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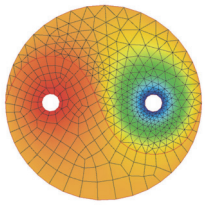

APPLICATION OF ISOGEOMETRIC ANALYSIS TO MODELING OF ELASTIC DEFORMATION IN DUAL PHASE MATERIALS BY USING STATISTICALLY SIMILAR REPRESENTATIVE VOLUME ELEMENT ON HETEROGENEOUS HARDWARE DEVICES

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Abstract

Statistically Similar Representative Volume Element (SSRVE) is a methodology applied for reduction of complexity of material microstructure representation for dual phase materials like DP steels or composites. It is based on assumption that typical RVE can be further reduced into simplified form, which joined together periodically behaves the same as its larger equivalent. SSRVE is based on Non-Uniform Rational B-Splines representation and determined by using optimization procedure, where objective function includes comparison of mechanical properties, shape coefficients and statistical characteristics. The first of these elements requires application of Finite Element Method (FEM) allowing to simulate deformation of pattern RVE and current SSRVE within elastic or elastic-plastic range. This paper presents approach allowing to replace time consuming FEM with more efficient Isogeometric Analysis (IGA). The performance of new approach is analysed and compared to conventional FEM-based methodology. Special attention is put on possibilities of IGA implementation on heterogeneous hardware devices allowing to improve computational efficiency and decrease overall power consumption.

Key words: isogeometric analysis, NURBS, representative volume element, heterogeneous hardware architectures

1. INTRODUCTION

Main techniques used in Computer Aided Design (CAD) to represent precisely complex geometries are splines and their modifications like Non-Uniform Rational B-Spline (NURBS), which were developed to improve designing process. A lot of scientific publications were written, including books and papers, focused on NURBS creation, applications and efficient algorithms for their evaluation (Piegl & Tiller, 1997; Farin, 1993; Rogers, 1995). To improve engineering analysis, Finite Element Method (FEM) was developed: the numerical technique for finding approximate solutions to boundary value problems for partial differential equations. Developments of CAD and FEM software were usually independent of each other, therefore, geometrical CAD description of an analysed objects had be converted to an Analysis Suitable Geometry (ASG) being input into FEM. The procedures of this conversion are well known and offer acceptable quality. Nevertheless, during creating of ASG some sophisticated geometrical details are only approximated and finally omitted to avoid numerical problems with convergence. The whole process of data preparation, conversion and further processing, including meshing or geometry discretization, can take up to 80% of the total time of analysis (Hughes et al., 2009). Minimization of the time spent for preparation of calculations input became the crucial part of scientific interests. The approach proposed by Hughes et al. (2005) was one of the first attempts to reorganize thinking of CAD and FEM integrated

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together as one approach to design manufacturing technologies. NURBS was proposed to be used not only to describe geometry of analyzed objects, but also to construct finite approximations for numerical analysis. The approach was called Isogeometric Analysis (IGA). It was proved that NURBS are well suited for numerical simulations of wide range of engineering problems (Bazilevs & Hughes, 2008; Bazilevs et al., 2008, Benson et al., 2010), and they offer high accuracy in comparison to conventional as well as adaptive finite elements (Wall et al., 2008). Many applications of IGA were also done for analysis of composite materials e.g.: steel-concrete beams (Lezgy-Nazargah, 2014) or in analysis of material microstructural phenomena (Casanova & Gallego, 2013; Deng et al., 2015).

More advanced analysis of materials uses multiscale modeling, where numerical simulations are performed in many scales, mainly using FEM, e.g.: FE^2 or Cellular Automata Finite Element (Madej et al., 2008). However, the main disadvantage of such approach is extremely high computing cost. Statistically Similar Representative Volume Element (SSRVE) is one of the new ideas allowing to reduce complexity of computational domain in micro scale by simplification of material microstructure (Balzani et al., 2010). The first approach to DP materials presented by Rauch et al. (2011) was designed and implemented by using FEM in microscale. In this work we apply IGA approach, which aims to increase computational efficiency in simulations of elastic deformation of material. The in-depth knowledge for deformation theories could be found in the books by Reddy (2004, 2007) and Bąk and Burczyński (2001). However, basic differences between the standard FEM and IGA will be briefly discussed in the next sections in application to the multi-scale modeling of composites deformation in combination with SSRVE.

2. IDEA OF SSRVE

The robust modeling of multi-phase materials deformations under loading conditions is very sophisticated problem and has remained as a scientific challenge for many years. One of the reasons is diversified distribution of mechanical properties characterizing microstructure. Most of the currently applied numerical methods are based on properties homogenization assuming average response of all material phases. However, in many cases homogenized model is not sufficient to get reliable response of material. Therefore, direct representation of material microstructure is applied. It is prepared on the basis of micrographs, which are analyzed and transformed by image and pattern recognition procedures to generate Digital Material Representation, DMR (Rauch & Madej, 2010). Direct usage of DMR for large areas of material can be very time consuming because of numerically intensive procedure and very fine mesh discretizing computing domain. Therefore, smaller Representative Volume Element (RVE) is used, which is determined by the smallest possible subdomain still able to represent the macroscopic behavior of the material. This approach usually is still too complex to simulate full multiscale solution even by using High Performance Computing (HPC) infrastructures. Therefore, we focus on development of SSRVE method, which allows stronger reduction of computing cost by reduction of computing domain complexity. SSRVE is usually defined in scientific literature as a significant simplification of RVE, which is characterized by the same behavior as RVE under loading conditions.

