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METAMODEL OF THE PLANE STRAIN COMPRESSION TEST AS A REPLACEMENT OF FE MODEL IN THE INVERSE ANALYSIS

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Abstract

The paper presents the application of plane strain compression (PSC) test to the identification of material model parameters. Moreover, an attempt to substitute the finite element (FE) model with the metamodel in the inverse analysis was made. There are several plastometric tests carried out in order to obtain material properties. Despite the fact that PCS test is very accurate, the most commonly used is uniaxial compression (UC) test. This is due to the difficulties in interpretation of results of the PSC test. Inverse analysis is used to overcome this problem. This analysis requires simulation results of adopted test. FE model is most commonly used in these simulations. However, due to mesh density in the area of greatest strain gradients, FE simulations can take unacceptably long time. Therefore, artificial neural network (ANN) was proposed in the present work as metamodel of the PSC test. This metamodel was used in the inverse analysis performed to obtain material properties for copper alloys. The results were compared with uniaxial compression test performed for the same material and good agreement was observed.

Key words: plain strain compression, inverse analysis, metamodel, artificial neural network

1. INTRODUCTION

Continuous progress in numerical techniques applied to modelling of metal forming is observed. In consequence, the accuracy of numerical simulations depends mainly on the correctness of the description of boundary conditions and properties of the deformed material (Lenard et al., 1999). The latter problem is the subject of the present work. A number of papers dealing with rheological models of materials subjected to plastic deformation can be found in the scientific literature. New models with specific features such as internal variable models, which account for the history of deformation (Pietrzyk, 2001) or multiscale models, (Madej et al., 2007) were developed. Potential extensive predictive capabilities of these models are useful only when proper identification of the models is performed on the basis of experimental tests. Interpretation of the results of plastometric tests, which are commonly used for estimation of models parameters, is the main part of the identification. Properly performed identification uses inverse analysis with FE simulation of plastometric test and allows eliminating influence of various disturbances in the test, such as an effect of friction or deformation heating. Various algorithms for the inverse analysis were developed in 1990s (Gelin & Ghouati, 1994; Boyer & Massoni, 2001; Forestier et al., 2002; Szeliga et al., 2006). The published results show that application of the inverse method improves accuracy of interpretation of plastometric tests. On the other hand, in conventional inverse analysis of plastometric tests, FE model is combined with optimization techniques and such combination involves long computing times. Therefore, a lot of effort is put into searching for more efficient inverse calculations. Application of the metamodel in the optimization was proposed by Sztangret et al. (2012). The metamodel substituted the FE solution and significantly reduced the computing costs. This approach was successfully tested for

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the uniaxial compression test. Possibility of application of the same approach to the plane strain compression test was investigated in the presented work. Plane strain compression is much more complicated when interpretation of results is considered, see next section. Due to a very non uniform strain distribution, FE simulation of this test requires fine mesh and much longer computations comparing to the UC test. In consequence, conventional inverse analysis for the PSC test is not efficient. The artificial neural network was used as the metamodel of the plastometric test.

the sample beyond the area under the die. These parts are not compressed, therefore, they do not have tendency to spread. Moreover, when the samples are heated by resistance heating (e.g. on Gleeble 3800) these parts are in lower temperature than the area under the dies and their resistance to deformation is higher.

Plane state of strains, which is not reachable in other plastometric tests, has for years inspired the scientists to various applications of the PSC tests. Identification of the flow stress model is one of such applications and investigation of the microstructure evolution is another example. Among several research laboratories involved in investigations based on the PSC tests a team led by Mark Sellars at the University of Sheffield should be mentioned. During the second half of the last century this test was commonly used there for investigation of materials and fundamental works on microstructure evolution (Sellars, 1979) and on flow stress models (Davenport et al., 1999) were a result of this research. Thorough analysis of temperature changes in the PSC test was performed by Hand et al. (2000) and analysis of the influence of the size of the sample on the test results was presented by Mirza and Sellars (2001). Application of the FE model of the PSC test to aid interpretation of results is described by Pietrzyk et al. (1993) and by Kowalski et al. (2000) and further analysis of the influence of the size of the sample on the test results was performed by Kowalski et al. (2006). All these discussed papers showed that PSC tests involve strong inhomogeneities of parameters. In spite of this, due to some spe-

