Publishing House
AKAPIT

COMPUTER METHODS IN MATERIALS SCIENCE
Informatyka w Technologii Materiałów

Vol. 15, 2015, No. 1

HOMOGENIZATION OF FIBER METAL LAMINATE STRUCTURES CHARACTERIZED BY ORTHOTROPIC AND ELASTIC-PLASTIC MATERIAL MODELS

TOMASZ NOWAK

ABB Corporate Research Center, Starowiślna 13A Street, 31-138 Krakow, Poland
**Corresponding author: tomasz.nowak@pl.abb.com*

Abstract

This paper gives a theoretical background and provides numerical calculations for the non-linear mechanical behavior of the panel structures consisting of fiber reinforced composite and aluminum laminates. Such structures offer high performance-to-weight ratio, therefore they are widely used in aerospace, subsea and high-pressure applications. The Classical Lamination Theory with orthotropic material properties is recalled and extended about elastic-plastic model for metal layers. The simplified stress-strain relation, as proposed by Hencky and Ilyushin, was applied to capture the influence of metal's plasticity on the mechanical performance of the hybrid structure. The numerical example showed, that at higher loads, the composite reinforcement provides a strong support for the aluminum layers when the metal approaches plastic deformation. In case of plastic flow within the aluminum, the bigger percentage of the external load is safely transferred to the composite fibers having much higher elastic limit. It prevents deformations of the aluminum laminate from being too large, and ensures the reliable operation of the Fiber Metal Laminates (FML) structure. Since the aluminum layers do not exhibit extensive strains, the application of Hencky-Ilyushin deformation theory seems to be reasonable for FMLs, even if aluminum layers are subjected to the anisotropic plastic flow. The proposed calculation method allows for very fast, but yet accurate, optimization of the Fiber Metal Laminate designs.

Key words: composite-reinforced metal structures, elastic-plastic orthotropic material, fiber-metal laminates, FML, Hencky-Ilyushin deformation theory

1. INTRODUCTION

Metal alloys are frequently used as the engineering materials for various industrial applications because they offer good mechanical strength, facilitate easy machining and assembling, and are relatively cheap. At high loads metals may exhibit a strong elastic-plastic behavior, thus the theory of plastic flow has been intensively investigated over last decades, (Hill, 1950; Chakrabarty, 1987). In contrast, the fiber-reinforced composites are generally expensive and difficult in manufacturing, however they offer the linear elastic behavior, and primarily - the superior strength-to-weight ratio (Jones, 1998; Gay et al., 2002). In the last three decades the Fiber Metal Laminates (FML), comprising the best properties of both materials, were studied extensively. The first FML was Arall®, a combination of aluminum and aramid/epoxy. In the 1980s, Delft University began developing a glass/epoxy FML called Glare®, which finally has been commercialized when Airbus decided to use it on the A380 aircraft. The mechanical properties of FML are typically studied by the Metal Volume Fraction, MVF, method, similar to the rule of mixtures for traditional composites (Vlot & Gunnik, 2001). Nowak (2013) and Nowak and Schmidt (2014) presented the analysis of equivalent mechanical properties for the FML tubes used in subsea and pressurized applications, respectively.

The work described in this paper considers a hybrid type of metal-composite laminate, aiming to

 ISSN 1641-8581

form a high-performance structure. A theoretical model for elasto-plastic composite reinforced aluminum panel is proposed, and validated by FEM calculations. The proposed approach assumes an orthotropic material model for the composite, and anisotropic plasticity model for aluminum layers. In the contrast to the earlier studies, author do not homogenize aluminum material with the composite structure into a single solid before the constitutive equation is applied. The elastic behavior of composite and the elastic-plastic behavior of metal are treated separately since the solution is found. It doubles the number of constitutive equations to be solved, but allows to isolate the anisotropic plastic flow in aluminum from purely elastic deformation of the composite layer. In addition, it was proved that a simple deformation theory of Hencky-Ilyushin may be successfully applied to FML structures, and high quality results may be found much faster. This facilitates an easy optimization of the hybrid structures if the fibers' angles and layers' thicknesses should be adjusted to the given load conditions.