The procedure of the SSRVE creation (figure 1) is a typical optimization technique, which starts with image recognition algorithms applied on original microstructure. After the separation of grains of the microstructure image is passed to the methods of shape analysis. Shape coefficients (e.g. ellipsoid fit, roundness, curvature or volume fraction) are calculated for each separated grain. The numerical procedures for most of the coefficients are based on body and contour analysis of grouped pixels.

After image analysis, optimization procedure is started to find the grains that are statistically similar to base grains according to shape coefficients. It generation begins with creation of an initial shapes, which are formed by using NURBS. Many optimization procedures were implemented, such as genetic algorithms, Hook–Jeeves or particle swarm optimization. Location of control points forming NURBS are taken as the optimization variables. They are modified in subsequent iterations to reduce value of assumed objective function. Finally, RVE composed of maximum three grains is obtained as a result, which as a representation of material in micro scale is further attached to each Gauss point in a coarse scale. SSRVE creation for specific material is performed only once and then obtained representation can be used many times in various applications. Moreover, the computing domain is drastically simplified which highly influences computational efficiency of the whole sophisticated multiscale approach.

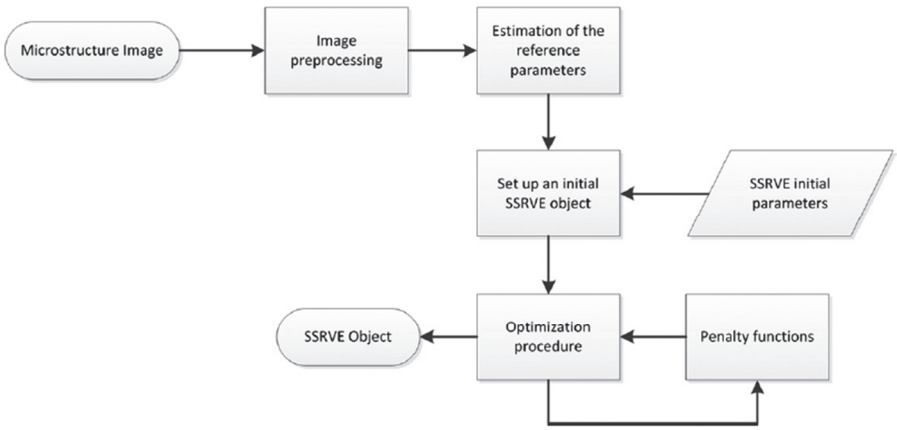

Fig. 1. *Scheme of SSRVE calculation, where the objective function is by Rauch et al. (2011).*

3. COMPARISON OF NUMERICAL APPROACHES

Obtained SSRVE with material properties assigned to each phase is used further by FEM or IGA approaches. In the first case NURBS representation is discretized and finite element mesh is created. In contrary to FEM, IGA requires pure NURBS representation. The problem description and procedures applied in both approaches are described in details in this section.

3.1. Finite Element Method 2D

Problem description

To create the formulation of the Finite Element Method for mentioned problem, elementary local equilibrium equations are used:

$$\begin{aligned} \frac{\partial \sigma_x}{\partial x}+\frac{\partial \sigma_{xy}}{\partial y}+X_\rho=0 \\ \frac{\partial \sigma_{yx}}{\partial x}+\frac{\partial \sigma_y}{\partial y}+Y_\rho=0 \end{aligned} \tag{1}$$

where X_ρ, Y_ρ – mass forces. Components of the state of stress(σ) and strain(ε) are dependent on each other by a physical relation which can be described in matrix form $\{\sigma\}=[D]\{\varepsilon\}$ or as follows:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix} = \begin{bmatrix} d_{11} & d_{12} & 0 \\ d_{21} & d_{22} & 0 \\ 0 & 0 & d_{33} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{Bmatrix},$$

$$\varepsilon_x=\frac{\partial u}{\partial x}, \varepsilon_y=\frac{\partial v}{\partial y}, \varepsilon_{xy}=\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x} \tag{2}$$

where $[D]$ is the elasticity matrix; u, v – displacements. After substitution of equations (2) to equation (1), the equilibrium equations can be expressed as follows:

$$\begin{aligned} &\frac{\partial}{\partial x}\overbrace{\left(d_{11}\frac{\partial u}{\partial x}+d_{12}\frac{\partial v}{\partial y}\right)}^{A}+\frac{\partial}{\partial y}\overbrace{\left[d_{33}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right]}^{B}+X_\rho=0 \\ &\frac{\partial}{\partial x}\overbrace{\left[d_{33}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right]}^{C}+\frac{\partial}{\partial y}\overbrace{\left(d_{12}\frac{\partial u}{\partial x}+d_{22}\frac{\partial v}{\partial y}\right)}^{D}+Y_\rho=0 \end{aligned} \tag{3}$$