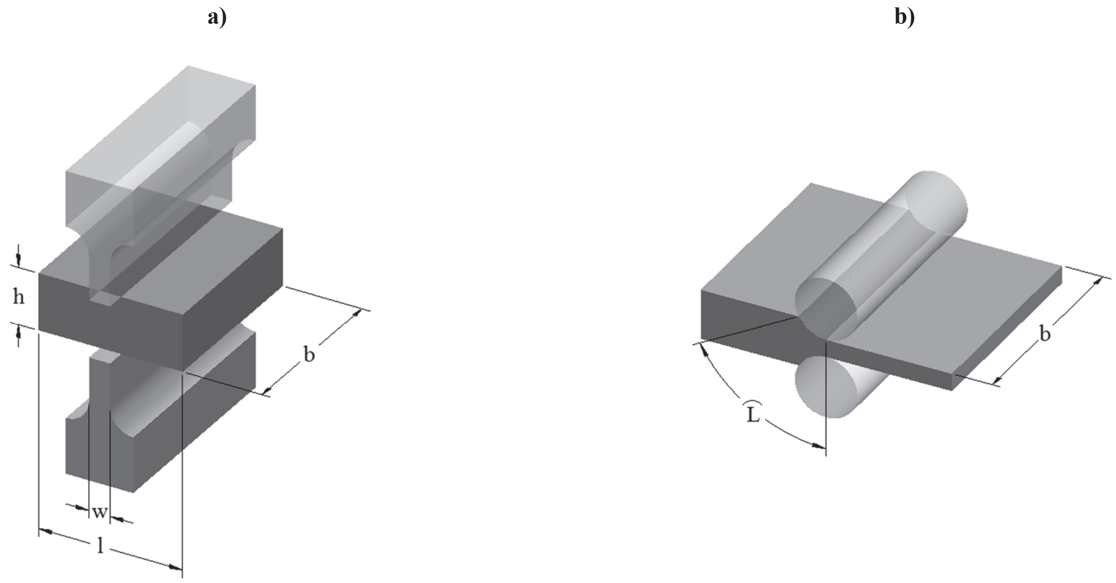


Fig. 1. Schematic illustration of the PCS test (a) and flat rolling process (b).

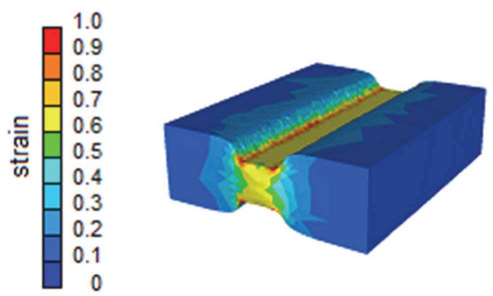
2. MODEL OF THE PLANE STRAIN COMPRESSION TEST

2.1. Plane Strain Compression

Plane strain compression is one of the plastometric tests, which is used for physical simulations of rolling processes. In this test a cuboid sample is compressed between two flat dies, see figure 1a. The PSC test allows large plastic deformation and the state of strains is similar to that, which occurs in the flat rolling process (figure 1b). The plane strain state is obtained due to two factors. The low width of the sample (b) – to width of the die (w) ratio prevents flow of the material in the width direction. It is similar to the flat rolling, where low length of contact (L) – to width of the strip (b) ratio fosters elongation and prevents spread. Influence of the so called rigid ends is another factor, which constrains spread and involves plane strain state. Rigid ends are the parts of cific advantageous features PSC test have been widely used during last half of a century for identification of models of various materials like steels (Davenportet al., 1999; Szeliga et al., 2002), aluminium alloys (Pietrzyk & Tibbals, 1995; Hosford, 1966; Silk & van der Winden, 1999; Kowalski et al., 2000) and magnesium alloys (Nave & Barnett, 2004). PSC tests were also widely used for investigation of various non-metallic materials, see for example applications for determination of strength and deformation characteristics of sand (Tatsuoka et al., 1986) or investigation of shear banding in sands (Finno et al., 1997). Discussion of the PSC tests features from the point of view of standards and good practice guides can be found in (Loveday et al., 2006).

