0040	0.0040	0.0021	0.0021	0.0022	0.0022	0.0034	0.0021	0.0001	0.0034	0.0021	0.0001	0.0034	0.0021	0.0001	0.0029	0.0029	0.0040	0.0040
0.0014	0.0026	0.0026	0.0036	0.0026	0.0032	0.0040	0.0040	0.0039	0.0052	0.0040	0.0027	0.0026	0.0036	0.0037	0.0026	0.0037	0.0034	0.0034	0.0024	0.0040	0.0040	0.0040	0.0040	0.0040	0.0021	0.0021	0.0022	0.0022	0.0110	0.0021	0.0001	0.0111	0.0021	0.0001	0.0111	0.0021	0.0001	0.0029	0.0029	0.0040	0.0040
0.745	0.752	0.752	0.717	0.752	0.643	0.843	0.843	0.862	0.802	0.843	0.529	0.529	0.764	0.764	0.529	0.764	0.815	0.815	0.876	0.720	0.635	0.720	0.720	0.778	0.909	0.909	0.817	0.817	0.860	0.244	0.017	0.860	0.244	0.017	0.860	0.244	0.017	0.611	0.611		111
0.912	1.032	1.032	1.032	1.032	1.032	1.365	1.365	1.365	1.365	1.365	1.101	1.073	1.035	1.073	1.075	1.070	1.096	1.096	1.096	1.817	1.817	1.888	1.656	1.888	1.229	1.229	1.477	1.477	1.283	1.283	1.283	1.295	1.295	1.295	1.295	1.295	1.295	0.994	0.994	///	///
38.3	62.0	62.0	70.8	62.0	89.2	70.8	70.8	62.0	89.2	70.8	67.2	67.2	67.2	67.2	67.2	67.2	55.6	55.6	37.2	70.1	91.1	70.1	70.1	55.4	27.4	27.4	27.4	27.4	186.0	124.4	98.3	186.0	124.4	98.3	186.0	124.4	98.3	6.69	6.69	111	111
0.0049	0.0024	0.0024	0.0060	0.0024	0.0033	0.0065	0.0072	0.0027	0.0038	0.0072	0.0074	0.0073	0.0073	0.0067	0.0067	0.0063	0.0065	0.0065	0.0046	0.0088	0.0100	0.0089	0.0085	0.0079	0.0035	0.0029	0.0029	0.0023	0.0034	0.0024	0.0020	0.0034	0.0024	0.0020	0.0034	0.0024	0.0020	0.0074	0.0074	///	
0.106	0.160	0.160	0.403	0.160	0.215	0.438	0.486	0.182	0.245	0.486	0.436	0.432	0.428	0.393	0.393	0.369	0.489	0.489	0.346	0.461	0.528	0.467	0.449	0.413	0.439	0.356	0.358	0.291	0.369	0.266	0.220	0.370	0.267	0.221	0.370	0.267	0.221	0.411	0.411		///
0.0133	0.0257	0.0257	0.0011	0.0257	0.0300	0.0011	0.0008	0.0257	0.0300	0.0008	0.0009	0.0009	0.0009	0.0013	0.0013	0.0015	0.0007	0.0007	0.0021	0.0012	0.0008	0.0012	0.0012	0.0018	0.0006	0.0012	0.0014	0.0028	0.0072	0.0144	0.0216	0.0072	0.0144	0.0216	0.0072	0.0144	0.0216	0.0010	0.0010	///	
12.2	54.4	54.4	54.4	54.4	54.4	104.2	104.2	104.2	104.2	104.2	45.4	44.5	43.4	44.5	44.6	44.5	73.0	73.0	73.0	127.1	127.1	130.8	118.6	130.8	51.5	51.3	27.4	27.2	28.5	28.5	28.5	28.7	28.7	28.7	28.7	28.7	28.7	12.2	12.2	12.2	12.2
43.2	83.9	102.2	112.3	120.4	100.3	141.1	136.5	186.5	147.8	164.8	49.9	48.8	52.1	60.7	55.2	67.3	99.0	0.99	126.6	197.0	158.8	199.7	160.1	252.3	55.2	73.0	43.1	66.3	26.1	25.6	24.5	35.4	34.8	33.8	39.9	39.4	38.3	28.2	28.2	30.4	30.4
65.9	61.8	83.2	93.5	104.6	76.9	91.1	78.9	148.0	94.5	112.2	23.6	23.0	28.7	39.5	31.7	48.6	59.0	59.0	97.3	145.6	92.6	145.6	103.0	218.4	24.9	49.7	32.3	64.5	7.7	6.9	5.5	18.3	17.6	16.2	23.7	23.0	21.5	24.5	24.5	26.0	26.0
43.3	61.8	61.8	50.6	61.8	34.0	91.1	45.5	114.7	61.2	45.5	23.6	23.0	28.7	39.5	31.7	48.6	30.5	30.5	68.8	145.6	92.6	145.6	103.0	218.4	24.9	49.7	32.3	64.5	7.7	6.9	5.5	7.7	7.0	5.5	7.7	7.0	5.5	10.9	10.9	12.4	12.4
0.0021	0.0039	0.0039	0.0040	0.0039	0.0040	0.0040	0.0040	0.0040	0.0040	0.0040	0.0033	0.0032	0.0040	0.0040	0.0032	0.0040	0.0040	0.0040	0.0028	0.0040	0.0040	0.0040	0.0040	0.0040	0.0024	0.0024	0.0026	0.0026	0.0040	0.0018	0.0010	0.0040	0.0018	0.0010	0.0040	0.0018	0.0010	0.0035	0.0035	0.0040	0.0040