Weak formulation

Two-dimensional area Ω was divided into a finite number of elements $\Omega=\bigcup_{e=1}^{N}\Omega_e$, where Ω_e are called finite elements of thickness h_e. For each element the approximated formulation was obtained by applying weighted residuals method. For this purpose we assumed that the weighted integrals (defined on volume) for each element are equal zero:

$$\begin{aligned} &\frac{\partial}{\partial x}(A)+\frac{\partial}{\partial y}(B)+X_\rho=R_x \\ &\frac{\partial}{\partial x}(C)+\frac{\partial}{\partial y}(D)+Y_\rho=R_y \end{aligned}, \quad \begin{aligned} &\int_{V^e} wR_x dV^e=0 \\ &\int_{V^e} wR_y dV^e=0 \end{aligned} \tag{4}$$

where w – weight function, V^e – volume of the e-th finite element. To obtain the weak formulation, integration by parts of weighted integrals and the Ostrogradsky's theorem were used, leading to weak formulation of boundary value problem in displacements for one element:

$$h_e \int_{\Omega^e} \left[\frac{\partial w}{\partial x}\left(d_{11}\frac{\partial u}{\partial x} + d_{12}\frac{\partial v}{\partial y} \right) + \frac{\partial w}{\partial y} d_{33}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) - wX\rho \right] dxdy - h_e \oint_{\Gamma^e} wq_{nx} ds = 0$$
$$h_e \int_{\Omega^e} \left[\frac{\partial w}{\partial x} d_{33}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) + \frac{\partial w}{\partial y}\left(d_{12}\frac{\partial u}{\partial x} + d_{22}\frac{\partial v}{\partial y} \right) - wY\rho \right] dxdy - h_e \oint_{\Gamma^e} wq_{ny} ds = 0 \quad (5)$$

where Γ^e is the set of edges along the Neumann part of the boundary. In the weak formulation the degree of differentiability of displacements has been reduced and expressions dependent on surface forces q_{nx} and q_{ny} appeared. To obtain the FEM equations for displacement boundary-value problem of linear elasticity, it was assumed that the displacement field can be approximated by polynomials as in classical FEM procedure.

3.2. IGA concept

Mesh and geometry discretization

Mesh differences between FEM and IGA come from the NURBS basis. The mesh in IGA is directly defined by NURBS parametrization. The geometry domain discretization requires a knot vector for each parametric direction i.e. $\Xi_\xi = \{\xi_1, \xi_2, ..., \xi_{n+p+1}\}$ and $\Xi_\eta = \{\eta_1, \eta_2, ..., \eta_{n+q+1}\}$, where p and q are the degree of basis functions in two independent directions. Then the new knot vectors Ξ_ξ' and Ξ_η' are constructed by removing repeated values and the result of $\Xi_\xi' \times \Xi_\eta'$ provides creation of new elements. Obtained elements are not new additional representation of the geometry as it has been observed in FEM, where description of geometry differs from finally generated mesh. In this case mesh elements with their basis functions and control points are a part of the original description of geometry. In consequence, obtained elements cover geometry precisely in 100% (figure 2). This method can reduce the error of geometric discretization, what significantly improves the computational accuracy.

Shape functions

One of the major differences between IGA and conventional FEM is that NURBS shape functions are generally not interpolating functions and can span several elements. In FEM, shape functions are only locally defined within one element, which leads to creation of local systems of equations. The degrees of freedom are called control variables instead of nodal variables.

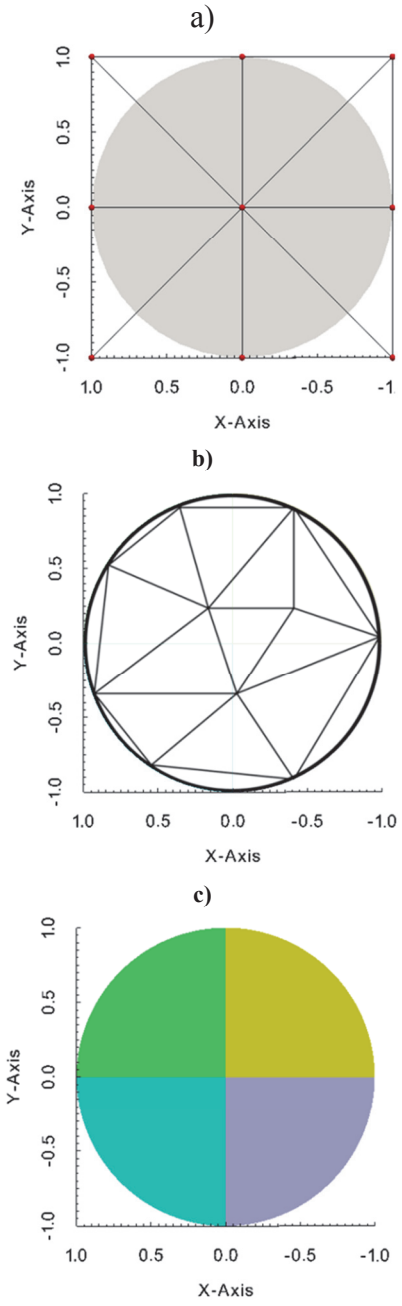

Fig. 2. *Comparison of the mesh element generated on circle geometry (a) by using conventional FEM discretization (b) and IGA approach (c).*

