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On the prognosis of the growth of a heterostructure from a gas phase to analyze the possibility of decreasing mismatch-induced stresses

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Abstract

An approach to decreasing mismatch-induced stress in a heterostructure by radiation processing during growth from the gas phase is introduced in this paper. Within the framework of the approach with decreasing mismatch-induced stresses, one can find the acceleration of the recombination and diffusion of radiation defects generated during radiation processing. An analytical approach for analyzing mass and heat transfer is also introduced. The approach provides the opportunity to simultaneously take into account spatial and temporal variations of mass transfer parameters. At the same time, the approach allows the nonlinearity of the considered processes to be taken into account.

Keywords: growth from gas phase; decreasing of mismatch-induced stresses; radiation processing; analytical approach for analyzing of mass and heat transfer

1. Introduction

The extensive use of multilayer structures for manufacturing solid-state electronics devices leads to the need to improve the properties of their constituent epitaxial layers. At the present time, several technological processes are used for the growth of multilayer structures: molecular beam epitaxy, epitaxy from the gas phase, and magnetron sputtering. A large number of papers have considered the growth of heterostructures (Bravo-García et al., 2015; Chakraborty et al., 2004; Gusev & Gusev, 1991; Lachin & Savelov, 2001; Li et al., 2006; Lundin et al., 2009; Mitsuhara, 1998; Vorob'ev et al., 2003; Sorokin et al., 2008; Stepanenko, 1980; Taguchia et al., 2016; Talalaev et al., 2001). It is known that heterostructures have mismatch-induced stress due to the mismatch of the crystal lattice constants. In this paper, the possibility of decreasing mismatch-induced stress due to radiation processing of the considered heterostructure during its growth in a horizontal reactor is considered (see Fig. 1). Using radiation processing during the growth of heterostructure provides the opportunity to remove the annealing of radiation defects: the growth of heterostructure from the gas phase is usually done at high temperatures. In this situation, one can obtain the acceleration of the recombination of radiation defects in the damaged region and their diffusion from the region simultaneously during the growth of the heterostructure. At the same time, an analytical approach for the analysis of mass and heat transport has

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been introduced. The approach provides the opportunity to simultaneously take into account spatial and temporal changes of the parameters of mass transfer. The approach also allows the nonlinearity of the considered processes to be taken into account.

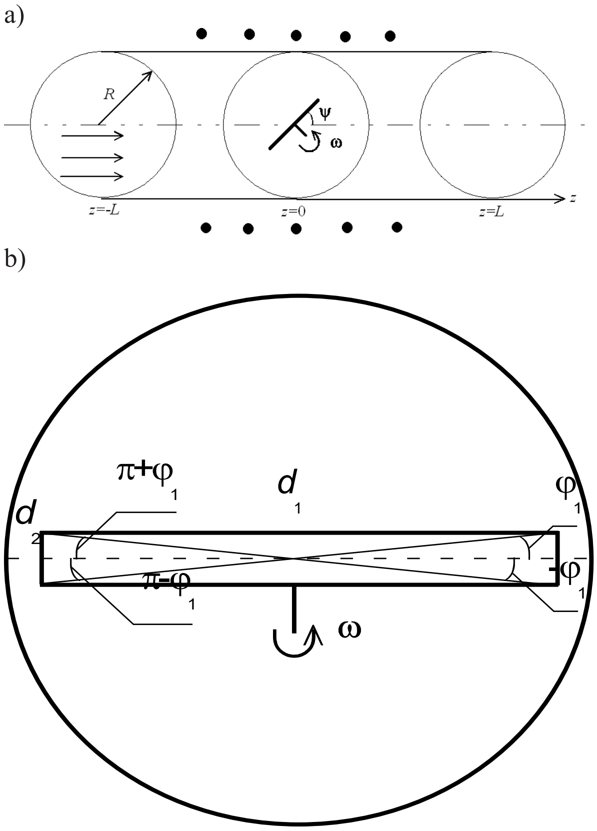

Fig. 1. Structure of the reactor for gas phase epitaxy with a sloping keeper of substrate (a) and view from the side of the keeper of substrate and approximation of the keeper by sloping lines with an angle of sloping φ_1 (b)

2. Method of solution

To obtain an appropriate solution, one should calculate the temperature distribution in space and time. The considered distribution has been calculated as a solution of the second Fourier law in the following form (Carslaw & Jaeger, 1964):

$$c\frac{\partial T(r,\varphi,z,t)}{\partial t}=p(r,\varphi,z,t)+ \\ div\left\{\lambda\cdot grad\left[T(r,\varphi,z,t)\right]- \\ \left[\vec{v}(r,\varphi,z,t)-\bar{\vec{v}}(r,\varphi,z,t)\right]\times \\ c(T)\cdot T(r,\varphi,z,t)\cdot C(r,\varphi,z,t)\right\} \quad (1)$$

where: $\vec{v}$ – speed of flow of the mixture of gases; c – parameter describing the capacity of heat; $T(r,\varphi,z,t)$ – function describing temperature distribution in space and time; $p(r,\varphi,z,t)$ – function describing power density in the considered system (substrate and keeper of the substrate); r, φ and z – the coordinates of a cylindrical system; t – the current time; $C(r,\varphi,z,t)$ – function describing the distribution of concentration of a mixture of gases in space and time; $\lambda=(\bar{v}\bar{l}c_v\rho)/3$ – conductivity of heat; $\bar{v}=\sqrt{2kT/m}$ – function describing the absolute value of the mean squared speed of molecules of gas; $\bar{l}$ – parameter describing average value of free path of molecules of a considered mixture of gases between collisions; c_v – parameter describing specific heat at a constant value of volume; ρ – parameter describing the value of gas density.

The solution of the considered boundary value problem leads to the necessity of accounting for the movement of mixture considered gases-reagents and gas-carrier with an account of the value of the concentration of the considered mixture. One can calculate the required values by the solution of a system of equations: the Navier–Stokes equation and the second law of Fourier. Also, the following approximation should be assumed: the radius of substrate keeper R is essentially larger in comparison with the thickness of near-boundary and diffusion layers. One can also consider a stream of gas as laminar. The considered assumptions leads to the need to solve the following systems of equations:

$$\frac{\partial\vec{v}}{\partial t}+(\vec{v}\cdot\nabla)\,\vec{v}=-\nabla\left(\frac{P}{\rho}\right)+\nu\Delta\vec{v} \quad (2)$$

$$\frac{\partial C(r,\varphi,z,t)}{\partial t}=div\left\{D\cdot grad\left[C(r,\varphi,z,t)\right]- \\ \left[\vec{v}(r,\varphi,z,t)-\bar{\vec{v}}\right]\cdot C(r,\varphi,z,t)\right\} \quad (3)$$

Here parameter D describes a mixture of gases of diffusion coefficient; parameter P describes pressure in the considered reactor; parameter ρ describes the density of the considered gases; parameter ν describes kinematic viscosity. Now one can consider the limiting flow regime. In this case, all molecules of the deposit material forthcoming to the substrate keeper are deposed on the considered substrate. One also consider homogenous and one dimension flow. In this situation, initial and boundary conditions could be written in the following form:

$$C(r,\varphi,-L,t)=C_0,\ C(r,\varphi,0,t)=0,\ C(r,0,z,t)= \\ C(r,2\pi,z,t),\ C(r,\varphi,z,0)=C_0\delta(z+L),\ C(0,\varphi,z,t)\neq\infty, \\ \left.\frac{\partial C(r,\varphi,z,t)}{\partial r}\right|_{r=R}=0,\ \left.\frac{\partial C(r,\varphi,z,t)}{\partial\varphi}\right|_{\phi=0}=\left.\frac{\partial C(r,\varphi,z,t)}{\partial\varphi}\right|_{\phi=2\pi} \\ T(r,\varphi,z,0)=T_r,\ v_r(r,\varphi,0,t)=0,\ T(0,\varphi,z,t)\neq\infty,$$

$$-\lambda \frac{\partial T(r,\varphi,z,t)}{\partial r}\bigg|_{r=R} = \sigma T^4 (R,\varphi,z,t)$$

$$\frac{\partial T(r,\varphi,z,t)}{\partial \varphi}\bigg|_{\varphi=0} = \frac{\partial T(r,\varphi,z,t)}{\partial \varphi}\bigg|_{\varphi=2\pi}$$

$$T(r,0,z,t) = T(r,2\pi,z,t),$$

$$-\lambda \frac{\partial T(r,\varphi,z,t)}{\partial z}\bigg|_{z=-L} = \sigma T^4 (r,\varphi,-L,t)$$

$$\frac{\partial v_r(r,\varphi,z,t)}{\partial r}\bigg|_{r=0} = 0$$

$$\frac{\partial v_\varphi(r,\varphi,z,t)}{\partial \varphi}\bigg|_{\varphi=0} = \frac{\partial v_\varphi(r,\varphi,z,t)}{\partial \varphi}\bigg|_{\varphi=2\pi} \quad (4)$$

$$\frac{\partial v_\varphi(r,\varphi,z,t)}{\partial \varphi}\bigg|_{\phi=0} = \frac{\partial v_\varphi(r,\varphi,z,t)}{\partial \varphi}\bigg|_{\varphi=2\pi}$$

$$\frac{\partial v_r(r,\varphi,z,t)}{\partial r}\bigg|_{r=R} = 0$$

$$-\lambda \frac{\partial T(r,\varphi,z,t)}{\partial z}\bigg|_{z=L} = \sigma T^4 (r,\varphi,z,t),\ v_r(r,\varphi,-L,t) = 0$$

$$v_r(r, \varphi, L, t) = 0,\ v_r(r, 0, z, t) = v_r(r, 2\pi, z, t),$$
$$v_r(0, \varphi, z, t) \neq \infty,\ v_\varphi(r, \varphi, 0, t) = \omega r,\ v_\varphi(r, \varphi, -L, t) = 0,$$
$$v_\varphi(r, \varphi, L, t) = 0,\ v_\varphi(r, 0, z, t) = v_\varphi(r, 2\pi, z, t),$$
$$v_\varphi(0, \varphi, z, t) \neq \infty,\ v_z(r, \varphi, -L, t) = V_0,\ v_z(r, \varphi, 0, t) = \overline{v}_z,$$
$$v_z(r, \varphi, L, t) = 0,\ v_z(r, 0, z, t) = v_z(r, 2\pi, z, t),$$
$$v_z(0, \varphi, z, t) \neq \infty,\ v_r(r, \varphi, z, 0) = 0,\ v_\varphi(r, \varphi, z, 0) = 0,$$
$$v_z(r, \varphi, -L, 0) = V_0$$

Here parameter σ is equal to $\sigma = 5{,}67 \cdot 10^{-8}\ W \cdot m^{-2} \cdot K^{-4}$; parameter T_r is equal to room temperature; parameter ω describes the frequency of rotation of substrate.

Equations to calculate components of the velocity of flow in the cylindrical system of coordinates take the form:

$$\frac{\partial v_r}{\partial t} = -v_r \frac{\partial v_r}{\partial r} - \frac{v_\varphi}{r}\frac{\partial v_\varphi}{\partial \varphi} - v_z \frac{\partial v_z}{\partial z} +$$
$$\nu\left(\frac{\partial^2 v_r}{\partial r^2} + \frac{\partial^2 v_r}{\partial r \partial z} - \frac{\partial^2 v_r}{\partial z^2} + \frac{\partial^2 v_z}{\partial r \partial z}\right) - \frac{\partial}{\partial r}\left(\frac{P}{\rho}\right)$$
$$\frac{\partial v_\varphi}{\partial t} = -v_r \frac{\partial v_r}{\partial r} - \frac{v_\varphi}{r}\frac{\partial v_\varphi}{\partial \varphi} - v_z \frac{\partial v_z}{\partial z} + \quad (5)$$
$$\nu\left(\frac{1}{r}\frac{\partial^2 v_r}{\partial r \partial \varphi} + \frac{2}{r^2}\frac{\partial^2 v_\varphi}{\partial \varphi^2} - \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \varphi \partial z} + \frac{\partial^2 v_\varphi}{\partial z^2}\right) - \frac{1}{r}\frac{\partial}{\partial \varphi}\left(\frac{P}{\rho}\right)$$
$$\frac{\partial v_z}{\partial t} = -v_r \frac{\partial v_r}{\partial r} - \frac{v_\varphi}{r}\frac{\partial v_\varphi}{\partial \varphi} - v_z \frac{\partial v_z}{\partial z} +$$
$$\nu\left(\frac{\partial^2 v_r}{\partial z^2} + \frac{\partial^2 v_z}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \varphi^2}\right) - \frac{\partial}{\partial z}\left(\frac{P}{\rho}\right)$$

Then the solution of the considered system of equations can be calculated by the standard method of averaging function corrections (Pankratov, 2012; Pankratov & Bulaeva, 2012, 2013; Sokolov, 1955). Using the approach for the calculation of the first-order approximation of the considered components of the speed of mixture of gases flow, one should replace the required functions on their not yet known average values $v_r \to \alpha_{1r}$, $v_\varphi \to \alpha_{1\varphi}$, $v_z \to \alpha_{1z}$ in right sides of the above equations of system (5). The replacement leads to the transformation of the above equations to the following form:

$$\frac{\partial v_{1r}}{\partial t} = -\frac{\partial}{\partial r}\left(\frac{P}{\rho}\right)$$
$$\frac{\partial v_{1\varphi}}{\partial t} = -\frac{1}{r}\frac{\partial}{\partial \varphi}\left(\frac{P}{\rho}\right) \quad (6)$$
$$\frac{\partial v_{1z}}{\partial t} = -\frac{\partial}{\partial z}\left(\frac{P}{\rho}\right)$$

The solutions of the above equations are the first-order approximations of the considered components:

$$v_{1r} = -\frac{\partial}{\partial r}\int_0^t \frac{P}{\rho}\, d\tau$$
$$v_{1\varphi} = -\frac{1}{r}\frac{\partial}{\partial \varphi}\int_0^t \frac{P}{\rho}\, d\tau \quad (7)$$
$$v_{1z} = -\frac{\partial}{\partial z}\int_0^t \frac{P}{\rho}\, d\tau$$

One can obtain approximations of components of the speed of flow of a mixture of gases with the second-order by replacement of the considered functions by the sums $v_r \to \alpha_{1r}$, $v_\varphi \to \alpha_{1\varphi}$, $v_z \to \alpha_{1z}$. Approximations for the components could be written as:

$$\frac{\partial v_{2r}}{\partial t} = \nu\left(\frac{\partial^2 v_{1r}}{\partial r^2} + \frac{\partial^2 v_{1r}}{\partial r \partial z} - \frac{\partial^2 v_{1r}}{\partial z^2} + \frac{\partial^2 v_{1z}}{\partial r \partial z}\right) - \frac{\partial}{\partial r}\left(\frac{P}{\rho}\right) -$$
$$(\alpha_{2r} + v_{1r})\frac{\partial v_{1r}}{\partial r} - \frac{(\alpha_{2\varphi} + v_{1\varphi})}{r}\frac{\partial v_{1r}}{\partial \varphi} - (\alpha_{2z} + v_{1z})\frac{\partial v_{1r}}{\partial z}$$
$$\frac{\partial v_{2\varphi}}{\partial t} = \nu\left(\frac{1}{r}\frac{\partial^2 v_{1r}}{\partial r \partial \varphi} + \frac{2}{r^2}\frac{\partial^2 v_{1\varphi}}{\partial \varphi^2} - \frac{1}{r^2}\frac{\partial^2 v_{1r}}{\partial \varphi \partial z} + \frac{\partial^2 v_{1\varphi}}{\partial z^2}\right) - \quad (8)$$
$$\frac{1}{r}\frac{\partial}{\partial \varphi}\left(\frac{P}{\rho}\right) - (\alpha_{2r} + v_{1r})\frac{\partial v_{1\varphi}}{\partial r} - \frac{(\alpha_{2\varphi} + v_{1\varphi})}{r}\frac{\partial v_{1\varphi}}{\partial \varphi}(\alpha_{2z} + v_{1z})\frac{\partial v_{1\varphi}}{\partial z}$$
$$\frac{\partial v_{2z}}{\partial t} = \nu\left(\frac{\partial^2 v_{1r}}{\partial z^2} + \frac{\partial^2 v_{1z}}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 v_{1z}}{\partial \varphi^2}\right) - \frac{\partial}{\partial z}\left(\frac{P}{\rho}\right) -$$
$$(\alpha_{2r} + v_{1r})\frac{\partial v_{1z}}{\partial r} - \frac{(\alpha_{2\varphi} + v_{1\varphi})}{r}\frac{\partial v_{1z}}{\partial \varphi} - (\alpha_{2z} + v_{1z})\frac{\partial v_{1z}}{\partial z}$$