It should be emphasized, however, that various disturbances still make interpretation of results of PSC tests very difficult. These tests are characterized by large inhomogeneity of deformation (figure 2a), which is caused by complex shape of the deformation zone (figure 1a) and by the effect of friction. Beyond this, heat generated due to plastic work and friction, as well as heat transfer to the tools and to the surrounding, cause strong inhomogeneity of the temperature in the sample (figure 2b).

a)



b)

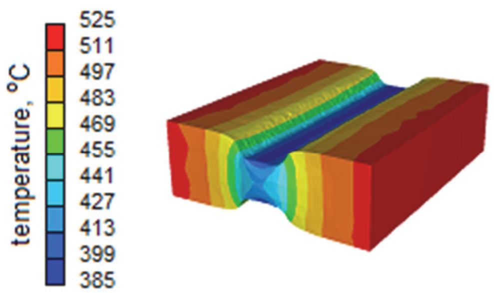


Fig. 2. *Distribution of strains (a) and temperatures (b) in the PSC test calculated by the Forge 3 code (Sztangret & Pietrzyk, 2012).*

Due to all these discussed features of the PSC test, and due to the fact that strain rate and temperature can be effectively controlled at the Gleeble simulator, this test is frequently used as physical simulation of the flat rolling process (Timothy et al., 1991). Since mentioned above inhomogeneities of strains, stresses and temperatures make determination of the flow stress from this test very difficult, researchers usually additionally performed uniaxial compression tests to identify the flow stress model. This procedure is very costly, therefore, researchers were searching for a possibility of application of numerical simulations to aid interpretation of the PSC test results. Results of this research are described in (Pietrzyk & Tibbals, 1995; Mirza & Sellars, 2001). Large progress in accuracy was obtained when inverse analysis was applied to identification of the flow stress model on the basis of the PSC test results (Gelin & Ghouati, 1994; Szeliga et al., 2002). Very good results of the inverse analysis were obtained. It was shown by Szeliga et al. (2002) that when inverse analysis was applied identical flow stress model were obtained from various tests (PSC and UC) and for various dimensions of the samples.

Long computing times were the main drawback of the inverse analysis for the PSC tests. As it has already been mentioned, FE simulation of the PSC test requires much finer mesh than that used in the UC test, what leads to very high computing costs. The objective of the present work was to investigate possibility of substitution of the FE model in the inverse analysis by the metamodel of the PSC test. Promising results obtained by Sztangret et al. (2012) for the UC test were the inspiration for this work. Artificial neural network was selected as a metamodel and the training data were generated using 2D FE code.

2.2. FE model and flow stress model

Large number of simulations has to be performed to generate the data for training of the Artificial Neural Network. These data could not be generated in a reasonable time using 3D FE code, which was applied to obtain results in figure 2. Therefore 2D finite element code based on the flow formulation (Kobayashi et al., 1989) was used for simulations of the test. The rigid-plastic constitutive law written in the form of relationship between stress tensor (σ) and the strain rate tensor ($\dot{\varepsilon}$) was the basis of the model:

$$\boldsymbol{\sigma} = \frac{2}{3}\frac{\sigma_p}{\dot{\varepsilon}_i}\dot{\boldsymbol{\varepsilon}} \qquad (1)$$

where: σ_p – flow stress, $\dot{\varepsilon}_i$ – effective strain rate.

Details of the mathematical formulation of the FE model are described by Lenard et al. (1999) and they are not repeated in the present work. 2D approach is a simplification of the PSC test. It is seen in figure 2 that some spread occurs under the contact with the die. Therefore, a correction of the force proposed by Kowalski et al. (2001) was used. The current width of the sample was calculated from the following equation:

$$b - b_0 = \left[1 + C - C\left(\frac{h}{h_0}\right)^{0,18}\right], \quad C = \frac{\frac{b_f}{b_0} - 1}{1 - \left(\frac{h_f}{h_0}\right)^{0,18}} \qquad (2)$$

where: b_0 – initial width, h_0 – initial height, b_f – final width, h_f – final height, b – current width, h – current height, C – coefficient.

This correction accounts for the spread and allowed using 2D model of the test to generate data for training the artificial neural network.

Flow stress σ_p in equation (1) has to be determined for the investigated material. Compression of the copper alloy in the temperature range 20-300°C was selected as a case study in the present work. Thorough analysis of plastic deformation of this alloy in high temperatures was performed by Pietrzyk et al. (2013). Extended Hollomon equation describes well behaviour of metals at lower temperatures:

$$\sigma_p = A\varepsilon_i^n \dot{\varepsilon}_i^m \exp\left[\frac{Q}{R(T+273)}\right] \qquad (3)$$

where: ε_i – effective strain, Q – activation energy of deformation, R – universal gas constant, T – temperature in °C.