NURBS are built by using of B-splines, where assumed control points and basis functions affect analyzed geometry. The number of the basis functions is equal to number of control points. The result of multiplication of control point coordinates and its basis function determines contribution of this control

point. B-spline basis functions are defined recursively by using a knot vector $\Xi = \{\xi_1, \xi_2, ..., \xi_{n+p+1}\}$ (coordinates in the parametric space) in the following way:

$$p = 0: \quad N_{i,0}(\xi) = \begin{cases} 1 & \xi_i \le \xi < \xi_{i+1}, \\ 0 & otherwise. \end{cases}$$

$$p > 0: \quad N_{i,p}(\xi) = \frac{\xi - \xi_i}{\xi_{i+p} - \xi_i} N_{i,p-1}(\xi) + \frac{\xi_{i+p+1} - \xi}{\xi_{i+p+1} - \xi_{i+1}} N_{i+1,p-1}(\xi). \tag{6}$$

where p represents order of i-th basis function. NURBS are alo equipped with weights of the control points, which allow to specify the influence of each control point more precisely. Due to such solution the defined functions can be more sophisticated for curves as well as for surfaces:

$$R_i^p(\xi) = \frac{N_{i,p}(\xi) w_i}{\sum_{\hat{i}=1}^{n} N_{\hat{i},p}(\xi) w_{\hat{i}}},$$

$$R_{i,j}^{p,q}(\xi,\eta) = \frac{N_{i,p}(\xi) M_{j,q}(\eta) w_{i,j}}{\sum_{\hat{i}=1}^{n} \sum_{\hat{j}=1}^{m} N_{\hat{i},p}(\xi) M_{\hat{j},q}(\eta) w_{\hat{i},\hat{j}}} \tag{7}$$

IGA mesh refinement

There are several methods to refine 2D/3D NURBS. It is possible to insert additional knots (*h*-refinement), to increase the order of the basis functions (*p*-refinement) or to apply both methods at the same time (*k*-refinement). In general, the refinement algorithms applied in IGA are much simpler than in the case of conventional FEM. More efficient, reliable and relatively not complicated algorithms are available in the literature (Piegl & Tiller, 1997). In this paper we focused on the first two approaches, i.e. *h*- and *p*-refinement. More detailed information about all three types of refinement can be found in [0].

Knot insertion: h-refinement

H-refinement is well known in classical FEM. However, in IGA it enriches the original approach by increasing the resolution of a parameter space. Based on initial knot vector $\Xi = \{\xi_1, \xi_2, ..., \xi_{n+p+1}\}$, new extended knot vector is introduced as $\bar{\Xi} = \{\bar{\xi}_1 = \xi_1, \bar{\xi}_2, ..., \bar{\xi}_{n+m+p+1} = \xi_{n+p+1}\}$, where $\Xi \subset \bar{\Xi}$. The new $n + m$ basis functions are formed according to equation (7) for a new knot vector ($\bar{\Xi}$). Afterwards, $n + m$ control points $\bar{P} = \{\bar{P}_1, \bar{P}_2, ..., \bar{P}_{n+m}\}$ are formed as linear combination of the original control points $P = \{P_1, P_2, ..., P_n\}$ as follows:

$$\bar{P} = \alpha_i P_i + (1 - \alpha_i) P_{i-1},$$

$$\alpha_i = \begin{cases} 1, & 1 \le i \le k - p, \\ \dfrac{\bar{\varepsilon} - \varepsilon_i}{\varepsilon_{i+p} - \varepsilon_i}, & k - p + 1 \le i \le k, \\ 0, & k + 1 \le i \le n + p + 2. \end{cases} \tag{8}$$

Degree elevation: p-refinement

Degree elevation is the second method of enriching the basis functions, which is equivalent to typical *p*-refinement in FEM. The first step is based on division of original NURBS (B-spline) to obtain separate Bézier curves by knot insertion (8). In effect it raises the multiplicity of knots to the polynomial degree. The procedure of increasing basis functions is applied for each Bézier element and finally the excess knots are removed to obtain new NURBS, which keeps original continuity of the basis functions (figure 3).