2. ANALYTICAL CALCULATION MODEL

2.1. Basic assumptions

The analytical model used to study a composite-reinforced aluminum panel assumes a general orthotropic laminate, subjected to in-plane loading. In the classical approach (Herakovitch, 1998; Crawford, 2002), the stress-strain relation is characterized by an equivalent generalized force (N, M) – generalized strain (ε^0, κ) system:

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} \varepsilon^0 \\ \kappa \end{Bmatrix} \tag{1}$$

where:
- N, M are vectors of forces and moments, respectively
- ε^0, κ are vectors of strains due to forces and moments, respectively
- A, D and B are called tension stiffness, bending stiffness and coupling stiffness matrices respectively. They are calculated as follows:

$$[A,B,D] = \int_{-t/2}^{t/2} [K_i](1, z, z^2)\,dz \tag{2}$$

where $[K_i]$ is the stiffness matrix of a single i lamina, which is spaced from the neutral axis of the laminate by distance z, and t is the total thickness of the laminate.

The matrix $[B]$ plays an important role in the lamination theory, since it causes the complex interaction between the in-plane loads and the bending effects (out-of-plane strains). However, composite structures are typically designed in such a way that all components of $[B]$ are zero, therefore generated stresses are only the result of in-plane strains, ε, driven by in-plane forces, N. In this case, the analysis simplifies with the following assumptions:

- The object under study is constructed out of two different materials, forming alternately several flat layers, which are perfectly bonded. The parts made of different materials are shown schematically in figure 1:
  - the inner part, marked as „c", represents the composite shell (having thickness e_c)
  - the outer parts, marked as „m", symbolize metal layers (having the total thickness e_m).
- The composite part consists of several layers of fibers (basalt, or carbon, e-glass, etc.), oriented at different angles, and embodied in epoxy resin matrix.
- The orientation, number, and thickness, t_i, of fiber layers are unrestricted, but practically it should form a symmetrical and balanced structure, e.g.: $[\alpha/\ \beta/\ ...]_S$, and the entire thickness of all plies constitutes the inner part ($\Sigma t_i = e_C$).
- For the metal part, the non-linear elastic-plastic material properties are assumed with anisotropic plasticity, while for the composite part – the orthotropic elastic model is used.

2.2. Derivation of the model

To determine the stress and strain fields in both the steel layers and the composite structure, one can apply equilibrium, constitutive, and continuity equations. On the basis of the first relation, shown in figure 1, it is possible to state:

$$\begin{cases} e_m \sigma_{mX} + e_c \sigma_{cX} = f_X \\ e_m \sigma_{mY} + e_c \sigma_{cY} = f_Y \\ e_m \tau_{mXY} + e_c \tau_{cXY} = f_{XY} \end{cases} \tag{3}$$

where: ε_i are the wall thicknesses, σ_{ij} are the stresses, and f_j are the external distributed forces [N/m]; indices m and c correspond to the metal and composite, respectively.

The constitutive equations are based on Hooke's law for orthotropic material (Lekhnitskii, 1981), but they must be provided for the metal part and the composite parts independently, with respective indices, $i = m, c$:

$$\begin{Bmatrix} \varepsilon_{iX} \\ \varepsilon_{iY} \\ \gamma_{iXY} \end{Bmatrix} = \begin{vmatrix} 1/E_{iX} & -\nu_{iYX}/E_{iY} & \eta_{iXY}/G_{iXY} \\ -\nu_{iXY}/E_{iX} & 1/E_{iY} & \mu_{iXY}/G_{iXY} \\ \eta_{iX}/E_{iX} & \mu_{iY}/E_{iY} & 1/G_{iXY} \end{vmatrix} \begin{Bmatrix} \sigma_{iX} \\ \sigma_{iY} \\ \tau_{iXY} \end{Bmatrix} \quad (4)$$

where: ε_{ij} is the elastic strain; E_{ij}, G_{ij} - are the values of Young and Kirchhoff modules, respectively; ν_{ij} is the Poisson ratio, while η_{ij} and μ_{ij} are coupling factors (known as Rabinovic's and Chentsov's) in corresponding directions.