To obtain the considered approximations, one should integrate the above equations on time. The integration leads to the following results:

$$v_{2r} = \nu \int_0^t \left(\frac{\partial^2 v_{1r}}{\partial r^2} + \frac{\partial^2 v_{1r}}{\partial r \partial z} - \frac{\partial^2 v_{1r}}{\partial z^2} + \frac{\partial^2 v_{1z}}{\partial r \partial z} \right) d\tau - \frac{\partial}{\partial r}\left(\int_0^t \frac{P}{\rho} d\tau \right) - \int_0^t \left(\alpha_{2r} + v_{1r} \right) \frac{\partial v_{1r}}{\partial r} d\tau - \int_0^t \frac{\left(\alpha_{2\varphi} + v_{1\varphi} \right)}{r} \frac{\partial v_{1r}}{\partial \varphi} d\tau - \int_0^t \left(\alpha_{2z} + v_{1z} \right) \frac{\partial v_{1r}}{\partial z} d\tau$$

$$v_{2\varphi} = \nu \int_0^t \left(\frac{1}{r} \frac{\partial^2 v_{1r}}{\partial r \partial \varphi} + \frac{2}{r^2} \frac{\partial^2 v_{1\varphi}}{\partial \varphi^2} - \frac{1}{r^2} \frac{\partial^2 v_{1r}}{\partial \varphi \partial z} + \frac{\partial^2 v_{1\varphi}}{\partial z^2} \right) d\tau - \frac{1}{r} \frac{\partial}{\partial \varphi}\left(\int_0^t \frac{P}{\rho} d\tau \right) - \int_0^t \left(\alpha_{2r} + v_{1r} \right) \frac{\partial v_{1\varphi}}{\partial r} d\tau - \int_0^t \frac{\left(\alpha_{2\varphi} + v_{1\varphi} \right)}{r} \frac{\partial v_{1\varphi}}{\partial \varphi} d\tau - \int_0^t \left(\alpha_{2z} + v_{1z} \right) \frac{\partial v_{1\varphi}}{\partial z} d\tau \quad (9)$$

$$v_{2z} = \nu \int_0^t \left(\frac{\partial^2 v_{1r}}{\partial z^2} + \frac{\partial^2 v_{1z}}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2 v_{1z}}{\partial \varphi^2} \right) d\tau - \frac{\partial}{\partial z}\left(\int_0^t \frac{P}{\rho} d\tau \right) - \int_0^t \left(\alpha_{2r} + v_{1r} \right) \frac{\partial v_{1z}}{\partial r} d\tau - \int_0^t \frac{\left(\alpha_{2\varphi} + v_{1\varphi} \right)}{r} \left(\alpha_{2\varphi} + v_{1\varphi} \right) \frac{\partial v_{1z}}{\partial \varphi} d\tau - \int_0^t \left(\alpha_{2z} + v_{1z} \right) \frac{\partial v_{1z}}{\partial z} d\tau$$

The calculation of averaged values α_{2r}, $\alpha_{2\varphi}$, α_{2z} of the considered approximation could be done by using the following standard relations:

$$\alpha_{2r} = \frac{1}{\pi \Theta R^2 L} \int_0^\Theta \int_0^R r \int_0^{2\pi} \int_{-L}^{L} \left(v_{2r} - v_{1r} \right) dz\, d\varphi\, dr\, dt$$

$$\alpha_{2\phi} = \frac{1}{\pi \Theta R^2 L} \int_0^\Theta \int_0^R r \int_0^{2\pi} \int_{-L}^{L} \left(v_{2\phi} - v_{1\phi} \right) dz\, d\varphi\, dr\, dt \quad (10)$$

$$\alpha_{2z} = \frac{1}{\pi \Theta R^2 L} \int_0^\Theta \int_0^R r \int_0^{2\pi} \int_{-L}^{L} \left(v_{2z} - v_{1z} \right) dz\, d\varphi\, dr\, dt$$

Parameter Θ describes the continuance of the technological process. Substitution of the considered approximations of the above components of speed to the above relation (10) leads to the possibility of obtaining a system of three equations to calculate the above-average values:

$$\begin{cases} A_1 \alpha_{2r} + B_1 \alpha_{2\varphi} + C_1 \alpha_{2z} = D_1 \\ A_2 \alpha_{2r} + B_2 \alpha_{2\varphi} + C_2 \alpha_{2z} = D_2 \\ A_3 \alpha_{2r} + B_3 \alpha_{2\varphi} + C_3 \alpha_{2z} = D_3 \end{cases} \quad (11)$$

Here:

$$A_1 = 1 + \int_0^\Theta (\Theta - t) \int_0^R r \int_0^{2\pi} \int_{-L}^{L} \frac{\partial v_{1r}}{\partial r} dz\, d\varphi\, dr\, dt$$

$$B_1 = \int_0^\Theta (\Theta - t) \int_0^R \int_0^{2\pi} \int_{-L}^{L} \frac{\partial v_{1r}}{\partial \varphi} dz\, d\varphi\, dr\, dt$$

$$C_1 = C_2 = \frac{\pi}{2} \Theta^2 R^2 V_0$$

$$D_1 = \nu \int_0^\Theta (\Theta - t) \int_0^R r \int_0^{2\pi} \int_{-L}^{L} \left(\frac{\partial^2 v_{1r}}{\partial r^2} + \frac{\partial^2 v_{1r}}{\partial r \partial z} - \frac{\partial^2 v_{1r}}{\partial z^2} + \frac{\partial^2 v_{1z}}{\partial r \partial z} \right) dz\, d\varphi\, dr\, dt - \int_0^\Theta (\Theta - t) \int_0^R r \int_0^{2\pi} \int_{-L}^{L} v_{1r} \frac{\partial v_{1r}}{\partial r} dz\, d\varphi\, dr\, dt - \frac{\pi}{8} \Theta^2 \times R^2 V_0^2 - \int_0^\Theta (\Theta - t) \int_0^R \int_0^{2\pi} \int_{-L}^{L} v_{1\phi} \frac{\partial v_{1r}}{\partial \varphi} dz\, d\varphi\, dr\, dt$$

$$A_2 = \int_0^\Theta (\Theta - t) \int_0^R r \int_0^{2\pi} \int_{-L}^{L} \frac{\partial v_{1r}}{\partial r} dz\, d\varphi\, dr\, dt$$

$$B_2 = \int_0^\Theta \int_0^R \int_0^{2\pi} \int_{-L}^{L} \frac{\partial v_{1r}}{\partial \phi} dz\, d\varphi\, dr \times (\Theta - t)\, dt$$

$$D_2 = \nu \int_0^\Theta (\Theta - t) \int_0^R r \int_0^{2\pi} \int_{-L}^{L} \left(\frac{1}{r} \frac{\partial^2 v_{1r}}{\partial r \partial \varphi} + \frac{2}{r^2} \frac{\partial^2 v_{1\varphi}}{\partial \varphi^2} - \frac{1}{r^2} \frac{\partial^2 v_{1r}}{\partial \varphi \partial z} + \frac{\partial^2 v_{1\varphi}}{\partial z^2} \right) dz\, d\varphi\, dr\, dt - \int_0^\Theta (\Theta - t) \int_0^R r \int_0^{2\pi} \int_{-L}^{L} v_{1r} \times \frac{\partial v_{1r}}{\partial r} dz\, d\varphi\, dr\, dt - \frac{\pi}{8} \Theta^2 R^2 V_0^2 - \int_0^\Theta (\Theta - t) \int_0^R \int_0^{2\pi} \int_{-L}^{L} v_{1\varphi} \frac{\partial v_{1r}}{\partial \varphi} dz\, d\varphi\, dr\, dt$$

$$A_3 = \int_0^\Theta (\Theta - t) \int_0^R r \int_0^{2\pi} \int_{-L}^{L} \frac{\partial v_{1z}}{\partial r} dz\, d\varphi\, dr\, dt$$

$$B_3 = \int_0^\Theta (\Theta - t) \int_0^R r \int_0^{2\pi} \int_{-L}^{L} \frac{\partial v_{1z}}{\partial \varphi} dz\, d\varphi\, dr\, dt$$

$$C_3 = 1 + \frac{\pi}{2} \Theta^2 R^2 V_0$$

$$D_3 = \nu \int_0^\Theta \int_0^R \int_0^{2\pi} \int_{-L}^{L} \left(\frac{\partial^2 v_{1r}}{\partial z^2} + \frac{\partial^2 v_{1z}}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2 v_{1z}}{\partial \varphi^2} \right) dz\, d\varphi \times r\, dr\, (\Theta - t)\, dt - \int_0^\Theta (\Theta - t) \int_0^R r \int_0^{2\pi} \int_{L} v_{1r} \frac{\partial v_{1z}}{\partial r} dz\, d\varphi\, dr\, dt - \int_0^\Theta (\Theta - t) \int_0^R r \int_0^{2\pi} \int_{-L}^{L} v_{1\varphi} \frac{\partial v_{1z}}{\partial \varphi} dz\, d\varphi\, dr\, dt - \frac{\pi}{8} \Theta^2 R^2 V_0^2$$

The solution of the considered system of equations (11) can be calculated by standard approaches for the solution of algebraic equations (Korn & Korn, 1968) and can be written in the following form:

$$\begin{aligned}\alpha_{2r} &= \Delta_r / \Delta \\ \alpha_{2\varphi} &= \Delta_\varphi / \Delta \\ \alpha_{2z} &= \Delta_z / \Delta\end{aligned} \tag{12}$$

Here:

$$\Delta = A_1\left(B_2C_3 - B_3C_2\right) - B_1\left(A_2C_3 - A_3C_2\right) + C_1\left(A_2B_3 - A_3B_2\right)$$

$$\Delta_r = D_1\left(B_2C_3 - B_3C_2\right) - B_1\left(D_2C_3 - D_3C_2\right) + C_1\left(D_2B_3 - D_3B_2\right)$$

$$\Delta_\varphi = D_1\left(B_2C_3 - B_3C_2\right) - B_1\left(D_2C_3 - D_3C_2\right) + C_1\left(D_2B_3 - D_3B_2\right)$$

$$\Delta_z = A_1\left(B_2D_3 - B_3D_2\right) - B_1\left(A_2D_3 - A_3D_2\right) + D_1\left(A_2B_3 - A_3B_2\right)$$

In the present section, components of the stream velocity of a mixture of gas-reagents and gas-carrier are calculated, and used for the growing of a heterostructure, by using the second-order approximation of the standard method of averaging function corrections. Usually, the considered approximation is good enough to make a qualitative analysis of considered processes and to obtain some quantitative results. Now equations (1) and (3) can be re-written in a cylindrical system of coordinates:

$$\begin{gathered} c\frac{\partial T(r,\varphi,z,t)}{\partial t} = \lambda\left[\frac{\partial^2 T(r,\varphi,z,t)}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 T(r,\varphi,z,t)}{\partial \varphi^2} + \right. \\ \left.\frac{\partial^2 T(r,\varphi,z,t)}{\partial z^2}\right] - c\cdot\frac{\partial}{\partial r}\left\{C(r,\varphi,z,t)\cdot T(r,\varphi,z,t)\times\right. \\ \left.\left[v_r(r,\varphi,z,t) - \bar{v}_r(r,\varphi,z,t)\right]\right\} - \frac{c}{r}\frac{\partial}{\partial\varphi}\left\{\left[v_\varphi(r,\varphi,z,t) - \right.\right. \\ \left.\left.\bar{v}_\varphi(r,\varphi,z,t)\right]\cdot C(r,\varphi,z,t)\cdot T(r,\varphi,z,t)\right\} - \\ c\cdot\frac{\partial}{\partial z}\left\{\left[v_z(r,\varphi,z,t) - \bar{v}_z(r,\varphi,z,t)\right]\cdot C(r,\varphi,z,t)\times\right. \\ \left. T(r,\varphi,z,t)\right\} + p(r,\varphi,z,t) \end{gathered} \tag{13}$$

$$\begin{gathered} \frac{\partial C(r,\varphi,z,t)}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left[rD\frac{\partial C(r,\varphi,z,t)}{\partial r}\right] + \\ \frac{1}{r^2}\frac{\partial}{\partial\varphi}\left[D\frac{\partial C(r,\varphi,z,t)}{\partial\varphi}\right] + \frac{\partial}{\partial z}\left[D\frac{\partial C(r,\varphi,z,t)}{\partial z}\right] + \\ \frac{1}{r}\frac{\partial}{\partial r}\left\{rC(r,\varphi,z,t)\cdot\left[v_r(r,\varphi,z,t) - \bar{v}_r(r,\varphi,z,t)\right]\right\} - \\ \frac{1}{r}\frac{\partial}{\partial\varphi}\left\{rC(r,\varphi,z,t)\cdot\left[v_\varphi(r,\varphi,z,t) - \bar{v}_\varphi(r,\varphi,z,t)\right]\right\} - \\ \frac{\partial}{\partial z}\left\{C(r,\varphi,z,t)\cdot\left[v_z(r,\varphi,z,t) - \bar{v}_z(r,\varphi,z,t)\right]\right\} \end{gathered} \tag{14}$$

To calculate the distribution of the temperature field and concentration of a mixture of gases in space and time, the method of an average of function corrections in the standard form is again used. To calculate the first-order approximations of the above functions, one should replace them on their average values α_{1T} and α_{1C}, which are not yet known, in the right sides of the considered equations. Now a recently considered algorithm for obtaining the considered first-order approximations of temperature and concentration of a mixture of gases can be used:

$$\begin{gathered} T_1(r,\varphi,z,t) = T_r + \int_0^t \frac{p(r,\varphi,z,\tau)}{c}d\tau - \\ \alpha_{1T}\alpha_{1C}\int_0^t \frac{\partial\left[v_r(r,\varphi,z,\tau) - \bar{v}_r(r,\varphi,z,\tau)\right]}{\partial r}d\tau - \\ \frac{\alpha_{1T}\alpha_{1C}}{r}\int_0^t \frac{\partial\left[v_\varphi(r,\varphi,z,\tau) - \bar{v}_\phi(r,\varphi,z,\tau)\right]}{\partial\varphi}d\tau - \\ \alpha_{1T}\alpha_{1C}\int_0^t \frac{\partial\left[v_z(r,\varphi,z,\tau) - \bar{v}_z(r,\varphi,z,\tau)\right]}{\partial z}d\tau \end{gathered} \tag{15}$$

$$\begin{gathered} C_1(r,\varphi,z,t) = \\ C_0 - \frac{\alpha_{1C}}{r}\int_0^t \frac{\partial\left\{r\left[v_r(r,\varphi,z,\tau) - \bar{v}_r(r,\varphi,z,\tau)\right]\right\}}{\partial r}d\tau - \\ \frac{\alpha_{1C}}{r}\int_0^t \frac{\partial\left[v_\varphi(r,\varphi,z,\tau) - \bar{v}_\varphi(r,\varphi,z,\tau)\right]}{\partial\varphi}d\tau - \\ \frac{\alpha_{1C}}{r}\int_0^t \frac{\partial\left[v_\varphi(r,\varphi,z,\tau) - \bar{v}_\varphi(r,\varphi,z,\tau)\right]}{\partial\varphi}d\tau - \\ \alpha_{1C}\int_0^t \frac{\partial\left[v_z(r,\varphi,z,\tau) - \bar{v}_z(r,\varphi,z,\tau)\right]}{\partial z}d\tau \end{gathered} \tag{16}$$