0.045	0.258	0.258	0.287	0.258	0.317	0.439	0.439	0.387	0.510	0.439	0.193	0.189	0.251	0.261	0.189	0.260	0.298	0.298	0.213	0.390	0.455	0.406	0.356	0.344	0.302	0.302	0.330	0.330	0.750	0.197	0.104	0.750	0.199	0.105	0.750	0.199	0.105	0.195	0.195		///
0.765	0.772	0.772	0.740	0.772	0.672	0.855	0.855	0.873	0.818	0.855	0.567	0.567	0.783	0.783	0.567	0.783	0.830	0.830	0.886	0.742	0.665	0.742	0.742	0.796	0.916	0.916	0.832	0.832	0.709	0.144	0.096	0.709	0.144	0.096	0.709	0.144	0.096	0.643	0.643		
0.927	1.049	1.049	1.049	1.049	1.049	1.387	1.387	1.387	1.387	1.387	1.118	1.090	1.051	1.090	1.092	1.087	1.114	1.114	1.114	1.846	1.846	1.919	1.683	1.919	1.248	1.248	1.501	1.501	1.303	1.303	1.303	1.316	1.316	1.316	1.316	1.316	1.316	1.010	1.010	11	111
35.2	56.9	56.9	65.1	56.9	81.9	65.1	65.1	56.9	81.9	65.1	61.7	61.7	61.7	61.7	61.7	61.7	51.1	51.1	34.2	64.4	83.8	64.4	64.4	50.9	25.2	25.2	25.2	25.2	170.9	114.4	90.4	170.9	114.4	90.4	170.9	114.4	90.4	64.3	64.3	11	11
22.6	0.0	21.4	42.9	42.9	42.9	0.0	33.3	33.3	33.3	66.6	0.0	0.0	0.0	0.0	0.0	0.0	28.5	28.5	28.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	10.7	10.7	10.7	16.0	16.0	16.0	13.6	13.6	13.6	13.6
10.4	46.3	46.3	46.3	46.3	46.3	88.6	88.6	88.6	88.6	88.6	38.6	37.9	36.9	37.9	37.9	37.8	62.0	62.0	62.0	108.0	108.0	111.2	100.8	111.2	43.8	43.6	23.3	23.1	24.2	24.2	24.2	24.4	24.4	24.4	24.4	24.4	24.4	10.3	10.3	10.3	10.3
50.0	159.0	205.6	219.1	225.6	242.7	213.6	272.8	297.5	316.7	309.8	61.6	59.7	60.0	80.7	64.3	90.0	124.4	130.0	148.0	306.1	246.7	257.2	260.6	334.3	61.0	89.8	55.6	71.5	66.0	69.0	75.0	88.0	90.0	91.0	90.0	93.0	98.0	65.0	45.9	66.1	66.7
239	240	241	242	243	244	245	246	247	248	249	250	251	252	253	254	255	256	257	258	259	260	261	262	263	264	265	266	267	268	269	270	271	272	273	274	275	276	277	278	279	280

$V_{\rm ANN}$	168.0	168.0	230.0	230.0	560.7	560.7	876.6	876.6	121.2	123.3	128.3	274.2	275.6	278.9	286.5	288.7	217.0	132.7	129.0	137.8	137.8	135.0	135.0	158.5	
$V_{r\text{-ABA}}$	101.9	101.9	114.6	114.6	402.9	402.9	482.6	482.6	57.4	70.6	89.5	129.2	142.4	159.6	159.6	159.6	180.6	70.5	54.7	83.8	102.4	200.7	200.7	180.6	
Check Equation (30)	73.1	73.1	85.7	85.7	278.3	278.3	357.7	357.7	25.4	38.6	57.4	97.2	110.3	129.1	182.1	200.9	119.9	31.5	15.7	44.8	63.3	50.1	124.4	42.7	
V_{fip}	34.0	34.0	49.6	49.6	100.2	100.2	198.5	198.5	31.5	47.7	71.0	31.5	47.7	71.0	47.7	71.0	66.7	39.0	19.5	55.5	78.4	141.5	141.5	116.7	
$\min_{\epsilon_{fipe}}$	0.0027	0.0027	0.0040	0.0040	0.0020	0.0020	0.0040	0.0040	0.0040	0.0034	0.0025	0.0040	0.0034	0.0025	0.0034	0.0025	0.0040	0.0025	0.0025	0.0024	0.0024	0.0018	0.0018	0.0040	
E _{fipe-2}	0.0027	0.0027	0.0040	0.0040	0.0020	0.0020	0.0040	0.0040	0.0040	0.0034	0.0025	0.0040	0.0034	0.0025	0.0034	0.0025	0.0040	0.0025	0.0025	0.0024	0.0024	0.0018	0.0018	0.0040	
$K_{_2}$	0.870	0.870	111	///	0.957	0.957	///	111	0.646	0.747	0.831	0.646	0.747	0.831	0.747	0.831	///	0.502	0.502	0.604	0.604	0.807	0.807	0.797	ıt.