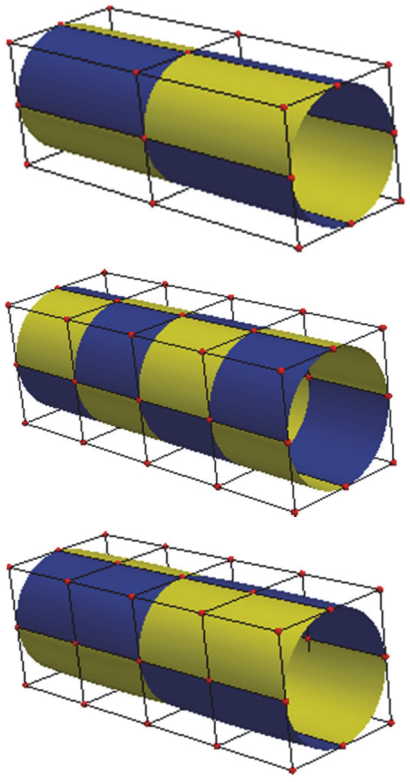

Fig. 3. *Base control net and surface division (left), base control net after knot insertion (in the middle) and base control net after degree elevation (right).*

4. COMPARISON OF SSRVE NUMERICAL SIMULATIONS

Numerical simulations of 10 percent deformation of SSRVE were selected as a benchmark numerical test. SSRVE was obtained as a result of optimization procedure for DP steel HTC600. The following material properties were assumed: E = 75000 MPa and v = 0.33 for soft ferritic matrix, E = 410000 MPa and v = 0.14 for martensitic grain inside soft matrix. The simulations were performed by using both FEM and IGA approaches, which were compared afterwards according to the quality of obtained results and number of generated elements. Two simulations were performed for each method, i.e. with coarse and refined meshes. After the first stimulation, the displacement values were collected in n points. Then the refinement algorithms were applied and the results from the same simulations were obtained for fine meshes (displacements were collected for the same n points). To calculate the differences between results obtained from both simulations for each method the following error coefficient was considered:

$$error\ coefficient = \frac{\sum_{i=1}^{n} \sqrt{(\Delta x_i)^2 + (\Delta y_i)^2}}{n} \qquad (9)$$

where n – number of test points, Δx_i – displacement difference in the x direction for point i before and after refinement, Δy_i – displacement difference in the y direction for point i before and after refinement. This procedure was repeated several times until a satisfactory small error values were obtained for both FEM and IGA concepts.

In the case of FEM simulations, four nodal elements were taken into account and commercial software Abaqus was used as a refinement tool. During matrix assembling step, the Gaussian quadrature approximation was used for integration. Numerical simulations were performed for different number of elements to investigate the quality of results and convergence. Two examples of simulation results are presented in figure 4 and figure 5 for 140 and 930 elements.

The same methodology was applied for IGA approach to analyze convergence investigating and the same simulations for different number of elements were performed. Starting number of elements was 16, which was assumed as the smallest number of elements able to cover geometry of SSRVE precisely. In opposite, in the case of FEM the calculations were started from 140 element. In the subsequent simulations the h-refinement method was used in order to increase the number of elements. The examples of results obtained for coarse and fine meshes are presented in figure 6 – figure 8 for 16, 128 and 256 element respectively.

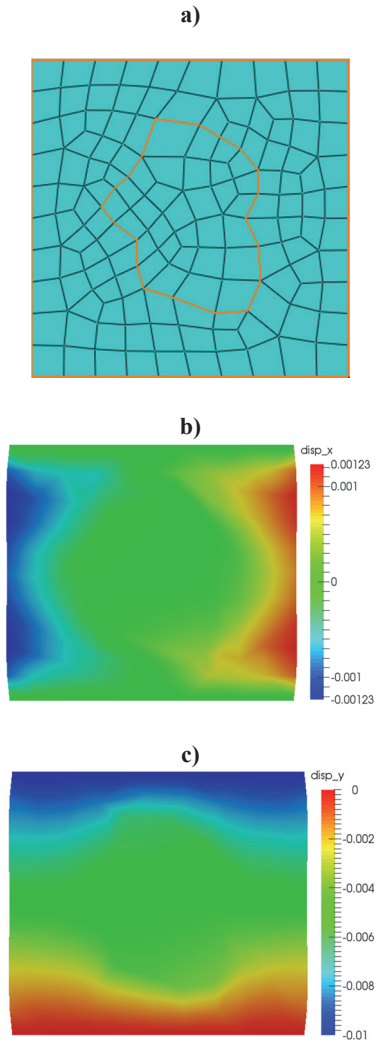

Fig. 4. *Results in form of displacements distribution for 161 nodes and 140 elements – (a) mesh for SSVE, (b) displacements in X axis, (c) displacements in Y axis.*

Based on the results from simulations performed by using both numerical approaches the error coefficient was compared (figure 9). It is possible to see that FEM needs much more elements to obtain similar error, while IGA approach convergences much faster for smaller number of elements. Moreover, the smaller number of elements causes that it is possible to obtain a significant simplification of the computational domain, which finally contributes to a smaller the final system of equations and acceleration of computing tasks.