$$d\varepsilon_{ij}^{p} = \frac{\partial f}{\partial \sigma_{ij}} d\lambda \quad (6)$$

where: ε_{ij} and σ_{ij} are the strain and stress tensors respectively, f is the plastic potential, and λ is the scaling factor. In most cases of the anisotropic elastic-plastic behavior of the material the Hill's yield criterion is assumed as the plastic potential:

$$f : F(\sigma_Y - \sigma_Z)^2 + G(\sigma_Z - \sigma_X)^2 + H(\sigma_X - \sigma_Y)^2 + 2L\tau_{YZ} + 2M\tau_{ZX} + 2N\tau_{XY} - 1 = 0 \quad (7)$$

where: F, G, H, L, M, N are constants proposed by Hill and obtained by tests of the anisotropic material in different orientations.

Based on equations (6) and (7), an exemplary, X component of the plastic strain increment may be

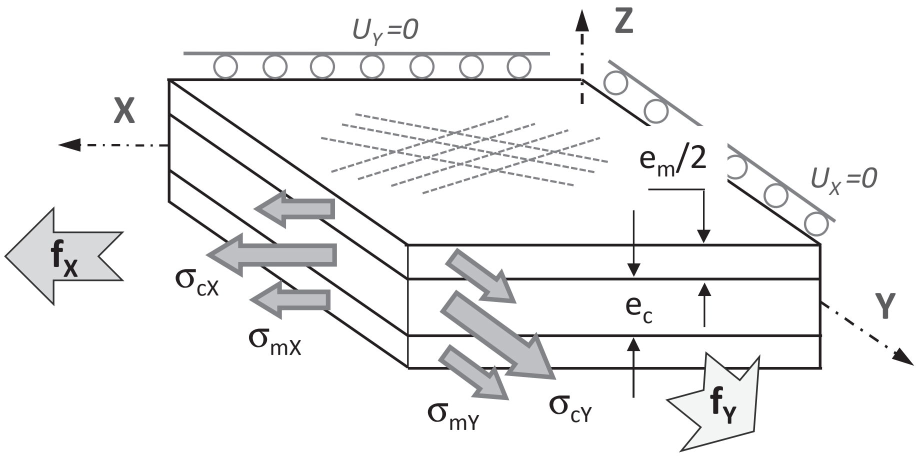

Fig. 1. Equilibrium relations for FML panel (shear load not shown).

If the isotropic material is considered $\nu_{XY} = \nu_{YX} = \nu$, $E_X = E_Y = E$, and $\eta_{XY} = \mu_{XY} = \eta_X = \mu_Y = 0$.

The continuity equations refer to the assumption, that metal and composite parts are perfectly bonded. Therefore one should also postulate that the strains in respective directions must be equal each other. While the composite can be considered as the linear elastic body, the elastic-plastic behavior of the aluminum should be assumed. Thus, the metal strains comprise the elastic parts ε^E, as given by the equation (4), and the plastic components, ε^P:

$$\begin{cases} \varepsilon_{cX}^{E} = \varepsilon_{mX}^{E} + \varepsilon_{mX}^{P} \\ \varepsilon_{cY}^{E} = \varepsilon_{mY}^{E} + \varepsilon_{mY}^{P} \\ \gamma_{cXY}^{E} = \gamma_{mXY}^{E} + \gamma_{mXY}^{P} \end{cases} \quad (5)$$

Based on Prandtl-Reuss material model (Hill, 1950), the plastic strain increment is expressed by the flow rule as:

expressed as:

$$d\varepsilon_{X}^{p} = \frac{\partial f}{\partial \sigma_X} d\lambda = [-2G(\sigma_Z - \sigma_X) + 2H(\sigma_X - \sigma_Y)] d\lambda \quad (8)$$