The recently considered but not yet known averaged value of temperature and components of the speed of gas flow could be calculated by using the recently considered standard relations:

$$\begin{aligned} \alpha_{1T} &= \frac{1}{\pi\Theta R^2 L}\int_0^\Theta\int_0^R r\int_0^{2\pi}\int_{-L}^{L} T_1(r,\varphi,z,\tau)\,dz\,d\varphi\,dr\,dt \\ \alpha_{1C} &= \frac{1}{\pi\Theta R^2 L}\int_0^\Theta\int_0^R r\int_0^{2\pi}\int_{-L}^{L} C_1(r,\varphi,z,\tau)\,dz\,d\varphi\,dr\,dt \end{aligned} \tag{17}$$

Substitution of the above first-order approximations into the considered relations (17) leads to the following results (Korn & Korn, 1968):

$$\begin{gathered} \alpha_{1C} = C_0 \Big/ L\cdot\left\{1 + \frac{1}{\pi\Theta RL}\int_0^\Theta(\Theta - t)\int_0^{2\pi}\int_{-L}^{L}\left[v_r(R,\varphi,z,t) - \right.\right. \\ \left.\left.\bar{v}_r(R,\varphi,z,t)\right]dz\,d\varphi\,dt + \frac{\Theta V_0}{RL}\right\} \end{gathered}$$

$$\alpha_{1T}=\left[T_r+\frac{1}{\pi\Theta R^2 L}\int_0^\Theta(\Theta-t)\int_0^R r\int_0^{2\pi}\int_{-L}^{L}\frac{p(r,\varphi,z,t)}{c}\,dzd\varphi drdt\right]\times$$
$$\left(1+\frac{C_0}{\pi\Theta}\left\{\int_0^\Theta\int_0^{2\pi}\int_{-L}^{L}\left[v_r(R,\varphi,z,\tau)-\bar{v}_r(R,\varphi,z,\tau)\right]dz\,d\varphi\times\right.\right.$$
$$\frac{(\Theta-t)}{RL^2}dt-\frac{1}{\pi\Theta R^2}\int_0^\Theta(\Theta-t)\int_0^R\int_0^{2\pi}\int_{-L}^{L}\left[v_r(r,\varphi,z,t)-\right.$$
$$\left.\left.\bar{v}_r(r,\varphi,z,t)\right]dz\,d\varphi\,dr\,dt+\frac{V_0}{2}\right\}\left\{1+\frac{1}{\pi\Theta RL}\times\right.$$
$$\left.\left.\int_0^\Theta\int_0^{2\pi}\int_{-L}^{L}\left[v_r(R,\varphi,z,\tau)-\bar{v}_r(R,\varphi,z,\tau)\right]dzd\varphi(\Theta-t)dt+\frac{\Theta V_0}{RL}\right\}^{-1}\right)$$

The second-order approximations of the considered functions are calculated by the standard method of averaging function corrections (Pankratov, 2012; Pankratov & Bulaeva, 2012a, 2013a, 2013b; Sokolov, 1955). In this case, one should replace the considered functions on the right sides of the above equations (13) and (14) on the standard sums $T\rightarrow\alpha_{2T}+T_1$, $C\ \alpha_{2C}+C_1$. After the replacement, the second-order approximations of the considered functions are obtained in the following form:

$$c\cdot T_2(r,\varphi,z,t)=\lambda\int_0^t\frac{\partial^2 T_1(r,\varphi,z,\tau)}{\partial r^2}d\tau+\lambda\frac{1}{r^2}\int_0^t\frac{\partial^2 T_1(r,\varphi,z,\tau)}{\partial\varphi^2}d\tau+$$
$$\lambda\int_0^t\frac{\partial^2 T_1(r,\varphi,z,\tau)}{\partial z^2}d\tau-c\cdot\frac{\partial}{\partial r}\int_0^t\left\{\left[\alpha_{2C}+C_1(r,\varphi,z,\tau)\right]\times\right.$$
$$\left.\left[v_r(r,\varphi,z,\tau)-\bar{v}_r(r,\varphi,z,\tau)\right]\cdot\left[\alpha_{2T}+T_1(r,\varphi,z,\tau)\right]\right\}d\tau-$$
$$\frac{c}{r}\frac{\partial}{\partial\varphi}\int_0^t\left\{\left[\alpha_{2C}+C_1(r,\varphi,z,\tau)\right]\cdot\left[v_\varphi(r,\varphi,z,\tau)-\bar{v}_\varphi(r,\varphi,z,\tau)\right]\times\right.$$
$$\left.\left[\alpha_{2T}+T_1(r,\varphi,z,\tau)\right]\right\}d\tau-\tag{18}$$
$$c\cdot\frac{\partial}{\partial z}\int_0^t\left\{\left[v_z(r,\varphi,z,\tau)-\bar{v}_z(r,\varphi,z,\tau)\right]\cdot\left[\alpha_{2C}+C_1(r,\varphi,z,\tau)\right]\times\right.$$
$$\left.\left[\alpha_{2T}+T_1(r,\varphi,z,\tau)\right]\right\}d\tau+T_r+\int_0^t p(r,\varphi,z,\tau)\,d\tau$$

$$C_2(r,\varphi,z,t)=\frac{1}{r}\frac{\partial}{\partial r}\int_0^t rD\frac{\partial C_1(r,\varphi,z,\tau)}{\partial r}d\tau+$$
$$\frac{1}{r^2}\frac{\partial}{\partial\varphi}\int_0^t D\frac{\partial C_1(r,\varphi,z,\tau)}{\partial\varphi}d\tau+\frac{\partial}{\partial z}\int_0^t D\frac{\partial C_1(r,\varphi,z,\tau)}{\partial z}d\tau-$$
$$\frac{1}{r}\frac{\partial}{\partial r}\left\{r\int_0^t\left[\alpha_{2C}+C_1(r,\varphi,z,\tau)\right]\cdot\left[v_r(r,\varphi,z,\tau)-\right.\right.\tag{19}$$
$$\left.\left.\bar{v}_r(r,\varphi,z,\tau)\,d\tau\right]\right\}-\frac{1}{r}\frac{\partial}{\partial\varphi}\int_0^t\cdot\left[v_\varphi(r,\varphi,z,\tau)-\bar{v}_\varphi(r,\varphi,z,\tau)\right]\times$$
$$\left[C_1(r,\varphi,z,\tau)+\alpha_{2C}\right]\}d\tau-\frac{\partial}{\partial z}\int_0^t\left[\alpha_{2C}+C_1(r,\varphi,z,\tau)\right]\times$$
$$\left[v_z(r,\varphi,z,\tau)-\bar{v}_z(r,\varphi,z,\tau)\right]d\tau+C_0\delta(z+L)$$

Averaged values of the considered approximations of concentration and temperature of a mixture of gases α_{2C} and α_{2T} were calculated by using these standard relations:

$$\alpha_{2T}=\frac{1}{\pi\Theta R^2 L}\int_0^\Theta\int_0^R r\int_0^{2\pi}\int_{-L}^{L}(T_2-T_1)\,dz\,d\varphi\,dr\,dt$$
$$\alpha_{2C}=\frac{1}{\pi\Theta R^2 L}\int_0^\Theta\int_0^R r\int_0^{2\pi}\int_{-L}^{L}(C_2-C_1)\,dz\,d\varphi\,dr\,dt\tag{20}$$

The substitution of both the considered approximations of concentration of a mixture of gases and temperature into Equation (20) leads to equations for the calculation of required averaged values of considered functions:

$$\alpha_{2T}=\left(\frac{\lambda\sigma}{c\pi\Theta RL}\int_0^\Theta(\Theta-t)\int_0^{2\pi}\int_{-L}^{L}T^4(R,\varphi,z,t)\,dz\,d\phi\,dt-\right.$$
$$\frac{\lambda\sigma}{c\pi\Theta R^2L}\int_0^\Theta(\Theta-t)\int_0^{2\pi}\int_{-L}^{L}T_1(R,\varphi,z,t)\,dz\,d\varphi\,dt+\frac{\lambda\sigma}{c\pi\Theta R^2L}\times$$
$$\int_0^\Theta(\Theta-t)\int_0^{2\pi}\int_{-L}^{L}T_1(0,\varphi,z,t)\,dz\,d\varphi\,dt-\frac{1}{\pi\Theta RL}\int_0^\Theta(\Theta-t)\times$$
$$\int_0^{2\pi}\int_{-L}^{L}\left\{\left[\alpha_{2C}+C_1(R,\varphi,z,t)\right]-\alpha_{1T}\alpha_{1C}\right\}\times$$
$$\left[v_r(R,\varphi,z,t)-\bar{v}_r(R,\varphi,z,t)\right]T_1(R,\varphi,z,t)\,dz\,d\varphi\,dt-$$
$$\int_0^\Theta\int_0^R r\int_0^{2\pi}\int_{-L}^{L}\left\{T_1(r,\varphi,z,t)\left[\alpha_{2C}+C_1(r,\varphi,z,t)\right]-\alpha_{1T}\alpha_{1C}\right\}\times$$
$$\left[v_r(r,\varphi,z,t)-\bar{v}_r(r,\varphi,z,t)\right]dz\,d\varphi\,dr\frac{(\Theta-t)dt}{\pi\Theta R^2L}-\frac{V_0}{\pi\Theta R^2L}\times$$
$$\int_0^\Theta(\Theta-t)\int_0^R r\int_0^{2\pi}\left[T_1(r,\varphi,L,t)(\alpha_{2C}+C_0)-\alpha_{1T}\alpha_{1C}\right]d\varphi\,dr\,dt\times$$
$$\left\{\frac{1}{\pi\Theta RL}\int_0^\Theta\int_0^{2\pi}\int_{-L}^{L}\left[\alpha_{2C}+C_1(R,\varphi,z,t)\right]\times\right.$$
$$\left[v_r(R,\varphi,z,t)-\bar{v}_r(R,\varphi,z,t)\right]dz\,d\varphi(\Theta-t)dt+1-$$
$$\frac{1}{\pi\Theta R^2L}\int_0^\Theta(\Theta-t)\int_0^R r\int_0^{2\pi}\int_{-L}^{L}\left[v_r(r,\varphi,z,t)-\bar{v}_r(r,\varphi,z,t)\right]\times$$
$$\left.\left[\alpha_{2C}+C_1(r,\varphi,z,t)\right]dz\,d\varphi\,dr\,dt+2\Theta V_0(\alpha_{2C}+C_0)L^{-1}\right\}^{-1}$$

$$\alpha_{2C}=\frac{1}{\pi\Theta R^2L}\int_0^\Theta(\Theta-t)\int_0^R r\int_0^{2\pi}D\left[\left.\frac{\partial C_1(r,\varphi,z,\tau)}{\partial z}\right|_{z=L}-\right.$$
$$\left.\left.\frac{\partial C_1(r,\varphi,z,\tau)}{\partial z}\right|_{z=-L}\right]d\varphi\,dr\,dt-$$
$$\frac{1}{\pi\Theta R^2L}\int_0^\Theta(\Theta-t)\int_0^{2\pi}\int_{-L}^{L}\left\{r\left[\alpha_{2C}-\alpha_{1C}+C_1(R,\varphi,z,\tau)\right]\times\right.$$
$$\left.\left[v_r(R,\varphi,z,\tau)-\bar{v}_r(R,\varphi,z,\tau)\right]\right\}dz\,d\varphi\,dt-$$
$$\frac{V_0}{\pi\Theta R^2L}\int_0^\Theta(\Theta-t)\int_0^R r\int_0^{2\pi}(\alpha_{2C}-\alpha_{1C}+C_0)\,dz\,d\varphi\,dr\,dt$$

Now a spatio-temporal distributions of concentrations of radiation defects can be determined by solving the following system of equations (Fahey et al., 1989; Landau & Lifshitz, 2001; Zhang & Bower, 1999):

$$\frac{\partial I(r,\varphi,z,t)}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}\left[r D_I\frac{\partial I(r,\varphi,z,t)}{\partial r}\right]-\frac{1}{r}\frac{\partial}{\partial r}\left[r I(r,\varphi,z,t)\, v_r(r,\varphi,z,t)\right]+\frac{1}{r^2}\frac{\partial}{\partial\varphi}\left[D_I\frac{\partial I(r,\varphi,z,t)}{\partial\varphi}\right]-\frac{1}{r}\frac{\partial}{\partial\varphi}\left[I(r,\varphi,z,t)v_\varphi(r,\varphi,z,t)\right]+\frac{\partial}{\partial z}\left[D_I\frac{\partial I(r,\varphi,z,t)}{\partial z}\right]-\frac{\partial}{\partial z}\left[I(r,\varphi,z,t)v_z(r,\varphi,z,t)\right]+\frac{\Omega}{r}\frac{\partial}{\partial r}\left[r\frac{D_{IS}}{kT}\nabla_S\mu_1(r,\varphi,z,t)\int_0^L I(r,\varphi,W,t)\,dW\right]+\frac{\Omega}{r}\frac{\partial}{\partial\varphi}\left[\frac{D_{IS}}{kT}\nabla_S\mu_1(r,\varphi,z,t)\int_0^L I(r,\varphi,W,t)\,dW\right]-k_{I,I}(r,\varphi,z,T)I^2(r,\varphi,z,t)-k_{I,V}(r,\varphi,z,T)I(r,\varphi,z,t)\,V(r,\varphi,z,t)$$

$$\frac{\partial V(r,\varphi,z,t)}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}\left[r D_V\frac{\partial V(r,\varphi,z,t)}{\partial r}\right]-\frac{1}{r}\frac{\partial}{\partial r}\left[r V(r,\varphi,z,t)v_r(r,\varphi,z,t)\right]+\frac{1}{r^2}\frac{\partial}{\partial\varphi}\left[D_V\frac{\partial V(r,\varphi,z,t)}{\partial\varphi}\right]-\frac{1}{r}\frac{\partial}{\partial\varphi}\left[V(r,\varphi,z,t)v_\varphi(r,\varphi,z,t)\right]+\frac{\partial}{\partial z}\left[D_V\frac{\partial V(r,\varphi,z,t)}{\partial z}\right]-\frac{\partial}{\partial z}\left[V(r,\varphi,z,t)v_z(r,\varphi,z,t)\right]+\frac{\Omega}{r}\frac{\partial}{\partial r}\left[r\frac{D_{VS}}{kT}\nabla_S\mu_1(r,\varphi,z,t)\int_0^L V(r,\varphi,W,t)\,dW\right]+\frac{\Omega}{r}\frac{\partial}{\partial\varphi}\left[\frac{D_{VS}}{kT}\nabla_S\mu_1(r,\varphi,z,t)\int_0^L V(r,\varphi,W,t)\,dW\right]-k_{V,V}(r,\varphi,z,T)V^2(r,\varphi,z,t)-k_{I,V}(r,\varphi,z,T)I(r,\varphi,z,t)\,V(r,\varphi,z,t) \quad (21)$$

with boundary and initial conditions:

$I(r, \varphi, z, 0) = f_I(r, \varphi, z)$, $I(0, \varphi, z, t) \neq \infty$, $V(r, \varphi, z, 0) = f_I(r, \varphi, z)$, $V(0, \varphi,z, t) \neq \infty$, $I(r, -\varphi_1, z, t) = I(r, \varphi_1, z, t) = I(r, \pi - \varphi_1, z, t) = I(r, \pi + \varphi_1, z, t)$, $V(r, -\varphi_1, z, t) = V(r, \varphi_1, z, t) = V(r, \pi - \varphi_1, z, t) = V(r, \pi + \varphi_1, z, t)$

$$\left.\frac{\partial I(r,\varphi,z,t)}{\partial r}\right|_{r=R}=0,\quad \left.\frac{\partial I(r,\varphi,z,t)}{\partial z}\right|_{z=0}=0 \quad (22)$$

$$\left.\frac{\partial I(r,\varphi,z,t)}{\partial z}\right|_{z=L}=0,\quad \left.\frac{\partial V(r,\varphi,z,t)}{\partial r}\right|_{r=R}=0$$

$$\left.\frac{\partial V(r,\varphi,z,t)}{\partial z}\right|_{z=0}=0,\quad \left.\frac{\partial V(r,\varphi,z,t)}{\partial z}\right|_{z=L}=0$$

where: $I(r, \varphi, z, t)$ – the spatio-temporal distribution of concentration of radiation interstitials with equilibrium distribution V^*; $V(r, \varphi, z, t)$ – the spatio-temporal distribution of concentration of radiation vacancies with equilibrium distribution V^*; $D_I(r, \varphi, z, T)$, $D_V(r, \varphi, z, T)$, $D_{IS}(r, \varphi, z, T)$, $D_{VS}(r, \varphi, z, T)$ – the coefficients of volumetric and surficial diffusions of interstitials and vacancies, respectively; Ω – the atomic volume of dopant; $V^2(r, \varphi, z, t)$, $I^2(r, \varphi, z, t)$ – terms corresponding to generation of divacancies and diinterstitials, respectively (see, for example, Fahey et al. 1989) and appropriate references in this book); $k_{I,V}(r, \varphi, z, T)$, $k_{I,I}(r, \varphi, z, T)$, $k_{V,V}(r, \varphi, z, T)$ – the parameters of recombination of point radiation defects and generation of their complexes.