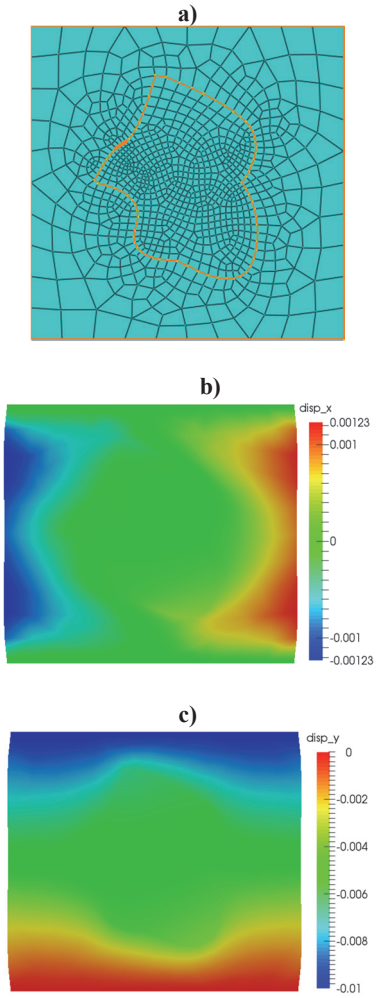

Fig. 5. *Results in form of displacements distribution for 951 nodes and 930 elements – (a) mesh for SSVE, (b) displacements in X axis, (c) displacements in Y axis.*

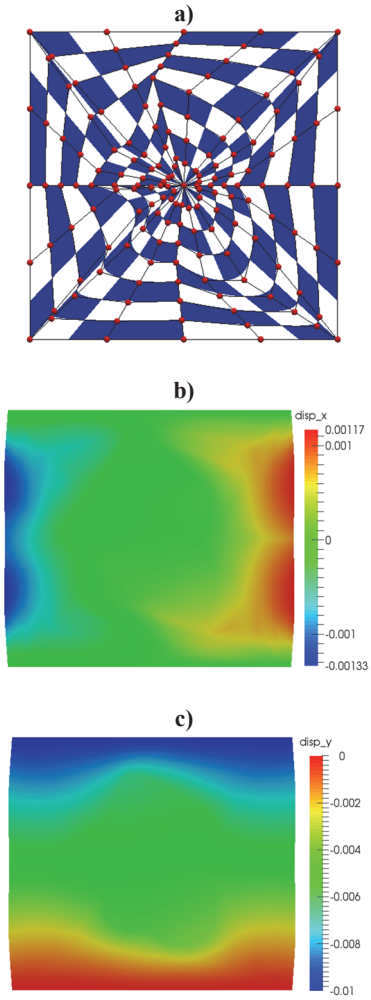

Fig. 7. *Results in form of displacements distribution for knots vectors* Ξ_ξ *= { 0, 0, 0, 0.0625, 0.125, 0.1875, 0.25, 0.25, 0.3125, 0.375, 0.4375, 0.5, 0.5, 0.5625, 0.625, 0.6875, 0.75, 0.75, 0.8125, 0.875, 0.9375, 1, 1, 1},* Ξ_η *={ 0, 0, 0.125, 0.25, 0.375, 0.5, 0.625, 0.75, 0.875, 1, 1 }, p=2, q=1 and 128 elements – (a) mesh for SSVE, (b) displacements in X axis, (c) displacements in Y axis.*

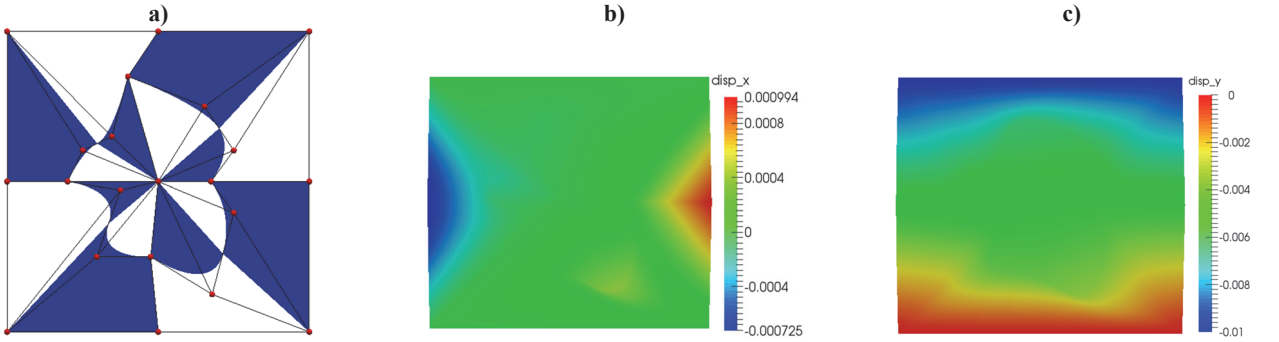

Fig. 6. *Results in form of displacements distribution for knots vectors* Ξ_ξ *= {0, 0, 0, 0.125, 0.25, 0.25, 0.375, 0.5, 0.5, 0.625, 0.75, 0.75, 0.875, 1, 1, 1},* Ξ_η *={ 0, 0, 0.5, 1, 1 }, p=2, q=1 and 16 elements – (a) mesh for SSVE, (b) displacements in X axis, (c) displacements in Y axis.*