Deriving other plastic strain components, focusing on plane stress conditions only ($\sigma_Z = \tau_{YZ} = \tau_{ZX} = 0$), and using the matrix notation one can write:

$$\begin{bmatrix} d\varepsilon_X^p \\ d\varepsilon_Y^p \\ d\varepsilon_Z^p \\ d\varepsilon_{XY}^p \end{bmatrix} = \begin{bmatrix} G+H & -H & 0 \\ -H & F+H & 0 \\ -G & -F & 0 \\ 0 & 0 & 2N \end{bmatrix} \begin{bmatrix} \sigma_X \\ \sigma_Y \\ \tau_{XY} \end{bmatrix} d\lambda \quad (9)$$

Note 1: the number "2" which appeared in the equation (8) is not introduced into the equation (9) since Hill's constants are typically multiplied by the factor "1/2". Note 2: The Hill's function reduces to the Mises stress potential in case of the isotropic material ($F=G=H=1/2 = 3L=3M=3N$).

The scaling factor increment $d\lambda$ involves the plastic strain intensity increment $d\varepsilon_{PL}$ and the stress intensity, $\bar{\sigma}$, and may be expressed as:

$$d\lambda = \frac{1}{C}\frac{d\varepsilon_{PL}}{\bar{\sigma}} \tag{10}$$

where: $\bar{\sigma}$ is the Hill's yield criterion, which extends the Mises function to allow anisotropic behavior:

$$\bar{\sigma} = \sigma_{HILL} = \frac{1}{\sqrt{C}}\sqrt{G\sigma_X^2 + F\sigma_Y^2 + H(\sigma_X - \sigma_Y)^2 + 2N\tau_{XY}^2} \tag{11}$$

and the effective plastic strain intensity increment reads:

$$d\varepsilon_{PL} = \sqrt{C}\sqrt{\left(\frac{Fd\varepsilon_X^{p\,2} + Gd\varepsilon_Y^{p\,2} + Hd\varepsilon_Z^{p\,2}}{HF + GF + HG} + 2\frac{d\varepsilon_{XY}^2}{N}\right)} \tag{12}$$

The constant C can be assigned quite freely, and it basically should simplify calculations. Gabryszewski and Gronostajski (1991), based on Hill's proposal, use $C = 2(H+F+G)/3$, while in ABAQUS (2013) the constant C is equal to one. Please note also that whichever factor C is used, the plastic work, corresponding to the product of strain and stress, is exactly the same.

Thus, based on equations (4) and (9), remembering $\gamma_{XY} = 2\varepsilon_{XY}$, and assuming that flow rule (6) could be replaced by the deformation theory of Hencky-Ilyushin (Chakrabarty, 1987), if the plastic strain increments developed at the loading path constitute the total plastic deformation without unloading, one can write in the matrix notation:

$$\begin{bmatrix} \frac{1}{\bar{E}_X} & -\frac{\bar{\nu}_{YX}}{\bar{E}_Y} & \frac{\bar{\eta}_{XY}}{\bar{G}_{XY}} \\ -\frac{\bar{\nu}_{XY}}{\bar{E}_X} & \frac{1}{\bar{E}_Y} & \frac{\bar{\mu}_{XY}}{\bar{G}_{XY}} \\ \frac{\bar{\eta}_X}{\bar{E}_X} & \frac{\bar{\mu}_Y}{\bar{E}_Y} & \frac{1}{\bar{G}_{XY}} \end{bmatrix} \begin{bmatrix} \sigma_{cX} \\ \sigma_{cY} \\ \tau_{cXY} \end{bmatrix} = \left(\begin{bmatrix} \frac{1}{E_X} & -\frac{\nu_{YX}}{E_Y} & 0 \\ -\frac{\nu_{YX}}{E_Y} & \frac{1}{E_Y} & 0 \\ 0 & 0 & \frac{1}{G_{XY}} \end{bmatrix} + \varphi \begin{bmatrix} (G+H) & -H & 0 \\ -H & (F+H) & 0 \\ 0 & 0 & 4N \end{bmatrix} \right) \begin{bmatrix} \sigma_{mX} \\ \sigma_{mY} \\ \tau_{mXY} \end{bmatrix} \tag{13}$$

where: φ is the plastic deformation potential, which is defined similarly to $d\lambda$, but the strain incremental forms in relations (9) and (12), $d\varepsilon_i$ take now their total values, ε_i, and:

$$\varphi = \frac{1}{C}\frac{\varepsilon_{PL}}{\bar{\sigma}} \tag{14}$$

Please note, that indices c and m in the equation (13) represent the composite and metal, respectively. The "bar" sign points out that the equivalent properties are being used for the composite, since it consists of several different plies. The material properties without "bar" sign refer to the metal.