Spatio-temporal distributions of $\Phi_V(r, \varphi, z, t)$ and $\Phi_I(r, \varphi, z, t)$ can be determined by solving the following system of equations (Fahey et al. 1989; Landau & Lifshitz, 2001):

$$\frac{\partial\Phi_I(r,\varphi,z,t)}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}\left[r\cdot D_{\Phi_I}(r,\varphi,z,T)\frac{\partial\Phi_I(r,\varphi,z,t)}{\partial r}\right]+k_I(r,\varphi,z,T)I(r,\varphi,z,t)++k_{I,I}(r,\varphi,z,T)I^2(r,\varphi,z,t)+\frac{1}{r}\frac{\partial}{\partial\varphi}\left[D_{\Phi_I}(r,\varphi,z,T)\frac{\partial\Phi_I(r,\varphi,z,t)}{\partial\varphi}\right]+\frac{\partial}{\partial z}\left[D_{\Phi_I}(r,\varphi,z,T)\frac{\partial\Phi_I(r,\varphi,z,t)}{\partial z}\right]+\frac{\Omega}{r}\frac{\partial}{\partial r}\left[r\frac{D_{\Phi_I S}}{kT}\nabla_S\mu_1(r,\varphi,z,t)\int_0^L\Phi_I(r,\varphi,W,t)\,dW\right]+\frac{\Omega}{r}\frac{\partial}{\partial\varphi}\left[\frac{D_{\Phi_I S}}{kT}\nabla_S\mu_1(r,\varphi,z,t)\int_0^L\Phi_I(r,\varphi,W,t)\,dW\right]$$

$$\frac{\partial\Phi_V(r,\varphi,z,t)}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}\left[r\cdot D_{\Phi_V}(r,\varphi,z,T)\frac{\partial\Phi_V(r,\varphi,z,t)}{\partial r}\right]+k_V(r,\varphi,z,T)V(r,\varphi,z,t)+k_{V,V}(r,\varphi,z,T)V^2(r,\varphi,z,t)+\frac{1}{r}\frac{\partial}{\partial\varphi}\left[D_{\Phi_V}(r,\varphi,z,T)\frac{\partial\Phi_V(r,\varphi,z,t)}{\partial\varphi}\right]+\frac{\partial}{\partial z}\left[D_{\Phi_V}(r,\varphi,z,T)\frac{\partial\Phi_V(r,\varphi,z,t)}{\partial z}\right]+\frac{\Omega}{r}\frac{\partial}{\partial r}\left[r\frac{D_{\Phi_V S}}{kT}\nabla_S\mu_1(r,\varphi,z,t)\int_0^L\Phi_V(r,\varphi,W,t)\,dW\right]+\frac{\Omega}{r}\frac{\partial}{\partial\varphi}\left[\frac{D_{\Phi_V S}}{kT}\nabla_S\mu_1(r,\varphi,z,t)\int_0^L\Phi_V(r,\varphi,W,t)\,dW\right] \quad (23)$$

with boundary and initial conditions:

$$\left.\frac{\partial\Phi_I(r,\varphi,z,t)}{\partial r}\right|_{r=R}=0,\quad \left.\frac{\partial\Phi_I(r,\varphi,z,t)}{\partial z}\right|_{z=0}=0$$

$$\left.\frac{\partial \Phi_I(r,\varphi,z,t)}{\partial z}\right|_{z=L}=0,\ \left.\frac{\partial \Phi_V(r,\varphi,z,t)}{\partial r}\right|_{r=R}=0$$

$$\left.\frac{\partial \Phi_V(r,\varphi,z,t)}{\partial z}\right|_{z=0}=0,\ \left.\frac{\partial \Phi_V(r,\varphi,z,t)}{\partial z}\right|_{z=L}=0 \quad (24)$$

$\Phi_V(r, \varphi, z, 0) = 0$, $\Phi_V(0, \varphi, z, t) \neq \infty$, $\Phi_I(r, \varphi, z, 0) = 0$, $\Phi_I(0, \varphi, z, t) \neq \infty$, $\Phi_I(r, -\varphi_1, z, t) = \Phi_I(r, \varphi_1, z, t) = \Phi_I(r, \pi - \varphi_1, z, t) = \Phi_I(r, \pi + \varphi_1, z, t)$, $\Phi_V(r, -\varphi_1, z, t) = \Phi_V(r, \varphi_1, z, t) = \Phi_V(r, \pi - \varphi_1, z, t) = \Phi_V(r, \pi + \varphi_1, z, t)$

Here $D_{\Phi I}(x, y, z, T)$, $D_{\Phi V}(x, y, z, T)$, $D_{\Phi IS}(x, y, z, T)$ and $D_{\Phi VS}(x, y, z, T)$ are the coefficients of volumetric and surficial diffusions of complexes of radiation defects; $k_I(x, y, z, T)$ and $k_V(x, y, z, T)$ are the parameters of decay of complexes of radiation defects. Chemical potential μ_1 in equations (21) and (23) could be determine by the following relation (Zhang & Bower, 1999):

$$\mu_1 = E(z)\Omega\sigma_{ij}\frac{u_{ij}(r,\varphi,z,t)+u_{ji}(r,\varphi,z,t)}{2} \quad (25)$$

where: $E(z)$ – the Young modulus, σ_{ij} – the stress tensor; $u_{ij} = [(\partial u_i/\partial x_j) + (\partial u_j/\partial x_i)] / 2$ – the deformation tensor; u_i, u_j – the components $u_r(r, \varphi, z, t)$, $u_\varphi(r, \varphi, z, t)$ and $u_z(r, \varphi, z, t)$ of the displacement vector; x_i, x_j – the coordinate r, φ, z.

The Equation (24) can be transform to the following form:

$$\mu_1(r,\varphi,z,t)=E(z)\frac{\Omega}{2}\left[\frac{\partial u_i(r,\varphi,z,t)}{\partial x_j}+\frac{\partial u_j(r,\varphi,z,t)}{\partial x_i}\right]\times \left\{\frac{1}{2}\left[\frac{\partial u_i(r,\varphi,z,t)}{\partial x_j}+\frac{\partial u_j(r,\varphi,z,t)}{\partial x_i}\right]- \varepsilon_0\delta_{ij}+\frac{\sigma(z)\delta_{ij}}{1-2\sigma(z)}\left[\frac{\partial u_k(r,\varphi,z,t)}{\partial x_k}-3\varepsilon_0\right]- K(z)\beta(z)\left[T(r,\varphi,z,t)-T_0\right]\delta_{ij}\right\}$$

where: σ is Poisson coefficient; $\varepsilon_0 = (a_s - a_{EL})/a_{EL}$ – the mismatch parameter; a_s, a_{EL} – lattice distances of the substrate and the epitaxial layer; K – the modulus of uniform compression; β – the coefficient of thermal expansion; T_r – the equilibrium temperature, which coincides (for our case) with room temperature.

Components of displacement vector can be obtained by the solution of the following equations (Landau & Lifshitz, 2001):

$$\rho(z)\frac{\partial^2 u_x(r,\varphi,z,t)}{\partial t^2}=\frac{1}{r}\frac{\partial\left[r\cdot\sigma_{rr}(r,\varphi,z,t)\right]}{\partial r}+ \frac{1}{r}\frac{\partial \sigma_{r\varphi}(r,\varphi,z,t)}{\partial \varphi}+\frac{\partial \sigma_{rz}(r,\varphi,z,t)}{\partial z}$$

$$\rho(z)\frac{\partial^2 u_\varphi(r,\varphi,z,t)}{\partial t^2}=\frac{1}{r}\frac{\partial\left[r\cdot\sigma_{\varphi r}(r,\varphi,z,t)\right]}{\partial r}+ \frac{1}{r}\frac{\partial \sigma_{\varphi\varphi}(r,\varphi,z,t)}{\partial y}+\frac{\partial \sigma_{\varphi z}(r,\varphi,z,t)}{\partial z}$$

$$\rho(z)\frac{\partial^2 u_z(r,\varphi,z,t)}{\partial t^2}=\frac{1}{r}\frac{\partial\left[r\cdot\sigma_{zx}(r,\varphi,z,t)\right]}{\partial r}+ \frac{1}{r}\frac{\partial \sigma_{zy}(r,\varphi,z,t)}{\partial \varphi}+\frac{\partial \sigma_{zz}(r,\varphi,z,t)}{\partial z}$$

where:

$$\sigma_{ij}=\frac{E(z)}{2\left[1+\sigma(z)\right]}\left[\frac{\partial u_i(r,\varphi,z,t)}{\partial x_j}+\frac{\partial u_j(r,\varphi,z,t)}{\partial x_i}- \frac{\delta_{ij}}{3}\frac{\partial u_k(r,\varphi,z,t)}{\partial x_k}\right]+\delta_{ij}\frac{\partial u_k(r,\varphi,z,t)}{\partial x_k}\times K(z)- \beta(z)K(z)\left[T(r,\varphi,z,t)-T_r\right]$$

$\rho(z)$ is the density of materials of heterostructure and δ_{ij} is the Kronecker symbol.

With the relation for σ_{ij} last system of equations can be written as:

$$\rho(z)\frac{\partial^2 u_r(r,\varphi,z,t)}{\partial t^2}=\frac{1}{r}\left\{K(z)+\frac{5E(z)}{6\left[1+\sigma(z)\right]}\right\}\times \frac{\partial}{\partial r}\left[r\frac{\partial u_r(r,\varphi,z,t)}{\partial x}\right]+\frac{\partial^2\left[r\cdot u_\varphi(r,\varphi,z,t)\right]}{\partial r\partial\varphi}\cdot\frac{1}{r^2}\times \left\{K(z)-\frac{E(z)}{3\left[1+\sigma(z)\right]}\right\}+\frac{E(z)}{2\left[1+\sigma(z)\right]}\left[\frac{1}{r^2}\frac{\partial^2 u_\varphi(r,\varphi,z,t)}{\partial\varphi^2}+ \frac{\partial^2 u_z(r,\varphi,z,t)}{\partial z^2}\right]+\frac{1}{r}\left\{K(z)+\frac{E(z)}{3\left[1+\sigma(z)\right]}\right\}\frac{\partial^2 u_z(r,\varphi,z,t)}{\partial r\partial z}- K(z)\beta(z)\frac{1}{r}\frac{\partial\left[r\cdot T(r,\varphi,z,t)\right]}{\partial r} \quad (26)$$

$$\rho(z)\frac{\partial^2 u_y(r,\varphi,z,t)}{\partial t^2}=\frac{E(z)}{2\left[1+\sigma(z)\right]}\left\{\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial u_\varphi(r,\varphi,z,t)}{\partial r}\right]+ \frac{\partial}{\partial r}\left[r\frac{\partial u_r(r,\varphi,z,t)}{\partial r}\right]\right\}-K(z)\frac{\beta(z)}{r}\frac{\partial\left[r\cdot T(r,\varphi,z,t)\right]}{\partial\varphi}+ \frac{\partial}{\partial z}\left\{\frac{E(z)}{2\left[1+\sigma(z)\right]}\left[\frac{\partial u_\varphi(r,\varphi,z,t)}{\partial z}+\frac{1}{r}\frac{\partial u_z(r,\varphi,z,t)}{\partial\varphi}\right]\right\}+ \frac{1}{r^2}\left\{\frac{5E(z)}{12\left[1+\sigma(z)\right]}+K(z)\right\}\frac{\partial^2 u_\varphi(r,\varphi,z,t)}{\partial\varphi^2}+ \frac{1}{r}\left\{K(z)-\frac{E(z)}{6\left[1+\sigma(z)\right]}\right\}\frac{\partial^2 u_y(r,\varphi,z,t)}{\partial\varphi\partial z}+ \frac{K(z)}{r}\frac{\partial^2\left[r\cdot u_\varphi(r,\varphi,z,t)\right]}{\partial r\partial\varphi}$$

$$\rho(z)\frac{\partial^2 u_z(r,\varphi,z,t)}{\partial t^2}=\frac{E(z)}{2[1+\sigma(z)]}\left\{\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial u_z(r,\varphi,z,t)}{\partial r}\right]+\frac{1}{r^2}\frac{\partial^2 u_z(r,\varphi,z,t)}{\partial\varphi^2}+\frac{\partial^2[r\cdot u_r(r,\varphi,z,t)]}{\partial r\partial z}+\frac{1}{r}\frac{\partial^2 u_\varphi(r,\varphi,z,t)}{\partial\varphi\partial z}\right\}+$$
$$\frac{\partial}{\partial z}\left\{K(z)\left[\frac{1}{r}\frac{\partial u_r(r,\varphi,z,t)}{\partial r}+\frac{1}{r}\frac{\partial u_\varphi(r,\varphi,z,t)}{\partial\varphi}+\frac{\partial u_r(r,\varphi,z,t)}{\partial z}\right]\right\}+\frac{1}{6}\frac{\partial}{\partial z}\left\{\frac{E(z)}{1+\sigma(z)}\left[6\frac{\partial u_z(r,\varphi,z,t)}{\partial z}-\frac{1}{r}\frac{\partial[r\cdot u_x(r,\varphi,z,t)]}{\partial x}-\frac{1}{r}\frac{\partial u_\varphi(r,\varphi,z,t)}{\partial y}-\frac{\partial u_z(r,\varphi,z,t)}{\partial z}\right]\right\}-$$
$$K(z)\beta(z)\frac{\partial T(r,\varphi,z,t)}{\partial z}$$