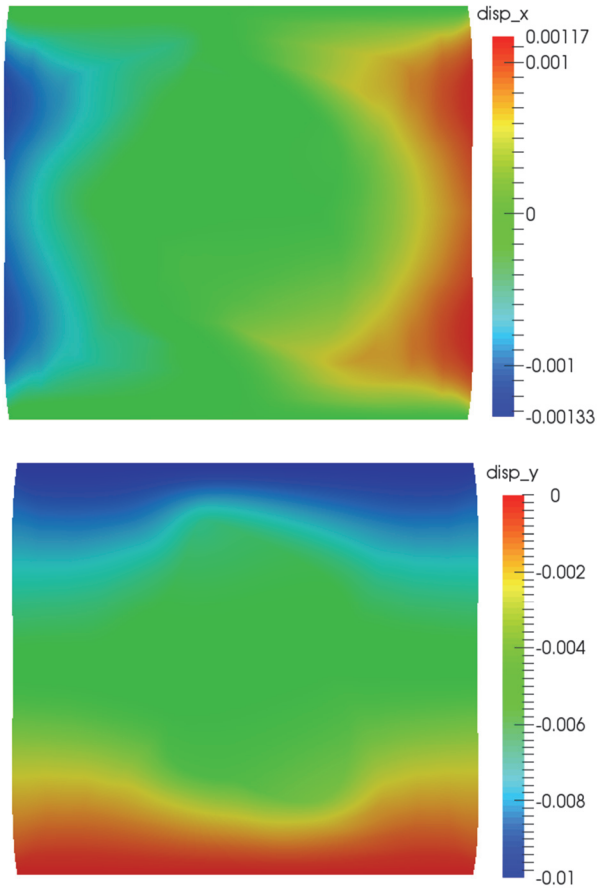

Fig. 8. Results as displacements for: p=2, q=1 and 256 elements

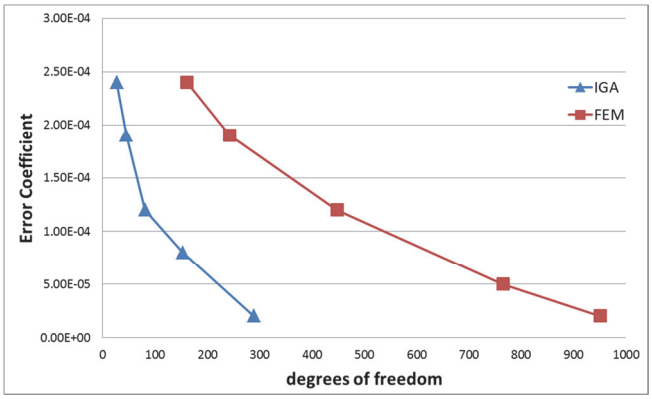

Fig. 9. Convergence comparison for IGA and FEM.

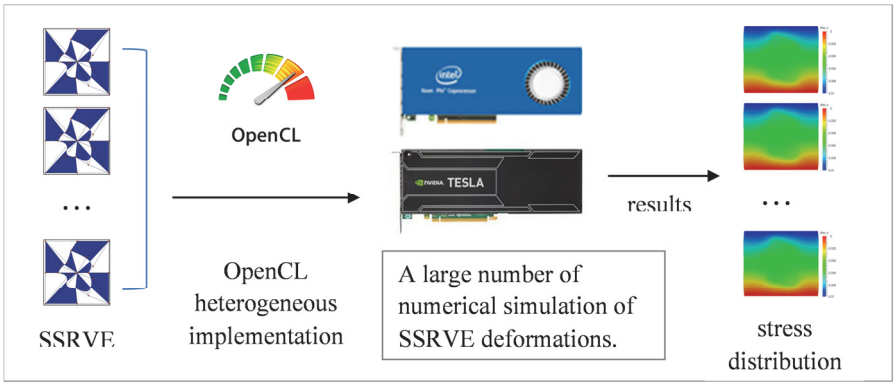

Fig. 10. Proposed heterogeneous approach.

5. SSRVE ON HETEROGENEOUS HARDWARE DEVICES

Heterogeneous hardware devices are very specific according to their internal architecture, which can be composed of two levels of parallelization, i.e. between Streaming Multiprocessors (SM) interconnected by using a common bus and inside SM between computing cores. Such architecture can be met in modern graphic cards, which are commonly supplied by major manufacturers of graphic devices e.g. NVIDIA, AMD. In opposite to GPGPUs, the architecture of Xeon Phi coprocessors is organized. It is based on Intel Many Integrated Core (Intel MIC) approach, which combines many Intel CPU cores (about 60 cores, depending on the device version) on a single chip. The interconnection between internal cores is implemented as a bidirectional ring. Each direction is comprised of three independent rings. The first, largest, and most expensive of these is the data block ring. The data block ring is 64 bytes wide to support the high bandwidth requirement due to the large number of cores. The address ring is much smaller and is used to send read/write commands and memory addresses. Finally, the smallest ring and the least expensive ring is the acknowledgement ring, which sends flow control and coherence messages. Intel MIC architecture is targeted for highly parallel computing in many fields, such as physics, chemistry or mechanics. Currently, the fastest supercomputers list in the world (TOP 500) also includes computing clusters designed with the use of Xeon Phi coprocessors.