Finally, relations (3) and (13) constitute a system of six equations with seven formal unknowns (σ_{mX}, σ_{mY}, τ_{mXY}, σ_{cX}, σ_{cY}, τ_{cXY}, and φ):

$$\begin{Bmatrix} \sigma_{mX} \\ \sigma_{mY} \\ \tau_{mXY} \\ \sigma_{cX} \\ \sigma_{cY} \\ \tau_{cXY} \end{Bmatrix} = \begin{bmatrix} e_m & 0 & 0 & e_c & 0 & 0 \\ 0 & e_m & 0 & 0 & e_c & 0 \\ 0 & 0 & e_m & 0 & 0 & e_c \\ \frac{1}{E_X} + \varphi(G+H) & -\frac{\nu_{YX}}{E_Y} - \varphi H & 0 & -\frac{1}{\bar{E}_X} & \frac{\bar{\nu}_{YX}}{\bar{E}_Y} & -\frac{\bar{\eta}_{XY}}{\bar{G}_{XY}} \\ -\frac{\nu_{YX}}{E_Y} - \varphi H & \frac{1}{E_Y} + \varphi(F+H) & 0 & \frac{\bar{\nu}_{XY}}{\bar{E}_X} & -\frac{1}{\bar{E}_Y} & -\frac{\bar{\mu}_{XY}}{\bar{G}_{XY}} \\ 0 & 0 & \frac{1}{G_{XY}} + 4\varphi N & -\frac{\bar{\eta}_X}{\bar{E}_X} & -\frac{\bar{\mu}_X}{\bar{E}_Y} & -\frac{1}{\bar{G}_{XY}} \end{bmatrix}^{-1} \begin{Bmatrix} f_X \\ f_Y \\ f_{XY} \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{15}$$

The closed-form solution of the equation (15) is generally not possible, since the plastic deformation potential φ is a function of unknown stresses (σ_{mX}, σ_{mY}, τ_{mXY}). However, it can be easily found numerically. Having the Hill's material constants determined, one has to select a test point (ε^*_{PL}, σ^*) on the true stress – plastic true strain curve managed for unidirectional tensile experiment, and calculate a "test plastic deformation potential", φ^* defined as:

$$\varphi^* = \frac{1}{G+H}\frac{\varepsilon^*_{PL}}{\sigma^*} \tag{16}$$

If the test plastic deformation potential φ^* is introduced into the equation (15) instead of φ, the set of equations may be solved for stresses, and finally, the actual value of can be found by the equation (14). The calculation procedure with selection of the next "test point" should be repeated, until the difference between plastic deformation potential φ and its test value φ^* reaches the assumed tolerance.

The solution of the equation (15) can be successfully found if the respective material properties for anisotropic plasticity material and the orthotropic composite layer are determined. Since the former is quite well elaborated in the metal industry (e.g. Gabryszewski & Gronostajski, 1991; ABAQUS, 2013), we will focus on the later in the next paragraph.