Conditions for the system of the above equations can be written in the form:

$$\left.\frac{\partial\vec{u}(r,\varphi,z,t)}{\partial r}\right|_{r=0}=0;\quad\left.\frac{\partial\vec{u}(r,\varphi,z,t)}{\partial r}\right|_{r=R}=0$$
$$\left.\frac{\partial\vec{u}(r,\varphi,z,t)}{\partial z}\right|_{z=0}=0;\quad\left.\frac{\partial\vec{u}(r,\varphi,z,t)}{\partial z}\right|_{z=L}=0$$
$$\vec{u}(r,0,z,t)=\vec{u}(r,2\pi,z,t);\quad\vec{u}(r,\varphi,z,0)=\vec{u}_0$$
$$\vec{u}(r,\varphi,z,\infty)=\vec{u}_0$$

The spatio-temporal distributions of concentrations of radiation defects are determined by solving the equations (20) and (22) with standard method of averaging of function corrections (Pankratov, 2012; Pankratov & Bulaeva, 2012, 2013a, 2013b; Sokolov, 1955). Previously the equations (20) and (22) were transformed to the following form with an account of initial distributions of the considered concentrations:

$$\frac{\partial I(r,\varphi,z,t)}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}\left[rD_I\frac{\partial I(r,\varphi,z,t)}{\partial r}\right]+\frac{1}{r^2}\frac{\partial}{\partial\varphi}\left[D_I\frac{\partial I(r,\varphi,z,t)}{\partial\varphi}\right]+\frac{\partial}{\partial z}\left[D_I\frac{\partial I(r,\varphi,z,t)}{\partial z}\right]-$$
$$\frac{1}{r}\frac{\partial}{\partial r}\left[rI(r,\varphi,z,t)v_r(r,\varphi,z,t)\right]-\frac{1}{r}\frac{\partial}{\partial\varphi}\left[I(r,\varphi,z,t)\times v_\varphi(r,\varphi,z,t)\right]-\frac{\partial}{\partial z}\left[I(r,\varphi,z,t)v_z(r,\varphi,z,t)\right]-$$
$$\frac{\Omega}{r}\frac{\partial}{\partial r}\left[r\frac{D_{IS}}{kT}\nabla_S\mu_1(r,\varphi,z,t)\int_0^L I(r,\varphi,W,t)dW\right]+\frac{\Omega}{r}\frac{\partial}{\partial\varphi}\left[\frac{D_{IS}}{kT}\nabla_S\mu_1(r,\varphi,z,t)\int_0^L I(r,\varphi,W,t)dW\right]-$$
$$k_{I,I}(r,\varphi,z,T)I^2(r,\varphi,z,t)-k_{I,V}(r,\varphi,z,T)I(r,\varphi,z,t)\times V(r,\varphi,z,t)+f_I(r,\varphi,z)\delta(t)\tag{27}$$

$$\frac{\partial V(r,\varphi,z,t)}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}\left[rD_V\frac{\partial V(r,\varphi,z,t)}{\partial r}\right]+\frac{1}{r^2}\frac{\partial}{\partial\varphi}\left[D_V\frac{\partial V(r,\varphi,z,t)}{\partial\varphi}\right]+\frac{\partial}{\partial z}\left[D_V\frac{\partial V(r,\varphi,z,t)}{\partial z}\right]-$$
$$\frac{1}{r}\frac{\partial}{\partial r}\left[rV(r,\varphi,z,t)v_r(r,\varphi,z,t)\right]-\frac{1}{r}\frac{\partial}{\partial\varphi}\left[V(r,\varphi,z,t)\times v_\varphi(r,\varphi,z,t)\right]-\frac{\partial}{\partial z}\left[V(r,\varphi,z,t)v_z(r,\varphi,z,t)\right]-$$
$$\frac{\Omega}{r}\frac{\partial}{\partial r}\left[r\frac{D_{IS}}{kT}\nabla_S\mu_1(r,\varphi,z,t)\int_0^L V(r,\varphi,W,t)dW\right]+\frac{\Omega}{r}\frac{\partial}{\partial\varphi}\left[\frac{D_{VS}}{kT}\nabla_S\mu_1(r,\varphi,z,t)\int_0^L V(r,\varphi,W,t)dW\right]-$$
$$k_{V,V}(r,\varphi,z,T)V^2(r,\varphi,z,t)-k_{I,V}(r,\varphi,z,T)\times I(r,\varphi,z,t)V(r,\varphi,z,t)+f_V(r,\varphi,z)\delta(t)$$

$$\frac{\partial\Phi_I(r,\varphi,z,t)}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}\left[r\cdot D_{\Phi_I}(r,\varphi,z,T)\frac{\partial\Phi_I(r,\varphi,z,t)}{\partial r}\right]+k_I(r,\varphi,z,T)I(r,\varphi,z,t)+\frac{1}{r}\frac{\partial}{\partial\varphi}\left[D_{\Phi_I}(r,\varphi,z,T)+\frac{\partial\Phi_I(r,\varphi,z,t)}{\partial\varphi}\right]+\frac{\partial}{\partial z}\left[D_{\Phi_I}(r,\varphi,z,T)\frac{\partial\Phi_I(r,\varphi,z,t)}{\partial z}\right]+$$
$$k_{I,I}(r,\varphi,z,T)I^2(r,\varphi,z,t)+\frac{\Omega}{r}\frac{\partial}{\partial r}\left[r\frac{D_{\Phi_I S}}{kT}\nabla_S\mu_1(r,\varphi,z,t)\times\int_0^L\Phi_I(r,\varphi,W,t)dW\right]+\frac{\Omega}{r}\frac{\partial}{\partial\varphi}\left[\frac{D_{\Phi IS}}{kT}\nabla_S\mu_1(r,\varphi,z,t)\times\int_0^L\Phi_I(r,\varphi,W,t)dW\right]\tag{28}$$

$$\frac{\partial\Phi_V(r,\varphi,z,t)}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}\left[r\cdot D_{\Phi_V}(r,\varphi,z,T)\frac{\partial\Phi_V(r,\varphi,z,t)}{\partial r}\right]+k_V(r,\varphi,z,T)V(r,\varphi,z,t)+\frac{1}{r}\frac{\partial}{\partial\varphi}\left[D_{\Phi_V}(r,\varphi,z,T)\times\frac{\partial\Phi_V(r,\varphi,z,t)}{\partial\varphi}\right]+\frac{\partial}{\partial z}\left[D_{\Phi_V}(r,\varphi,z,T)\frac{\partial\Phi_V(r,\varphi,z,t)}{\partial z}\right]+$$
$$k_{V,V}(r,\varphi,z,T)V^2(r,\varphi,z,t)+\frac{\Omega}{r}\frac{\partial}{\partial r}\left[r\frac{D_{\Phi_V S}}{kT}\nabla_S\mu_1(r,\varphi,z,t)\times\int_0^L\Phi_V(r,\varphi,W,t)dW\right]+\frac{\Omega}{r}\frac{\partial}{\partial\varphi}\left[\frac{D_{\Phi_{VS}}}{kT}\nabla_S\mu_1(r,\varphi,z,t)\times\int_0^L\Phi_V(r,\varphi,W,t)dW\right]$$

Further, concentrations of radiation defects are replaced on the right sides of equations (27) and (28) on their not yet known average values $\alpha_{1\rho}$. In this situation, equations for the first-order approximations of the required concentrations are obtained in the following form:

$$\frac{\partial I_1(r,\varphi,z,t)}{\partial t}=\alpha_{1I}L\frac{\Omega}{r}\frac{\partial}{\partial r}\left[r\frac{D_{IS}}{kT}\nabla_S\mu_1(r,\varphi,z,t)\right]+\alpha_{1I}L\frac{\Omega}{r}\frac{\partial}{\partial\varphi}\left[\frac{D_{IS}}{kT}\nabla_S\mu_1(r,\varphi,z,t)\right]-\frac{\alpha_{1I}}{r}\frac{\partial\left[r\cdot v_r(r,\varphi,z,t)\right]}{\partial r}-\frac{\alpha_{1I}}{r}\frac{\partial v_\varphi(r,\varphi,z,t)}{\partial\varphi}-\alpha_{1I}\frac{\partial v_z(r,\varphi,z,t)}{\partial z}-\alpha_{1I}^2k_{I,I}(r,\varphi,z,T)-\alpha_{1I}\alpha_{1V}k_{I,V}(r,\varphi,z,T)+f_I(r,\varphi,z)\,\delta(t) \quad (29)$$

$$\frac{\partial V_1(r,\varphi,z,t)}{\partial t}=\alpha_{1V}L\frac{\Omega}{r}\frac{\partial}{\partial r}\left[r\frac{D_{VS}}{kT}\nabla_S\mu_1(r,\varphi,z,t)\right]+\alpha_{1V}L\frac{\Omega}{r}\frac{\partial}{\partial\varphi}\left[\frac{D_{VS}}{kT}\nabla_S\mu_1(r,\varphi,z,t)\right]-\frac{\alpha_{1V}}{r}\frac{\partial\left[r\cdot v_r(r,\varphi,z,t)\right]}{\partial r}-\frac{\alpha_{1V}}{r}\frac{\partial v_\varphi(r,\varphi,z,t)}{\partial\varphi}-\alpha_{1V}\frac{\partial v_z(r,\varphi,z,t)}{\partial z}-\alpha_{1V}^2k_{V,V}(r,\varphi,z,T)-\alpha_{1I}\alpha_{1V}k_{I,V}(r,\varphi,z,T)+f_V(r,\varphi,z)\,\delta(t)$$

$$\frac{\partial\Phi_{1I}(r,\varphi,z,t)}{\partial t}=\alpha_{1\Phi_I}L\frac{\Omega}{r}\left\{\frac{\partial}{\partial r}\left[r\frac{D_{\Phi_I S}}{kT}\nabla_S\mu_1(r,\varphi,z,t)\right]+\frac{\Omega}{r}\frac{\partial}{\partial\varphi}\left[r\frac{D_{\Phi_I S}}{kT}\nabla_S\mu_1(r,\varphi,z,t)\right]\right\}+k_I(r,\varphi,z,T)I(r,\varphi,z,t)+k_{I,I}(r,\varphi,z,T)I^2(r,\varphi,z,t) \quad (30)$$

$$\frac{\partial\Phi_{1V}(r,\varphi,z,t)}{\partial t}=\alpha_{1\Phi_V}L\frac{\Omega}{r}\left\{\frac{\partial}{\partial r}\left[r\frac{D_{\Phi_V S}}{kT}\nabla_S\mu_1(r,\varphi,z,t)\right]+\frac{\Omega}{r}\frac{\partial}{\partial\varphi}\left[r\frac{D_{\Phi_V S}}{kT}\nabla_S\mu_1(r,\varphi,z,t)\right]\right\}+k_V(r,\varphi,z,T)V(r,\varphi,z,t)+k_{V,V}(r,\varphi,z,T)V^2(r,\varphi,z,t)$$

Integration of the left and right sides of the equations (29) and (30) on time gives a possibility to obtain relations for the above approximation in the final form:

$$I_1(r,\varphi,z,t)=\alpha_{1I}L\frac{\Omega}{r}\left\{\frac{\partial}{\partial r}\left[r\frac{D_{IS}}{kT}\nabla_S\int_0^t\mu_1(r,\varphi,z,\tau)d\tau\right]+\frac{\partial}{\partial\varphi}\left[\frac{D_{IS}}{kT}\nabla_S\int_0^t\mu_1(r,\varphi,z,\tau)d\tau\right]\right\}-\frac{\alpha_{1I}}{r}\frac{\partial}{\partial r}\left[r\cdot\int_0^t v_r(r,\varphi,z,\tau)d\tau\right]-\frac{\alpha_{1I}}{r}\frac{\partial}{\partial\varphi}\int_0^t v_\phi(r,\varphi,z,\tau)d\tau-\alpha_{1I}\frac{\partial}{\partial z}\int_0^t v_z(r,\varphi,z,\tau)d\tau-\alpha_{1I}^2\int_0^t k_{I,I}(r,\varphi,z,T)d\tau-\alpha_{1I}\alpha_{1V}\int_0^t k_{I,V}(r,\varphi,z,T)d\tau+f_I(r,\varphi,z)$$

$$V_1(r,\varphi,z,t)=\alpha_{1V}L\frac{\Omega}{r}\left\{\frac{\partial}{\partial r}\left[r\frac{D_{VS}}{kT}\nabla_S\int_0^t\mu_1(r,\varphi,z,\tau)d\tau\right]+\frac{\partial}{\partial\varphi}\left[\frac{D_{VS}}{kT}\nabla_S\int_0^t\mu_1(r,\varphi,z,\tau)d\tau\right]\right\}-\frac{\alpha_{1V}}{r}\frac{\partial}{\partial r}\left[r\cdot\int_0^t v_r(r,\varphi,z,\tau)d\tau\right]-\frac{\alpha_{1V}}{r}\frac{\partial}{\partial\varphi}\int_0^t v_\varphi(r,\varphi,z,\tau)d\tau-\alpha_{1V}\frac{\partial}{\partial z}\int_0^t v_z(r,\varphi,z,\tau)d\tau-\alpha_{1V}^2\int_0^t k_{V,V}(r,\varphi,z,T)d\tau-\alpha_{1I}\alpha_{1V}\int_0^t k_{I,V}(r,\varphi,z,T)d\tau+f_V(r,\varphi,z) \quad (31)$$

$$\Phi_{1I}(r,\varphi,z,t)=\alpha_{1\Phi_I}L\frac{\Omega}{r}\frac{\partial}{\partial r}\left[r\frac{D_{\Phi_I S}}{kT}\nabla_S\int_0^t\mu_1(r,\varphi,z,\tau)d\tau\right]+\int_0^t k_I(r,\varphi,z,T)I(r,\varphi,z,\tau)d\tau+\alpha_{1\Phi_I}L\frac{\Omega}{r}\frac{\partial}{\partial\varphi}\left[r\frac{D_{\Phi_I S}}{kT}\nabla_S\int_0^t\mu_1(r,\varphi,z,\tau)d\tau\right]+\int_0^t k_{I,I}(r,\varphi,z,T)I^2(r,\varphi,z,\tau)d\tau \quad (32)$$

$$\Phi_{1V}(r,\varphi,z,t)=\alpha_{1\Phi_V}L\frac{\Omega}{r}\frac{\partial}{\partial r}\left[r\frac{D_{\Phi_V S}}{kT}\nabla_S\int_0^t\mu_1(r,\varphi,z,\tau)d\tau\right]+\int_0^t k_V(r,\varphi,z,T)V(r,\varphi,z,\tau)d\tau+\alpha_{1\Phi_V}L\frac{\Omega}{r}\frac{\partial}{\partial\varphi}\left[r\frac{D_{\Phi_V S}}{kT}\nabla_S\int_0^t\mu_1(r,\varphi,z,\tau)d\tau\right]+\int_0^t k_{V,V}(r,\varphi,z,T)V^2(r,\varphi,z,\tau)d\tau$$