Both presented architectures, i.e. Intel Xeon Phi and GPGPUs, obtain the highest performance, while using Single Instruction Multiple Data (SIMD) instruction set allowing to execute massive parallelization of one computing kernel for different data. In case of SSRVE it can be obtained for the part of computing task focused on numerical simulations of elastic deformation. The highest numerical intensity is related to solver of the linear equations system. Therefore, we propose parallel performance of multiple individual numerical simulations of SSRVE on the computing device, while the preparation of mesh and description of data on the host side (figure 10).

According to the various architectures of different computing accelerators the OpenCL implementation technology was proposed to overcome the problems with heterogeneous implementation for different devices. Due to this approach it is possible to implement homogeneous source codes and deploy them on various computing accelerators like Xeon Phi coprocessors or GPGPUs. The proposition of parallel computing approach presented for CAFE method by Rauch et al. (2012) was adjusted to the needs FE^2 method with SSRVE in microscale. It was applied for one computing device inside the hardware node. Initially obtained results proved that the proposed approach is much more efficient for heterogeneous hardware than for conventional distribution between multicore CPUs. The obtained speedup and efficiency were similar to parallel CAFE approach presented in Rauch et al. (2012).

6. CONCLUSIONS

In this work the approach based on IGA was proposed to improve the computational efficiency of multiscale modelling based on FE^2 methods for DP steels. The main attention was put on microscale level, which is realized by using SSRVE methodology allowing to simplify representation of computing domain. Two approaches were implemented, i.e. conventional FEM and IGA. Afterwards the numerical simulations of elastic deformation were performed for different meshes, including coarse and fine cases. The obtained results were compared according to the proposed error coefficient, pointing out the IGA as the approach characterized by much faster convergence than conventional FEM. IGA offered better computational efficiency, while maintaining high quality of results. Additionally, realization of FE^2 multiscale simulations on heterogeneous hardware architectures was proposed on the basis of similar approach presented for CAFE method. Similar parallel speedup and efficiency were obtained in both cases.

The presented usage of IGA will be developed further for plastic deformations, which should lead to higher efficiency in solving practical problems. This should allow to increase performance of sophisticated computing tasks and application of the proposed approach in industry.

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ZASTOSOWANIE ANALIZY IZOGEOMETRYCZNEJ W MODELOWANIU ODKSZTAŁCEŃ MATERIAŁÓW SPRĘŻYSTYCH Z WYKORZYSTANIEM STATYCZNIE PODOBNYCH REPREZENTATYWNYCH ELEMENTÓW OBJĘTOŚCIOWYCH NA HETEROGENICZNYCH ARCHITEKTURACH SPRZĘTOWYCH

Streszczenie

Statystycznie Podobny Reprezentatywny Element Objętościowy (ang. Statistically Similar Representative Volume Element, SSRVE) jest metodyką stosowaną w celu redukcji złożoności obliczeniowej reprezentacji mikrostruktury materiałów wielofazowych jak np.: stale DP oraz kompozyty. Podejście to bazuje na założeniu, że typowa reprezentacja RVE może być jeszcze bardziej uproszczona do elementu, który połączony periodycznie z samym sobą będzie zachowywał się podobnie jak jego bardziej złożony odpowiednik. SSRVE konstruowany jest w oparciu o krzywe NURBS (ang. Non-Uniform Rational B-Splines), a wyznaczany za pomocą procedury optymalizacji, gdzie funkcja celu zawiera własności mechaniczne, współczynniki kształtu oraz charakterystykę statystyczną analizowanej mikrostruktury. Pierwszy z wymienionych elementów funkcji celu wymaga zastosowania Metody Elementów Skończonych (MES), umożliwiającej symulację odkształcenia zarówno wzorca RVE jak i kolejnych rozwiązań SSRVE w zakresie sprężystym oraz sprężysto-plastycznym. Niniejszy artykuł przedstawia podejście pozwalające na podmianę kosztownej obliczeniowo procedury MES bardziej wydajną metodą analizy Izogeometrycznej (ang. Isogeometric Analysis, IGA). Wydajność nowego podejścia została przeanalizowana i porównana z konwencjonalnym rozwiązaniem opartym o MES. Ponadto, w artykule omówiona została możliwość zastosowania rozwiązania IGA z wykorzystaniem heterogenicznych architektur sprzętowych, co umożliwi poprawę wydajności obliczeniowej całego podejścia.

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