2.3. Equivalent material properties for the composite

Derivation of the equivalent material properties for the composite part refers basically to the Classical Lamination Theory. The mechanical behavior of a single ply is characterized by its local stiffness matrix, $[k_i]$, which is oriented along the roving fibers, according to the local co-ordinate system, (Herakovitch, 1998; Crawford, 2002):

$$[k_i]=\begin{bmatrix} \frac{E_L}{1-\nu_{LT}\nu_{TL}} & \frac{\nu_{LT}E_T}{1-\nu_{LT}\nu_{TL}} & 0 \\ \frac{\nu_{TL}E_L}{1-\nu_{LT}\nu_{TL}} & \frac{E_T}{1-\nu_{LT}\nu_{TL}} & 0 \\ 0 & 0 & G_{LT} \end{bmatrix} \tag{17}$$

Indices L, T and LT, TL refer to the longitudinal direction, transverse direction and shear, respectively. The corresponding values of the Young modules, E, Poisson ratios, ν, and Kirchhoff modulus, G, should be provided by the composite manufacturer, but if not given – they can be easily derived by the commonly used Rule of Mixtures, which incorporates the fiber volume fraction (Jones, 1998). Furthermore, because the longitudinal direction of a single i-th ply may be arbitrarily oriented in the global co-ordinate system - its Z-axis is rotated by the α angle, therefore it is also needed to transform the local stiffness matrix into a global one, $[K_i]$:

$$[K_i]=T^{-1}[k_i]T^{-1^T} \tag{18}$$

where $[K_i]$ is the stiffness matrix in the global co-ordinate system, GCS, and T – is the transformation matrix, given as:

$$T=\begin{bmatrix} c^2 & s^2 & 2sc \\ s^2 & c^2 & -2sc \\ -sc & sc & c^2-s^2 \end{bmatrix} \tag{19}$$

where: s – $sin\alpha$, c – $cos\alpha$; α – rotation angle about Z axis from LCS to GCS.

If the composite consists of several plies (which is normally the case), then the global stiffness matrix $[K_C]$ for the whole composite structure is calculated with respect to the thickness of individual plies:

$$[K_C]=\frac{1}{t}\sum_i^N t_i[K_i] \tag{20}$$

where: t_i – thickness of i-th ply, t – total thickness of all N plies (t is equal to e_c, according to the figure 1).

Finally, one can determine the equivalent mechanical properties of the multi-layer composite, as used in the equation (4). It is necessary to calculate the inverse of the global stiffness matrix, $[K_C]^{-1}$ first, and to perform some simple operations on its components:

$$[K_C]^{-1}=[S_C]=$$

$$\begin{vmatrix} s_C^{11} & s_C^{12} & s_C^{13} \\ s_C^{21} & s_C^{22} & s_C^{23} \\ s_C^{31} & s_C^{32} & s_C^{33} \end{vmatrix} \rightarrow \begin{cases} \bar{E}_X=\frac{1}{s_C^{11}} & \bar{\nu}_{YX}=-\frac{s_C^{12}}{s_C^{22}} & \bar{\eta}_{XY}=\frac{s_C^{13}}{s_C^{33}} \\ \bar{\nu}_{XY}=-\frac{s_C^{21}}{s_C^{11}} & \bar{E}_Y=\frac{1}{s_C^{22}} & \bar{\mu}_{XY}=\frac{s_C^{23}}{s_C^{33}} \\ \bar{\eta}_X=\frac{s_C^{31}}{s_C^{11}} & \bar{\mu}_Y=\frac{s_C^{32}}{s_C^{22}} & \bar{G}_{XY}=\frac{1}{s_C^{33}} \end{cases} \tag{21}$$

3. HOMOGENIZATION OF FIBER METAL LAMINATE

The solution of the equation (15) gives the values of stresses in both: metal layers and the composite structure directly (without homogenization). However, the same result can be achieved using more common notation of the homogenization formalism. One may consider equation (20) as a simpler form of the generalized definition for an effective material properties <h> represented as the ensemble average of variable, h, (Kroner, 1972):

$$\langle h\rangle=\langle h(x)\rangle=\frac{1}{V}\int_V h(x)dV=\frac{1}{N}\sum_{i=1}^N h_i(x) \tag{22}$$

Applying the equation (22) to the layered structures of composite and metal, one may define the global stiffness matrix as:

$$[K_G]=\frac{[K_M]e_M+[K_C]e_C}{e_M+e_C} \tag{23}$$

where the metal stiffness matrix $[K_M]$ reads:

$$[K_M]=[S_M]^{-1}=$$

$$\begin{bmatrix} \frac{1}{E_X}+\varphi(G+H) & -\frac{\nu_{YX}}{E_Y}-\varphi H & 0 \\ -\frac{\nu_{YX}}{E_Y}-\varphi H & \frac{1}{E_Y}+\varphi(F+H) & 0 \\ 0 & 0 & \frac{1}{G_{XY}}+4\varphi N \end{bmatrix}^{-1} \tag{24}$$

and the composite stiffness matrix $[K_C]$ is defined by the equation (20). Naturally, the metal stiffness matrix $[K_M]$ is state-dependent and relates to the actual value of the plastic deformation potential, φ.