Average values of the first-order approximations of concentrations of radiation defects can be determined by the relation, analogous to (20) (Pankratov, 2012; Pankratov & Bulaeva, 2012, 2013a, 2013b; Sokolov, 1955). Substitution of the relations (31) and (32) into relation (19) gives a possibility to obtain required average values in the following form:

$$\alpha_{1I}=\sqrt{\frac{(a_3+A)^2}{4a_4^2}-4\left(B+\frac{\Theta a_3B+2\pi LR^2\Theta^2a_1}{a_4}\right)}-\frac{a_3+A}{4a_4}$$

$$\alpha_{1V}=\frac{1}{S_{IV\,00}}\left[\frac{\Theta}{\alpha_{1I}}\int_0^{2\pi}\int_0^L\int_0^R r\cdot f_\rho(r,\varphi,z)\,dr\,dz\,d\varphi-\alpha_{1I}S_{II\,00}-2\pi LR^2\Theta\right]$$

where:

$$S_{\rho\rho\,ij}=\int_0^\Theta(\Theta-t)\int_0^{2\pi}\int_0^L\int_0^R k_{\rho,\rho}(r,\varphi,z,T)I_1^i(r,\varphi,z,t)\times V_1^j(r,\varphi,z,t)\,dr\,dz\,d\varphi\,dt$$

$$a_4 = S_{II00} \times \left(S_{IV00}^2 - S_{II00} S_{VV00}\right)$$

$$a_3 = S_{IV00} S_{II00} + S_{IV00}^2 - S_{II00} S_{VV00}$$

$$a_2 = S_{IV00} S_{IV00}^2 \int_0^{2\pi}\int_0^{L}\int_0^{R} r \cdot f_\rho\left(r,\varphi,z\right) d\,r\, d\,z\, d\,\varphi +$$

$$S_{IV00} S_{IV00}^2 - S_{IV00}^2 \int_0^{2\pi}\int_0^{L}\int_0^{R} r \cdot f_\rho\left(r,\varphi,z\right) d\,r\, d\,z\, d\,\varphi +$$

$$2\, S_{II00} S_{VV00} \int_0^{2\pi}\int_0^{L}\int_0^{R} r \cdot f_\rho\left(r,\varphi,z\right) d\,r\, d\,z\, d\,\varphi - 4\pi^2 L^2\, S_{VV00} \times$$

$$R^2\Theta + 4\pi^2 L^2 R^2 \Theta\, S_{IV00}$$

$$a_1 = S_{IV00} \int_0^{2\pi}\int_0^{L}\int_0^{R} r \cdot f_\rho\left(r,\varphi,z\right) d\,r\, d\,z\, d\,\varphi$$

$$A = \sqrt{8\,y + \Theta^2 \frac{a_3^2}{a_4^2} - 4\Theta \frac{a_2}{a_4}}$$

$$a_0 = S_{VV00} \times \left[\int_0^{2\pi}\int_0^{L}\int_0^{R} r \cdot f_\rho\left(r,\varphi,z\right) d\,r\, d\,z\, d\,\varphi\right]^2$$

$$B = \frac{\Theta\, a_2}{6\, a_4} + \sqrt[3]{\sqrt{q^2 + p^3} - q} - \sqrt[3]{\sqrt{q^2 + p^3} + q}$$

$$q = \frac{\Theta^3 a_2}{24\, a_4^2} \times \left(4 a_0 - 2\pi L R \Theta \frac{a_1 a_3}{a_4}\right) -$$

$$\frac{a_0 \Theta^2}{8 a_4^2}\left(4\,\Theta\, a_2 - \Theta^2 \frac{a_3^2}{a_4}\right) - \frac{\Theta^3}{54\, a_4^3}\left(a_2^3 + 27\,\pi^2 L^2 R^2 \Theta\, a_1^2 a_4\right)$$

$$p = \left[3 a_4 \Theta^2 \left(2 a_0 a_4 - \pi L R \Theta\, a_1 a_3\right) - \Theta\, a_2 a_4\right] / 18\, a_4$$

$$\alpha_{1\Phi_I} = \frac{R_{I1}}{2\pi L R \Theta} + \frac{S_{II20}}{2\pi L R \Theta} +$$

$$\frac{1}{2\pi L R}\int_0^{2\pi}\int_0^{L}\int_0^{R} r \cdot f_\rho\left(r,\varphi,z\right) d\,r\, d\,z\, d\,\varphi$$

$$\alpha_{1\Phi_V} = \frac{R_{V1}}{2\pi L R \Theta} + \frac{S_{VV20}}{2\pi L R \Theta} +$$

$$\frac{1}{2\pi L R}\int_0^{2\pi}\int_0^{L}\int_0^{R} r \cdot f_\rho\left(r,\varphi,z\right) d\,r\, d\,z\, d\,\varphi$$

where:

$$R_{\rho\, i} = \int_0^{\Theta}\left(\Theta - t\right)\int_0^{2\pi}\int_0^{L}\int_0^{R} k_I\left(r,\varphi,z,T\right) I_1^i\left(r,\varphi,z,t\right) d\,r\, d\,z\, d\,\varphi$$

Approximations of the second and higher orders of concentrations of radiation defects are determined by the standard iterative procedure of a method of averaging of function corrections (Pankratov, 2012; Pankratov & Bulaeva, 2012, 2013a, 2013b; Sokolov, 1955). With this procedure to determine approximations of the *n*-th order of concentrations of radiation defects $I(r, \varphi, z, t)$, $V(r, \varphi, z, t)$, $\Phi_I(r, \varphi, z, t)$ and $\Phi_V(r, \varphi, z, t)$ one can replace the required concentrations in the equations (31), (32) on the following sum $\alpha_{n\rho} + \rho_{n-1}(r, \varphi, z, t)$. The replacement leads to the following transformation of the appropriate equations:

$$\frac{\partial\, I_2\left(r,\varphi,z,t\right)}{\partial\, t} = \frac{1}{r}\frac{\partial}{\partial\, r}\left[r \cdot D_I\left(r,\varphi,z,T\right)\frac{\partial\, I_1\left(r,\varphi,z,t\right)}{\partial\, r}\right] +$$

$$\frac{1}{r^2}\frac{\partial}{\partial\varphi}\left[D_I\left(r,\varphi,z,T\right)\frac{\partial\, I_1\left(r,\varphi,z,t\right)}{\partial\varphi}\right] +$$

$$\frac{\partial}{\partial\, z}\left[D_I\left(r,\varphi,z,T\right)\frac{\partial\, I_1\left(r,\varphi,z,t\right)}{\partial\, z}\right] - k_{I,V}\left(r,\varphi,z,T\right) \times$$

$$\left[\alpha_{1I} + I_1\left(r,\varphi,z,t\right)\right]\left[\alpha_{1V} + V_1\left(r,\varphi,z,t\right)\right] +$$

$$\frac{\Omega}{r}\frac{\partial}{\partial\, r}\left\{r \cdot \frac{D_{IS}}{kT}\nabla_S \mu\left(r,\varphi,z,t\right)\int_0^{L}\left[\alpha_{2I} + I_1\left(r,\varphi,W,t\right)\right] d\,W\right\} -$$

$$k_{I,I}\left(r,\varphi,z,T\right)\left[\alpha_{1I} + I_1\left(r,\varphi,z,t\right)\right]^2 +$$

$$\frac{\Omega}{r}\frac{\partial}{\partial\varphi}\left\{\frac{D_{IS}}{kT}\nabla_S \mu\left(r,\varphi,z,t\right)\int_0^{L}\left[\alpha_{2I} + I_1\left(r,\varphi,W,t\right)\right] d\,W\right\}$$

$$\frac{\partial\, V_2\left(r,\varphi,z,t\right)}{\partial\, t} = \frac{1}{r}\frac{\partial}{\partial r}\left[r \cdot D_V\left(r,\varphi,z,T\right)\frac{\partial V_1\left(r,\varphi,z,t\right)}{\partial\, r}\right] +$$

$$\frac{1}{r^2}\frac{\partial}{\partial\varphi}\left[D_V\left(r,\varphi,z,T\right)\frac{\partial V_1\left(r,\varphi,z,t\right)}{\partial\varphi}\right] + \qquad (33)$$

$$\frac{\partial}{\partial\, z}\left[D_V\left(r,\varphi,z,T\right)\frac{\partial V_1\left(r,\varphi,z,t\right)}{\partial\, z}\right] - k_{I,V}\left(r,\varphi,z,T\right) \times$$

$$\left[\alpha_{1I} + I_1\left(r,\varphi,z,t\right)\right]\left[\alpha_{1V} + V_1\left(r,\varphi,z,t\right)\right] +$$

$$\frac{\Omega}{r}\frac{\partial}{\partial\, r}\left\{r \cdot \frac{D_{VS}}{kT}\nabla_S \mu\left(r,\varphi,z,t\right)\int_0^{L}\left[\alpha_{2I} + V_1\left(r,\varphi,W,t\right)\right] d\,W\right\} -$$

$$k_{V,V}\left(r,\varphi,z,T\right)\left[\alpha_{1V} + V_1\left(r,\varphi,z,t\right)\right]^2 +$$

$$\frac{\Omega}{r}\frac{\partial}{\partial\varphi}\left\{\frac{D_{VS}}{kT}\nabla_S \mu\left(r,\varphi,z,t\right)\int_0^{L}\left[\alpha_{2V} + V_1\left(r,\varphi,W,t\right)\right] d\,W\right\}$$

$$\frac{\partial\, \Phi_{2I}\left(r,\varphi,z,t\right)}{\partial\, t} = \frac{1}{r}\frac{\partial}{\partial r}\left[r \cdot D_{\Phi_I}\left(r,\varphi,z,T\right)\frac{\partial \Phi_{1I}\left(r,\varphi,z,t\right)}{\partial\, r}\right] +$$

$$\frac{1}{r^2}\frac{\partial}{\partial\varphi}\left[D_{\Phi_I}\left(r,\varphi,z,T\right) \times \frac{\partial \Phi_{1I}\left(r,\varphi,z,t\right)}{\partial\,\varphi}\right] +$$

$$\frac{\Omega}{r}\frac{\partial}{\partial\, r}\left\{\frac{D_{\Phi_I S}}{kT}\nabla_S \mu\left(r,\varphi,z,t\right)\int_0^{L}\left[\alpha_{2\Phi_I} + \Phi_{1I}\left(r,\varphi,W,t\right)\right] d\,W\right\} +$$

$$\frac{\Omega}{r}\frac{\partial}{\partial\varphi}\left\{\frac{D_{\Phi_I S}}{kT}\nabla_S \mu\left(r,\varphi,z,t\right)\int_0^{L_z}\left[\alpha_{2\Phi_I} + \Phi_{1I}\left(r,\varphi,W,t\right)\right] d\,W\right\} +$$

$$k_{I,I}\left(r,\varphi,z,T\right) I^2\left(r,\varphi,z,t\right) +$$

$$\frac{\partial}{\partial\, z}\left[D_{\Phi_I}\left(r,\varphi,z,T\right)\frac{\partial \Phi_{1I}\left(r,\varphi,z,t\right)}{\partial\, z}\right] +$$

$$k_I\left(r,\varphi,z,T\right) I\left(r,\varphi,z,t\right) + f_{\Phi_I}\left(r,\varphi,z\right)\delta\left(t\right)$$

$$\frac{\partial \Phi_{2V}(r,\varphi,z,t)}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}\left[r\cdot D_{\Phi_V}(r,\varphi,z,T)\frac{\partial \Phi_{1V}(r,\varphi,z,t)}{\partial r}\right]+$$
$$\frac{1}{r^2}\frac{\partial}{\partial \varphi}\left[D_{\Phi_V}(r,\varphi,z,T)\times\frac{\partial \Phi_{1V}(r,\varphi,z,t)}{\partial \varphi}\right]+$$
$$\frac{\Omega}{r}\frac{\partial}{\partial r}\left\{\frac{D_{\Phi_V S}}{kT}\nabla_S\mu(r,\varphi,z,t)\int_0^L\left[\alpha_{2\Phi_V}+\Phi_{1V}(r,\varphi,W,t)\right]dW\right\}+$$
$$\frac{\Omega}{r}\frac{\partial}{\partial \varphi}\left\{\frac{D_{\Phi_V S}}{kT}\nabla_S\mu(r,\varphi,z,t)\int_0^{L_z}\left[\alpha_{2\Phi_V}+\Phi_{1V}(r,\varphi,W,t)\right]dW\right\}+$$
$$k_{V,V}(r,\varphi,z,T)V^2(r,\varphi,z,t)+$$
$$\frac{\partial}{\partial z}\left[D_{\Phi_V}(r,\varphi,z,T)\frac{\partial \Phi_{1V}(r,\varphi,z,t)}{\partial z}\right]+ \qquad (34)$$
$$k_V(r,\varphi,z,T)V(r,\varphi,z,t)+f_{\Phi_V}(r,\varphi,z)\delta(t)$$

Integration of the left and the right sides of equations (33) and (34) gives a possibility to obtain relations for the required concentrations in the final form:

$$I_2(r,\varphi,z,t)=\frac{1}{r}\frac{\partial}{\partial r}\left[r\cdot\int_0^t D_I(r,\varphi,z,T)\frac{\partial I_1(r,\varphi,z,\tau)}{\partial r}d\tau\right]+$$
$$\frac{1}{r^2}\frac{\partial}{\partial \varphi}\left[\int_0^t D_I(r,\varphi,z,T)\times\frac{\partial I_1(r,\varphi,z,\tau)}{\partial \varphi}d\tau\right]+$$
$$\frac{\partial}{\partial z}\left[\int_0^t D_I(r,\varphi,z,T)\frac{\partial I_1(r,\varphi,z,\tau)}{\partial z}d\tau\right]-$$
$$\int_0^t\left[\alpha_{1I}+I_1(r,\varphi,z,\tau)\right]^2k_{I,I}(r,\varphi,z,T)\,d\tau+$$
$$\frac{\Omega}{r}\frac{\partial}{\partial r}\left\{r\cdot\int_0^t\frac{D_{IS}}{kT}\nabla_S\mu(r,\varphi,z,\tau)\int_0^L\left[\alpha_{2I}+I_1(r,\varphi,W,\tau)\right]dW\,d\tau\right\}+$$
$$\frac{\Omega}{r}\frac{\partial}{\partial \varphi}\left\{\int_0^t\frac{D_{IS}}{kT}\nabla_S\mu(r,\varphi,z,\tau)\int_0^L\left[\alpha_{2I}+I_1(r,\varphi,W,\tau)\right]dW\,d\tau\right\}-$$
$$\int_0^t k_{I,V}(r,\varphi,z,T)\times\left[\alpha_{1V}+I_1(r,\varphi,z,\tau)\right]\left[\alpha_{1V}+V_1(r,\varphi,z,\tau)\right]d\tau$$

$$V_2(r,\varphi,z,t)=\frac{1}{r}\frac{\partial}{\partial r}\left[r\cdot\int_0^t D_V(r,\varphi,z,T)\frac{\partial V_1(r,\varphi,z,\tau)}{\partial r}d\tau\right]+$$
$$\frac{1}{r^2}\frac{\partial}{\partial \varphi}\left[\int_0^t D_V(r,\varphi,z,T)\times\frac{\partial V_1(r,\varphi,z,\tau)}{\partial \varphi}d\tau\right]+ \qquad (35)$$
$$\frac{\partial}{\partial z}\left[\int_0^t D_V(r,\varphi,z,T)\frac{\partial V_1(r,\varphi,z,\tau)}{\partial z}d\tau\right]-$$
$$\int_0^t\left[\alpha_{1V}+V_1(r,\varphi,z,\tau)\right]^2\times k_{V,V}(r,\varphi,z,T)\,d\tau+$$
$$\frac{\Omega}{r}\frac{\partial}{\partial r}\left\{r\cdot\int_0^t\frac{D_{VS}}{kT}\nabla_S\mu(r,\varphi,z,\tau)\int_0^L\left[\alpha_{2V}+V_1(r,\varphi,W,\tau)\right]dW\,d\tau\right\}+$$
$$\frac{\Omega}{r}\frac{\partial}{\partial \varphi}\left\{\int_0^t\frac{D_{VS}}{kT}\nabla_S\mu(r,\varphi,z,\tau)\int_0^L\left[\alpha_{2V}+V_1(r,\varphi,W,\tau)\right]dW\,d\tau\right\}-$$
$$\int_0^t k_{I,V}(r,\varphi,z,T)\left[\alpha_{1V}+I_1(r,\varphi,z,\tau)\right]\left[\alpha_{1V}+V_1(r,\varphi,z,\tau)\right]d\tau$$