K_l	0.994	0.994	///	///	0.994	0.994	///	///	0.935	0.935	0.935	0.935	0.935	0.935	0.935	0.935	///	1.164	1.164	1.164	1.164	1.313	1.313	1.313	statemei
L_e	46.8	46.8	111	///	31.3	31.3		111	100.4	71.8	48.0	100.4	71.8	48.0	71.8	48.0	///	62.3	62.3	49.4	49.4	24.6	24.6	57.0	ditional
Efrpe-1	0.0074	0.0074	///	///	0.0074	0.0074	///	///	0.0061	0.0051	0.0041	0.0061	0.0051	0.0041	0.0051	0.0041	///	0.0080	0.0098	0.0056	0.0056	0.0058	0.0058	0.0051	of the con
R	0.411	0.411	///	///	0.411	0.411	///	///	0.433	0.364	0.295	0.433	0.364	0.295	0.364	0.295	///	0.419	0.516	0.372	0.372	0.278	0.278	0.429	gatives o
ρ_{frp}	0.0010	0.0010	///	///	0.0010	0.0010	///	///	0.0008	0.0014	0.0028	0.0008	0.0014	0.0028	0.0014	0.0028	///	0.0016	0.0008	0.0016	0.0016	0.0071	0.0071	0.0009	ate the ne
V_c	47.9	47.9	47.9	47.9	207.3	207.3	207.3	207.3	53.2	53.2	53.2	53.2	53.2	53.2	53.2	53.2	100.8	64.8	64.8	64.8	64.8	80.1	80.1	80.1	nk indic
$\omega V_{\rm n-ACI}$	109.0	109.0	120.3	120.3	431.2	431.2	507.5	507.5	61.2	79.0	98.8	133.0	150.8	170.6	187.7	187.7	192.7	82.0	64.4	95.6	115.9	216.5	216.5	180.2	ked in pi
Check Equation (17)	93.5	93.5	103.3	103.3	348.6	348.6	430.8	430.8	31.5	56.2	83.5	115.9	140.6	168.0	225.0	252.4	144.4	48.7	24.4	67.6	95.6	211.6	211.6	161.5	ne cells mai
V_{f}	39.9	39.9	49.6	49.6	116.3	116.3	198.5	198.5	31.5	56.2	83.5	31.5	56.2	83.5	56.2	83.5	66.7	48.7	24.4	67.6	95.6	166.8	166.8	116.7	ant and t
ϵ_{fe}	0.0032	0.0032	0.0040	0.0040	0.0023	0.0023	0.0040	0.0040	0.0040	0.0040	0.0030	0.0040	0.0040	0.0030	0.0040	0.0030	0.0040	0.0031	0.0031	0.0029	0.0029	0.0021	0.0021	0.0040	al stateme
K_{v}	0.179	0.179	///	///	0.130	0.130	///	///	0.355	0.289	0.213	0.355	0.289	0.213	0.289	0.213	///	0.162	0.162	0.192	0.192	0.099	0.099	0.398	ondition
$K_{_{2}}$	0.881	0.881	111	111	0.960	0.960	111	111	0.675	0.768	0.845	0.675	0.768	0.845	0.768	0.845	///	0.542	0.542	0.636	0.636	0.823	0.823	0.813	of the co
$K_{_{I}}$	1.010	1.010	///	///	1.010	1.010	///	///	0.950	0.950	0.950	0.950	0.950	0.950	0.950	0.950	///	1.183	1.183	1.183	1.183	1.334	1.334	1.334	ositives
L_e	43.0	43.0	///	///	28.8	28.8	111	///	92.3	66.0	44.1	92.3	66.0	44.1	66.0	44.1	///	57.2	57.2	45.5	45.5	22.6	22.6	52.4	ate the p
V_s	53.7	53.7	53.7	53.7	232.2	232.2	232.2	232.2	0.0	0.0	0.0	84.4	84.4	84.4	168.9	168.9	77.7	0.0	0.0	0.0	0.0	44.8	44.8	44.8	en indic
V_c	40.7	40.7	40.7	40.7	176.2	176.2	176.2	176.2	45.2	45.2	45.2	45.2	45.2	45.2	45.2	45.2	85.7	55.1	55.1	55.1	55.1	68.1	68.1	68.1	ed in gre
V_{U-exp}	154.6	159.8	236.4	250.3	563.4	559.8	871.6	881.2	102.4	120.0	121.7	282.0	255.0	267.2	309.4	297.2	207.0	154.0	94.5	108.0	110.0	137.0	141.0	146.0	lls mark
No.	281	282	283	284	285	286	287	288	289	290	291	292	293	294	295	296	297	298	299	300	301	302	303	304	The cei

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