Since the global stiffness matrix of the whole composite-metal structure is found by the equation (23), the respective equivalent, or homogenized material properties (effective Young and Kirchhoff modules, Poisson ratios, and coupling factors) may be calculated by the procedure described by the relation (21).

other more structured iterative method, aiming to minimize the difference between φ and its test value φ^*, as described by equations (14) and (16), respectively.

The calculated values of stresses [MPa], plastic strains [-] and deformation potential [-], as well as composite-to-aluminum stress ratio for different load levels were compared with the numerical simulation results delivered by ABAQUS software package, table 1.

Table 1. *Comparison of the results provided by the proposed procedure and ABAQUS.*

Load	f_X	f_Y		Aluminum			Composite	Comp./AL.
[%]	[N/mm]			σ_{MISES}	ε_{PL}	ϕ	σ_{MISES}	Mises ratio
30	600	-300	**This work**	215,6	0	0	191,5	0,89
			ABAQUS	215,5	0		191,5	
60	1200	-600	**This work**	317,0	3,60E-03	1,01E-05	498,4	
			ABAQUS	316,9	3,55E-03		498,9	1,57
100	2000	-1000	**This work**	340,1	1,24E-02	3,24E-05	1019,4	
			ABAQUS	339,5	1,22E-02		1024,8	3,02
			error [%]	*-0,2*	*-1,1*		*0,5*	

4. NUMERICAL EXAMPLE

The numerical example was conducted for the flat plate (100x100 mm) consisting of two external layers of 2024 T3 aluminum (having the thickness of 1.0 mm, each) and the [0/45/-45/0] composite structure made of carbon fibers reinforced polymer (having the fiber content at the level of 60% vol., and each of four plies is 0.5 mm thick). The material properties in the elastic range were assumed to be the typical values, as provided by material suppliers for aluminum (E = 67.5 GPa), carbon fibers (E = 240 GPa), and epoxy resin (E = 4.5 GPa). Important to note, that 2024 T3 aluminum shows the plastic behavior starting from the yield level of about 337.5 MPa ($\bar{\sigma} = 337.5+1107.3\varepsilon_{PL}$^0.7285), and exhibits anisotropic flows (the following Hill's factors were assumed: $F = 0.53$, $G = 0.295$, $H = 0.705$, $N = 1.5$). The composite structure, made of plies at different orientation of fibers, forms the orthotropic material ($\bar{E}_X$ = 81 GPa; $\bar{E}_Y$ = 24 GPa; $\bar{G}_{XY}$ = 20.8 GPa; $\bar{v}_{XY}$ =.72; $\bar{v}_{YX}$= .21). The in-plane distributed forces were set at full load as 2 kN/mm and -1 kN/mm in X and Y directions, respectively, as shown in figure 1. Introducing above material properties into the equation (15) one could solve it for unknown stress components in composite and aluminum layers. The only unidentified factor - the plastic deformation potential φ can be found by a trial-and-error approach, or any

Interesting to note that the increased plastic flow within aluminum, caused by the higher load level, affects the stress ratio between composite and aluminum layers. Since the composite structure protects the aluminum strains from being too large, the application of Hencky-Ilyushin theory of small elastic-plastic deformations into FML plates is reasonable.