$$\Phi_{2I}(r,\varphi,z,t)=\frac{1}{r}\frac{\partial}{\partial r}\left[r\cdot\int_0^t D_{\Phi_I}(r,\varphi,z,T)\frac{\partial \Phi_{1I}(r,\varphi,z,\tau)}{\partial r}d\tau\right]+$$
$$\frac{1}{r^2}\frac{\partial}{\partial \varphi}\left[\int_0^t D_{\Phi_I}(r,\varphi,z,T)\times\frac{\partial \Phi_{1I}(r,\varphi,z,\tau)}{\partial \varphi}d\tau\right]+$$
$$\frac{\partial}{\partial z}\left[\int_0^t D_{\Phi_I}(r,\varphi,z,T)\frac{\partial \Phi_{1I}(r,\varphi,z,\tau)}{\partial z}d\tau\right]+\frac{\Omega}{r}\frac{\partial}{\partial r}\int_0^t\frac{D_{\Phi_I S}}{kT}\times$$
$$\nabla_S\mu(r,\varphi,z,\tau)\int_0^L\left[\alpha_{2\Phi_I}+\Phi_{1I}(r,\varphi,W,\tau)\right]dW\,d\tau+$$
$$\int_0^t k_{I,I}(r,\varphi,z,T)I^2(r,\varphi,z,\tau)\,d\tau+f_{\Phi_I}(r,\varphi,z)+$$
$$\frac{\Omega}{r}\frac{\partial}{\partial \varphi}\int_0^t\frac{D_{\Phi_I S}}{kT}\nabla_S\mu(r,\varphi,z,\tau)\int_0^L\left[\alpha_{2\Phi_I}+\Phi_{1I}(r,\varphi,W,\tau)\right]dW\,d\tau+$$
$$\int_0^t k_I(r,\varphi,z,T)I(r,\varphi,z,\tau)\,d\tau \qquad (36)$$

$$\Phi_{2V}(r,\varphi,z,t)=\frac{1}{r}\frac{\partial}{\partial r}\left[r\cdot\int_0^t D_{\Phi_V}(r,\varphi,z,T)\frac{\partial \Phi_{1V}(r,\varphi,z,\tau)}{\partial r}d\tau\right]+$$
$$\frac{1}{r^2}\frac{\partial}{\partial \varphi}\left[\int_0^t D_{\Phi_V}(r,\varphi,z,T)\times\frac{\partial \Phi_{1V}(r,\varphi,z,\tau)}{\partial \varphi}d\tau\right]+$$
$$\frac{\partial}{\partial z}\left[\int_0^t D_{\Phi_V}(r,\varphi,z,T)\frac{\partial \Phi_{1V}(r,\varphi,z,\tau)}{\partial z}d\tau\right]+\frac{\Omega}{r}\frac{\partial}{\partial r}\int_0^t\frac{D_{\Phi_V S}}{kT}\times$$
$$\nabla_S\mu(r,\varphi,z,\tau)\int_0^L\left[\alpha_{2\Phi_V}+\Phi_{1V}(r,\varphi,W,\tau)\right]dW\,d\tau+$$
$$\int_0^t k_{V,V}(r,\varphi,z,T)V^2(r,\varphi,z,\tau)\,d\tau+f_{\Phi_V}(r,\varphi,z)+$$
$$\frac{\Omega}{r}\frac{\partial}{\partial \varphi}\int_0^t\frac{D_{\Phi_V S}}{kT}\nabla_S\mu(r,\varphi,z,\tau)\int_0^L\left[\alpha_{2\Phi_V}+\Phi_{1V}(r,\varphi,W,\tau)\right]dW\,d\tau+$$
$$\int_0^t k_V(r,\varphi,z,T)V(r,\varphi,z,\tau)\,d\tau$$

Average values of the second-order approximations of required approximations are obtained following standard relation (Pankratov, 2012; Pankratov & Bulaeva, 2012, 2013a, 2013b; Sokolov, 1955), which is analogous to relation (21). Substitution of the relations (35) and (36) into relation (21) gives a possibility to obtain relations for required average values $\alpha_{2\rho}$:

$$\alpha_{2C}=0,\ \alpha_{2\Phi I}=0,\ \alpha_{2\Phi V}=0$$

$$\alpha_{2V}=\sqrt{\frac{(b_3+E)^2}{4b_4^2}-4\left(F+\frac{\Theta a_3F+2\pi LR\,\Theta^2b_1}{b_4}\right)}-\frac{b_3+E}{4b_4}$$

$$\alpha_{2I}=\frac{C_V-\alpha_{2V}^2S_{VV00}-\alpha_{2V}\left(2S_{VV01}+S_{IV10}+2\pi LR\Theta\right)}{S_{IV01}+\alpha_{2V}S_{IV00}}+\frac{-S_{VV02}-S_{IV11}}{S_{IV01}+\alpha_{2V}S_{IV00}}$$

where:

$$b_4=\frac{S_{IV00}^2 S_{VV00}-S_{VV00}^2 S_{II00}}{2\pi L R^2\Theta}$$

$$b_3=\left(S_{IV01}+2S_{II10}+S_{IV01}+2\pi L R^2\Theta\right)\frac{S_{IV00}S_{VV00}}{2\pi L R^2\Theta}-\frac{S_{II00}}{2\pi L}\times\frac{S_{VV00}}{\Theta R^2}\left(2S_{VV01}+S_{IV10}+2\pi L R^2\Theta\right)+S_{IV00}^2\frac{2S_{VV01}+S_{IV10}+2\pi L R^2\Theta}{2\pi L R^2\Theta}-\frac{S_{IV00}^2 S_{IV10}}{6\pi^3 L^3 R^6\Theta^3}$$

$$b_2=S_{II00}S_{VV00}\frac{S_{VV02}+S_{IV11}+C_V}{2\pi L R^2\Theta}-\left(2\pi L R^2\Theta-2S_{VV01}+S_{IV10}\right)^2+\left(2\pi L R^2\Theta+2S_{II10}+S_{IV01}\right)\frac{S_{IV01}S_{VV00}}{2\pi L R^2\Theta}+S_{IV00}\left(2\pi L R^2\Theta+S_{IV01}+2S_{II10}+2S_{IV01}\right)\times\frac{2S_{VV01}+2\pi L R^2\Theta+S_{IV10}}{2\pi L R^2\Theta}-S_{IV00}^2\frac{C_V-S_{VV02}-S_{IV11}}{2\pi L R^2\Theta}+\frac{C_I S_{IV00}^2}{4\pi^2 L^2 R^2\Theta^2}-2\frac{S_{IV10}S_{IV00}S_{IV01}}{\pi L R^2\Theta}$$

$$b_2=S_{II00}S_{VV00}\frac{S_{VV02}+S_{IV11}+C_V}{2\pi L R^2\Theta}-\left(2\pi L R^2\Theta-2S_{VV01}+S_{IV10}\right)^2+\left(2\pi L R^2\Theta+2S_{II10}+S_{IV01}\right)\frac{S_{IV01}S_{VV00}}{2\pi L R^2\Theta}+S_{IV00}\left(2\pi L R^2\Theta+S_{IV01}+2S_{II10}+2S_{IV01}\right)\times\frac{2S_{VV01}+2\pi L R^2\Theta+S_{IV10}}{2\pi L R^2\Theta}-S_{IV00}^2\frac{C_V-S_{VV02}-S_{IV11}}{2\pi L R^2\Theta}+\frac{C_I S_{IV00}^2}{4\pi^2 L^2 R^2\Theta^2}-2\frac{S_{IV10}S_{IV00}S_{IV01}}{\pi L R^2\Theta}$$

$$b_1=S_{II00}\frac{S_{IV11}+S_{VV02}+C_V}{2\pi L R^2\Theta}\left(2S_{VV01}+S_{IV10}+2\pi L R^2\right)+S_{IV01}\times\frac{2\pi L R^2\Theta+2S_{II10}+S_{IV01}}{2\pi L R^2\Theta}\times\left(2S_{VV01}+S_{IV10}+2\pi L R^2\Theta\right)+2S_{IV01}C_I S_{IV00}-\frac{S_{IV10}S_{IV01}^2}{2\pi L R^2\Theta}-\frac{C_V-S_{VV02}-S_{IV11}}{2\pi L R^2\Theta}\times S_{IV00}\left(3S_{IV01}+2S_{II10}+2\pi L R^2\Theta\right)$$

$$b_0=S_{II00}\frac{\left(S_{IV00}+S_{VV02}\right)^2}{2\pi L R^2\Theta}-S_{IV01}\frac{C_V-S_{VV02}-S_{IV11}}{2\pi L R^2\Theta}\times\left(2S_{II10}+2\pi L R^2\Theta+S_{IV01}\right)-S_{IV01}\frac{C_V-S_{VV02}-S_{IV11}}{2\pi L R^2\Theta}\times\left(2\pi L R^2\Theta+2S_{II10}+S_{IV01}\right)+2C_I S_{IV01}^2$$

$$C_V=\alpha_{1V}^2\times S_{VV00}+\alpha_{1I}\alpha_{1V}S_{IV00}-S_{IV11}-S_{VV02}$$

$$C_I=\frac{\alpha_{1I}\alpha_{1V}S_{IV00}+\alpha_{1I}^2 S_{II00}-S_{II20}S_{II20}-S_{IV11}}{2\pi L R^2\Theta}$$

$$r=\frac{\Theta^3 b_2}{12\,b_4^2}\times\left(2b_0-\pi L R^2\Theta\frac{b_1 b_3}{b_4}\right)-b_0\frac{\Theta^2}{8b_4^2}\left(4\Theta b_2-\Theta^2\frac{b_3^2}{b_4}\right)-4\pi^2 L^2 R^4\frac{\Theta^4 b_1^2}{8b_4^2}-4\pi^2 L^2 R^4\frac{\Theta^4 b_1^2}{8b_4^2}-\frac{\Theta^3 b_2^3}{54b_4^3}$$

$$F=\frac{\Theta a_2}{6a_4}+\sqrt[3]{\sqrt{r^2+s^3}-r}-\sqrt[3]{\sqrt{r^2+s^3}+r}$$

$$E=\sqrt{8y+\Theta^2\frac{a_3^2}{a_4^2}-4\Theta\frac{a_2}{a_4}}$$

$$s=\frac{\Theta}{18b_4^2}\left[3\Theta\left(2b_0 b_4-\pi L R^2\Theta b_1 b_3\right)-b_2 b_4\right]$$

Further, solutions of equations (25), i.e., components of the displacement vector are determined. To calculate the first-order approximations of the considered components, the required functions in the right sides of the equations are replaced by their not yet known average values α_i. The substitution leads to the following result:

$$\rho(z)\frac{\partial^2 u_{1x}(r,\varphi,z,t)}{\partial t^2}=-K(z)\frac{\beta(z)}{r}\frac{\partial\left[r\cdot T(r,\varphi,z,t)\right]}{\partial r}$$

$$\rho(z)\frac{\partial^2 u_{1y}(r,\varphi,z,t)}{\partial t^2}=-K(z)\frac{\beta(z)}{r}\frac{\partial T(r,\varphi,z,t)}{\partial\varphi}$$

$$\rho(z)\frac{\partial^2 u_{1z}(r,\varphi,z,t)}{\partial t^2}=-K(z)\beta(z)\frac{\partial T(r,\varphi,z,t)}{\partial z}$$

Integration of the left and the right sides of the above relations on time t leads to the following result:

$$u_{1r}(r,\varphi,z,t)=K(z)\frac{\beta(z)}{r\cdot\rho(z)}\times\frac{\partial}{\partial r}\left[r\cdot\int_0^t\int_0^\vartheta T(r,\varphi,z,\tau)\,d\tau\,d\vartheta\right]-K(z)\frac{\beta(z)}{r\cdot\rho(z)}\frac{\partial}{\partial r}\left[r\cdot\int_0^t\int_0^\vartheta T(r,\varphi,z,\tau)\,d\tau\,d\vartheta\right]+u_{0r}$$

$$u_{1\varphi}(r,\varphi,z,t)=K(z)\frac{\beta(z)}{r\cdot\rho(z)}\frac{\partial}{\partial\varphi}\int_0^t\int_0^\vartheta T(r,\varphi,z,\tau)\,d\tau\,d\vartheta-K(z)\frac{\beta(z)}{r\cdot\rho(z)}\frac{\partial}{\partial\varphi}\int_0^\infty\int_0^\vartheta T(r,\varphi,z,\tau)\,d\tau\,d\vartheta+u_{0\varphi}$$

$$u_{1z}(r,\varphi,z,t)=K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial z}\int_0^t\int_0^\vartheta T(r,\varphi,z,\tau)\,d\tau\,d\vartheta-K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial z}\int_0^\infty\int_0^\vartheta T(r,\varphi,z,\tau)\,d\tau\,d\vartheta+u_{0z}$$

Approximations of the second and higher orders of the components of the displacement vector can be determined by the standard replacement of the required components on the following sums $\alpha_i + u_i(r, \varphi, z, t)$ (Pankratov, 2012; Pankratov & Bulaeva, 2012, 2013a, 2013b; Sokolov, 1955). The replacement leads to the following result:

$$\rho(z)\frac{\partial^2 u_{2r}(r,\varphi,z,t)}{\partial t^2}=\frac{1}{r}\left\{K(z)+\frac{5E(z)}{6[1+\sigma(z)]}\right\}\times\frac{\partial}{\partial r}\left[r\frac{\partial u_{1r}(r,\varphi,z,t)}{\partial r}\right]-\frac{1}{r^2}\left\{\frac{E(z)}{3[1+\sigma(z)]}-K(z)\right\}\times\frac{\partial^2[r\cdot u_{1\varphi}(r,\varphi,z,t)]}{\partial r\partial\varphi}+\frac{E(z)}{2[1+\sigma(z)]}\times\left[\frac{1}{r^2}\frac{\partial^2 u_{1y}(r,\varphi,z,t)}{\partial\varphi^2}+\frac{\partial^2 u_{1z}(r,\varphi,z,t)}{\partial z^2}\right]-\frac{\partial[r\cdot T_{1z}(r,\varphi,z,t)]}{\partial r}\times K(z)\beta(z)+\frac{1}{r}\left\{K(z)+\frac{E(z)}{3[1+\sigma(z)]}\right\}\frac{\partial^2[r\cdot u_{1z}(r,\varphi,z,t)]}{\partial r\partial z}$$

$$\rho(z)\frac{\partial^2 u_{2\varphi}(r,\varphi,z,t)}{\partial t^2}=\frac{E(z)}{2[1+\sigma(z)]}\left\{\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial u_{1\varphi}(r,\varphi,z,t)}{\partial r}\right]+\left[\frac{\partial^2 r\cdot u_{1r}(r,\varphi,z,t)}{\partial r\partial\varphi}\right]\right\}-K(z)\beta(z)\frac{\partial T(r,\varphi,z,t)}{\partial y}+\frac{\partial}{\partial z}\left\{\frac{E(z)}{2[1+\sigma(z)]}\left[\frac{\partial u_{1\varphi}(r,\varphi,z,t)}{\partial z}+\frac{1}{r}\frac{\partial u_{1z}(r,\varphi,z,t)}{\partial\varphi}\right]\right\}+\frac{1}{r^2}\left\{\frac{5E(z)}{12[1+\sigma(z)]}+K(z)\right\}\frac{\partial^2 u_{1\varphi}(r,\varphi,z,t)}{\partial\varphi^2}+\frac{1}{r}\left\{K(z)-\frac{E(z)}{6[1+\sigma(z)]}\right\}\frac{\partial^2 u_{1\varphi}(r,\varphi,z,t)}{\partial\varphi\partial z}+\frac{K(z)}{r}\frac{\partial^2 u_{1\varphi}(r,\varphi,z,t)}{\partial r\partial\varphi}$$

$$\rho(z)\frac{\partial^2 u_{2z}(r,\varphi,z,t)}{\partial t^2}=\frac{E(z)}{2[1+\sigma(z)]}\left\{\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial u_{1z}(r,\varphi,z,t)}{\partial r}\right]+\frac{1}{r^2}\frac{\partial^2 u_{1z}(r,\varphi,z,t)}{\partial\varphi^2}+\frac{\partial^2[r\cdot u_{1x}(r,\varphi,z,t)]}{\partial r\partial z}+\frac{1}{r}\frac{\partial^2 u_{1\varphi}(r,\varphi,z,t)}{\partial\varphi\partial z}\right\}+\frac{\partial}{\partial z}\left(\left\{\frac{1}{r}\frac{\partial[r\cdot u_{1r}(r,\varphi,z,t)]}{\partial r}+\frac{1}{r}\frac{\partial u_{1\varphi}(r,\varphi,z,t)}{\partial\varphi}+\frac{\partial u_{1r}(r,\varphi,z,t)}{\partial z}\right\}\right)+\frac{1}{6}\frac{\partial}{\partial z}\left(\left\{6\frac{\partial u_{1z}(r,\varphi,z,t)}{\partial z}-\frac{1}{r}\frac{\partial[r\cdot u_{1r}(r,\varphi,z,t)]}{\partial r}-\frac{1}{r}\frac{\partial u_{1y}(r,\varphi,z,t)}{\partial\varphi}-\frac{\partial u_{1z}(r,\varphi,z,t)}{\partial z}\right\}\times\frac{E(z)}{1+\sigma(z)}\right)-K(z)\beta(z)\frac{\partial T(r,\varphi,z,t)}{\partial z}$$