5. CONCLUSIONS

This paper introduced a structured approach for the design of composite reinforced metal plates used in many industrial applications. The provided theory captured the elastic-plastic behavior of the aluminum layers, by use of the Hencky-Ilyushin theory. It was proved, that this theory can be successfully applied to the materials exhibiting the anisotropic plastic flow, if small elastic-plastic deformations are ensured. Since it is the case for FML structures, the proposed calculation method allows for very fast, but yet accurate, optimization of the analyzed structures in the industrial conditions.

REFERENCES

ABAQUS Analysis, version 6.12, 2013, *User's Manual. Volume III*, Dassault Systémes (access October 2014)

Chakrabarty, J., 1987, *Theory of Plasticity*, McGraw-Hill, New York.

Crawford, R.J., 2002, *Plastics Engineering*, Elsevier, New York.

Gabryszewski, Z., Gronostajski, J., 1991, *Mechanika Procesów Obróbki Plastycznej*, Wydawnictwo Naukowe PWN, Warszawa (in Polish).

Gay, D., Hoa. S.V., Tsai, S.W., 2002, *Composite Materials: Design and Applications*, CRC Press, New York.

Herakovitch, C., 1998, *Mechanics of Fibrous Composites,* John Wiley & Sons, New York.

Hill, R., 1950, *The Mathematical Theory of Plasticity*, Clarendon Press, Oxford.

Jones, R., 1998, *Mechanics of Composite Materials 2nd ed*., CRC Press, Oxford.

Kroner, E., 1972, *Statistical Continuum Mechanics*, Springer Verlag, New York.

Lekhnitskii, S.G., 1981, *Theory of Elasticity of an Anisotropic Body*, Mir Publishers, Moscow (English translation).

Nowak, T., 2013, Modeling and Optimization of Fiber Reinforced Pressure Chamber for Subsea Applications, *Composites Theory and Practice*, 13, 275-281.

Nowak, T., Schmidt, J., 2014, Prediction of Elasto-Plastic Behavior of Pressurized Composite Reinforced Metal Tube by Means of Acoustic Emission Measurements and Theoretical Investigation, *Journal of Composite Structures*, 118, 49-56.

Vlot, A., Gunnink, J.W., 2001, *Fibre Metal Laminates: An Introduction*, Kluwer Academic Publishers, Dordrecht.

HOMOGENIZACJA STRUKTUR KOMPOZYTOWO-METALOWYCH Z WYKORZYSTANIEM MODELI MATERIAŁU ORTOTROPOWEGO ORAZ SPRĘŻYSTO-PLASTYCZNEGO

Streszczenie

W artykule przedstawiono podstawy teoretyczne, zilustrowane przykładem numerycznym, opisujące model laminatu metalowo-włóknistego składającego się z kompozytu polimerowego wzmacnianego włóknami węglowymi oraz warstw aluminium. Przedstawione struktury hybrydowe (Fiber Metal Laminate, FML) charakteryzują się wysokimi własnościami mechanicznymi w stosunki do ciężaru, dlatego są chętnie używane w przemyśle lotniczymi oraz stosowane w aplikacjach wysokociśnieniowych. W artykule klasyczną teorię laminatów (Classical Lamination Theory) uzupełniono o model sprężysto-plastyczny dla warstw metalowych. Zastosowano uproszczony model plastyczności zaproponowany przez Hencky'ego i Iljuszyna. Podano rozwiązanie numeryczne dla przypadku panelu FML poddanego płaskiemu obciążeniu dwuosiowemu. Wykazano, że w przypadku znacznego wytężenia, gdy dochodzi do uplastycznienia aluminium, znaczna część obciążeń zewnętrznych przekazywana jest do warstw kompozytu włóknistego, który charakteryzuje się znacząco wyższą granicą sprężystości. Takie zachowanie zabezpiecza warstwy aluminium przed nadmiernym płynięciem i umożliwia bezpieczną eksploatację struktury hybrydowej, nawet w przypadku wysokich obciążeń. Wykazano, że przyjęta metoda obliczeniowa charakteryzuje się wystarczającą dokładnością, a dzięki swojej szybkości umożliwia przemysłową optymalizację hybrydowych struktur FML.

Received: October 1, 2014
Received in a revised form: November 5, 2014
Accepted: November 19, 2014