Integration of the left and right sides of the above relations on time t leads to the following result:

$$u_{2r}(r,\varphi,z,t)=\frac{1}{\rho(z)}\left\{K(z)+\frac{5E(z)}{6[1+\sigma(z)]}\right\}\frac{\partial}{\partial r}\left[r\frac{\partial}{\partial r}\int_0^t\int_0^\vartheta u_{1r}(r,\varphi,z,\tau)\,d\tau\,d\vartheta\right]+\frac{1}{\rho(z)}\times\left\{K(z)-\frac{E(z)}{3[1+\sigma(z)]}\right\}\times\frac{\partial^2}{\partial r\partial\varphi}\left[r\int_0^t\int_0^\vartheta u_{1\varphi}(r,\varphi,z,\tau)\,d\tau\,d\vartheta\right]+\frac{E(z)}{2r^2\rho(z)[1+\sigma(z)]}\times\left[\frac{\partial^2}{\partial\varphi^2}\int_0^t\int_0^\vartheta u_{1\varphi}(r,\varphi,z,\tau)\,d\tau\,d\vartheta+\frac{\partial^2}{\partial z^2}\int_0^t\int_0^\vartheta u_{1z}(r,\varphi,z,\tau)\,d\tau\,d\vartheta\right]+\left\{K(z)+\frac{E(z)}{3[1+\sigma(z)]}\right\}\times\frac{1}{\rho(z)}\frac{\partial^2}{\partial r\partial z}\left[r\int_0^t\int_0^\vartheta u_{1z}(r,\varphi,z,\tau)\,d\tau\,d\vartheta\right]-K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial r}\left[r\int_0^t\int_0^\vartheta T(r,\varphi,z,\tau)\,d\tau\,d\vartheta\right]-\frac{1}{\rho(z)}\times\left\{K(z)+\frac{5E(z)}{6[1+\sigma(z)]}\right\}\frac{\partial}{\partial r}\left[r\frac{\partial^2}{\partial r^2}\int_0^\infty\int_0^\vartheta u_{1r}(r,\varphi,z,\tau)\,d\tau\,d\vartheta\right]-\left\{K(z)-\frac{E(z)}{3[1+\sigma(z)]}\right\}\times\frac{1}{\rho(z)}\frac{\partial^2}{\partial r\partial\varphi}\left[r\int_0^\infty\int_0^\vartheta u_{1y}(r,\varphi,z,\tau)\,d\tau\,d\vartheta\right]-\frac{E(z)}{2\rho(z)}\left[\frac{1}{r^2}\frac{\partial^2}{\partial\varphi^2}\int_0^\infty\int_0^\vartheta u_{1\varphi}(r,\varphi,z,\tau)\,d\tau\,d\vartheta+\frac{\partial^2}{\partial z^2}\int_0^\infty\int_0^\vartheta u_{1z}(r,\varphi,z,\tau)\,d\tau\,d\vartheta\right]\frac{1}{1+\sigma(z)}-\frac{\partial^2}{\partial r\partial z}\left[r\int_0^\infty\int_0^\vartheta u_{1z}(r,\varphi,z,\tau)\,d\tau\,d\vartheta\right]\left\{\frac{E(z)}{3[1+\sigma(z)]}+K(z)\right\}\frac{1}{\rho(z)}+K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial r}\left[r\int_0^\infty\int_0^\vartheta T(r,\varphi,z,\tau)\,d\tau\,d\vartheta\right]+u_{0r}$$

$$
\begin{aligned}
u_{2\varphi}(r,\varphi,z,t) = &\left\{\frac{\partial}{\partial r}\left[r\frac{\partial}{\partial r}\int_0^t\int_0^\vartheta u_{1r}(r,\varphi,z,\tau)\,d\tau\,d\vartheta\right]+\frac{\partial^2}{\partial r\,\partial\varphi}\left[r\int_0^t\int_0^\vartheta u_{1r}(r,\varphi,z,\tau)\,d\tau\,d\vartheta\right]\right\}\times\frac{E(z)}{2r\rho(z)\left[1+\sigma(z)\right]}+\\
&\frac{1}{r^2\rho(z)}\left\{\frac{5E(z)}{12\left[1+\sigma(z)\right]}+K(z)\right\}\frac{\partial^2}{\partial\varphi^2}\int_0^t\int_0^\vartheta u_{1r}(r,\varphi,z,\tau)\,d\tau\,d\vartheta+\frac{K(z)}{r\rho(z)}\frac{\partial^2}{\partial r\,\partial\varphi}\left[r\int_0^t\int_0^\vartheta u_{1y}(r,\varphi,z,\tau)\,d\tau\,d\vartheta\right]-\\
&K(z)\frac{\beta(z)}{\rho(z)}\int_0^t\int_0^\vartheta T(r,\varphi,z,\tau)\,d\tau\,d\vartheta+\frac{1}{2\rho(z)}\frac{\partial}{\partial z}\left\{\frac{E(z)}{1+\sigma(z)}\left[\frac{\partial}{\partial z}\int_0^t\int_0^\vartheta u_{1\varphi}(r,\varphi,z,\tau)\,d\tau\,d\vartheta+\frac{1}{r}\frac{\partial}{\partial\varphi}\int_0^t\int_0^\vartheta u_{1z}(r,\varphi,z,\tau)\,d\tau\,d\vartheta\right]\right\}+\\
&\frac{1}{r\rho(z)}\left\{K(z)-\frac{E(z)}{6\left[1+\sigma(z)\right]}\right\}\frac{\partial^2}{\partial\varphi\,\partial z}\int_0^t\int_0^\vartheta u_{1\varphi}(r,\varphi,z,\tau)\,d\tau\,d\vartheta-\frac{E(z)}{2r\rho(z)\left[1+\sigma(z)\right]}\times\\
&\left\{\frac{\partial}{\partial r}\left[r\frac{\partial}{\partial r}\int_0^\infty\int_0^\vartheta u_{1r}(r,\varphi,z,\tau)\,d\tau\,d\vartheta\right]+\frac{\partial^2}{\partial r\,\partial\varphi}\left[r\int_0^\infty\int_0^\vartheta u_{1r}(r,\varphi,z,\tau)\,d\tau\,d\vartheta\right]\right\}-K(z)\frac{\beta(z)}{\rho(z)}\times\\
&\int_0^\infty\int_0^\vartheta T(r,\varphi,z,\tau)\,d\tau\,d\vartheta-\frac{1}{r^2\rho(z)}\frac{\partial^2}{\partial\varphi^2}\int_0^\infty\int_0^\vartheta u_{1r}(r,\varphi,z,\tau)\,d\tau\,d\vartheta\left\{K(z)+\frac{5E(z)}{12\left[1+\sigma(z)\right]}\right\}-\\
&\frac{1}{2\rho(z)}\frac{\partial}{\partial z}\left\{\frac{E(z)}{1+\sigma(z)}\left[\frac{\partial}{\partial z}\int_0^\infty\int_0^\vartheta u_{1\varphi}(r,\varphi,z,\tau)\,d\tau\,d\vartheta+\frac{1}{r}\frac{\partial}{\partial\varphi}\int_0^\infty\int_0^\vartheta u_{1z}(r,\varphi,z,\tau)\,d\tau\,d\vartheta\right]\right\}-\\
&\frac{K(z)}{r\rho(z)}\frac{\partial^2}{\partial r\,\partial\varphi}\left[r\int_0^\infty\int_0^\vartheta u_{1\varphi}(r,\varphi,z,\tau)\,d\tau\,d\vartheta\right]-\frac{1}{r\rho(z)}\frac{\partial^2}{\partial\varphi\,\partial z}\int_0^\infty\int_0^\vartheta u_{1\varphi}(r,\varphi,z,\tau)\,d\tau\,d\vartheta\times\left\{K(z)-\frac{E(z)}{6\left[1+\sigma(z)\right]}\right\}+u_{0\varphi}
\end{aligned}
$$

$$
\begin{aligned}
u_{2z}(r,\varphi,z,t) = &\frac{E(z)}{2r\rho(z)}\left\{\frac{\partial}{\partial r}\left[r\frac{\partial}{\partial r}\int_0^\infty\int_0^\vartheta u_{1z}(r,\varphi,z,\tau)\,d\tau\,d\vartheta\right]+\frac{1}{r}\frac{\partial^2}{\partial\varphi^2}\int_0^\infty\int_0^\vartheta u_{1z}(r,\varphi,z,\tau)\,d\tau\,d\vartheta+\right.\\
&\left.\frac{\partial^2}{\partial r\,\partial z}\left[r\int_0^\infty\int_0^\vartheta u_{1r}(r,\varphi,z,\tau)\,d\tau\,d\vartheta\right]+\frac{\partial^2}{\partial\varphi\,\partial z}\int_0^\infty\int_0^\vartheta u_{1y}(r,\varphi,z,\tau)\,d\tau\,d\vartheta\right\}\frac{1}{1+\sigma(z)}+\frac{1}{r\rho(z)}\times\\
&\frac{\partial}{\partial z}\left(K(z)\left\{\frac{\partial}{\partial r}\left[r\int_0^\infty\int_0^\vartheta u_{1r}(r,\varphi,z,\tau)\,d\tau\,d\vartheta\right]+\frac{1}{r}\frac{\partial}{\partial\varphi}\int_0^\infty\int_0^\vartheta u_{1r}(r,\varphi,z,\tau)\,d\tau\,d\vartheta+\frac{\partial}{\partial z}\int_0^\infty\int_0^\vartheta u_{1r}(r,\varphi,z,\tau)\,d\tau\,d\vartheta\right\}\right)+\\
&\frac{1}{6\rho(z)}\frac{\partial}{\partial z}\left(\frac{E(z)}{1+\sigma(z)}\left\{6\frac{\partial}{\partial z}\int_0^\infty\int_0^\vartheta u_{1z}(r,\varphi,z,\tau)\,d\tau\,d\vartheta-\frac{1}{r}\frac{\partial}{\partial r}\left[r\int_0^\infty\int_0^\vartheta u_{1r}(r,\varphi,z,\tau)\,d\tau\,d\vartheta\right]-\right.\right.\\
&\left.\left.\frac{1}{r}\frac{\partial}{\partial\varphi}\int_0^\infty\int_0^\vartheta u_{1\phi}(r,\varphi,z,\tau)\,d\tau\,d\vartheta-\frac{\partial}{\partial z}\int_0^\infty\int_0^\vartheta u_{1z}(r,\varphi,z,\tau)\,d\tau\,d\vartheta\right\}\right)-K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial z}\int_0^\infty\int_0^\vartheta T(r,\varphi,z,\tau)\,d\tau\,d\vartheta+u_{0z}
\end{aligned}
$$

Then concentration of radiation defects, components of displacement vector and speed of flow of a mixture of gases-reagents, as well as the temperature of growth, are calculated by the second-order approximation method of averaging of function corrections. This approximation is usually satisfactory to make qualitative analysis and to obtain some quantitative results. All of the obtained results have been checked by comparison with the results of numerical simulations.

3. Discussion

In this section, the dynamics of mass and heat transfer during the growth of films in reactors for epitaxy from the gas phase were analyzed to determine conditions to improve the properties of epitaxial layers. The main aim of this paper was to analyze the influence of radiation processing of materials on the value of mismatch-induced stress in the grown heterostructure. Figures 2 and 3 show distributions of concentrations of vacancies in the considered heterostructure (*a* is the thickness of the epitaxial layer after finishing of growth) and dependences of the component of the displacement vector u_z on the coordinate z.

It can be seen that the interstitial atoms which are generated during radiation processing leave the damaged region faster due to the larger value of the diffusion coefficient. After that, under the influence of mechanical stresses in the neighborhood of the interface between the layers of the heterostructure becomes their compression with decreasing quantity. At the same time, the value of mismatch-induced stress stresses decreases. All of the considered processes will be activated with increasing temperature of growth.

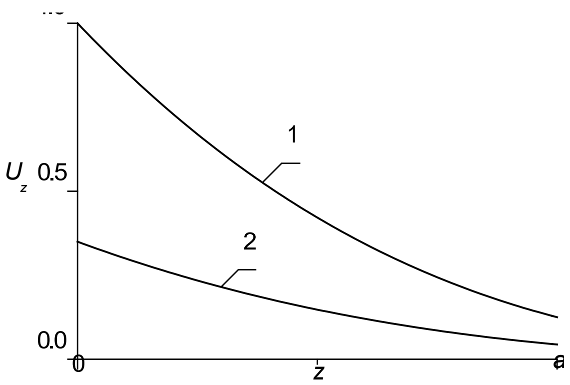

Fig. 2. Normalized dependences of the component of the displacement vector u_z on the *z* coordinate (*a* is the thickness of the epitaxial layer) for unirradiated (curve 1) and irradiated (curve 2) epitaxial layers

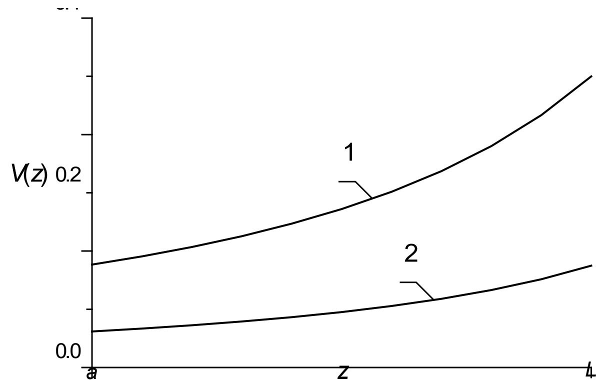

Fig. 3. Normalized dependences of concentration of vacancies on coordinate *z* in unstressed (curve 1) and stressed (curve 2) irradiated epitaxial layers

4. Conclusion

In this paper, we considered the possibility of decreasing mismatch-induced stress in a heterostructure by using radiation processing during growth from the gas phase. To decrease the considered stress, radiation processing of the heterostructure during the growth was introduced. The combination of the growth of the heterostructure and radiation processing provides the opportunity to remove the annealing of radiation defects because the growth of the heterostructure from the gas phase is usually done at high temperatures. In this situation, one can obtain the acceleration of the recombination of radiation defects in the damaged region and their diffusion from the region simultaneously during the growth of the heterostructure. An analytical approach for analyzing mass and heat transfer was also introduced. The approach provides the opportunity to simultaneously take into account spatial and temporal variations of mass transfer parameters. At the same time, the approach allows the nonlinearity of the considered processes to be taken into account.

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