Since analysed copper alloy shows large sensitivity to temperature, equation (3) could be not flexible enough to describe behaviour of this alloy in a range of temperatures. Therefore, Hansel-Spittel (1979) equation was additionally considered in the present paper.

$$\sigma_p = A\varepsilon_i^n \exp(-q\varepsilon)\dot{\varepsilon}_i^m \exp(-BT) \qquad (4)$$

Coefficients A, n, m and Q in equation (3) and A, n, q, m and B in equation (4) were determined using inverse analysis of the compression tests.

2.3. Metamodel

The FE models provide the nearest approximation of simulated processes, but they are usually time consuming. Lower computing costs are possible to achieve by applying the metamodel instead of the FE model. Metamodel is an abstraction created on the basis of lower level model of the analysed process built with an application of the selected methods of mathematical modelling (Kusiak et al., 2009). Any approximation of the model of the real process that gives reasonably reliable, approximate description of the considered process and allows significant decrease of the computing costs, can be considered as a metamodel. Various techniques can be used to build a metamodel, and the artificial intelligence techniques, in particular artificial neural networks, are the most frequently used. The ANN is capable to reproduce very complex relations, while the computing costs remain low. Full description of the Neural Networks can be found in (Tadeusiewicz, 1993; Bishop, 1995; Haykin, 1999).

Four ANN based metamodels were applied in the present work. Two of them were used to determine coefficients in equation (3) (one for the uniaxial compression test and second for the plane strain compression test). The other two metamodels were used to determine coefficients in equation (4) for both tests. Each metamodel consists of ten artificial neural networks built with the multi layer perceptrons (MLPs), which are frequently used in metamodelling of static processes. Each metamodel predicts the load values recorded at ten, specific deformation time steps (the number of time intervals is selected arbitrary by the user).

The number of neurons in the input layer corresponds to the number of the parameters of the flow stress equation and parameters of the test conditions (temperature, strain rate, friction), while a single neuron of the output layer corresponds to the load value of the compression test. The user chooses the number of hidden layers and the number of neurons in the hidden layers, arbitrarily. A root mean square error (RMS) was used as a measure of the accuracy of the neural network metamodel.

Trained metamodel predicts the loads during compression test based on the information regarding the characteristic material parameters (parameters of the flow stress equation) and the test (temperature and strain rate). The accuracy of metamodels is presented in table 3 (paragraph 4.2).

3. IDENTIFICATION OF THE FLOW STRESS EQUATION USING INVERSE APPROACH

3.1. Experiment

Plane strain compression of the copper alloy in the temperature range 20-300°C and strain rate range 0.1-10 s^{-1} was considered in the present work. Sample dimensions were 10×15×20 mm and the width of the die was 5 mm. Chemical composition of the alloy was 0.81%Cr, <0.001%Ni, <0.001%Si, 0.026%Fe, <0.001%As, <5 ppm Bi and balance Cu. Uniaxial compression tests were performed in the same conditions for comparison. The cylindrical samples dimensions were ϕ10×12 mm. Both types of tests were performed at constant strain rate (decreasing velocity of the die). Figure 3 shows all loads recorded in the PSC and UC tests for the investigated copper alloy. Analysis of these results allows to conclude that the investigated copper alloy shows large sensitivity to the temperature and very small sensitivity to the strain rate.

3.2. Inverse analysis

The identification of the plane strain compression test results was performed by the inverse calculations. The problem of the parameter identification was defined as the inverse problem. The commonly used method to solve such a task, where the direct problem is formulated with nonlinear differential equations, is to transform it to the optimization task with the goal function introduced as the distance between calculated and measured data in the norm defined in the functional value space. The minimum of the goal function is the solution of the inverse problem (Kirsch, 1996; Szeliga, 2013). The inverse algorithm dedicated to such identification consists of three steps (figure 4):

- Performing physical experiments and collecting the obtained data.
- Performing simulations of the experiments at constant physical conditions (temperatures, strain rates) and elaborating the collected data.
- Running optimization procedure to minimize the difference between measured and calculated data with respect to the parameters of the material flow stress model.

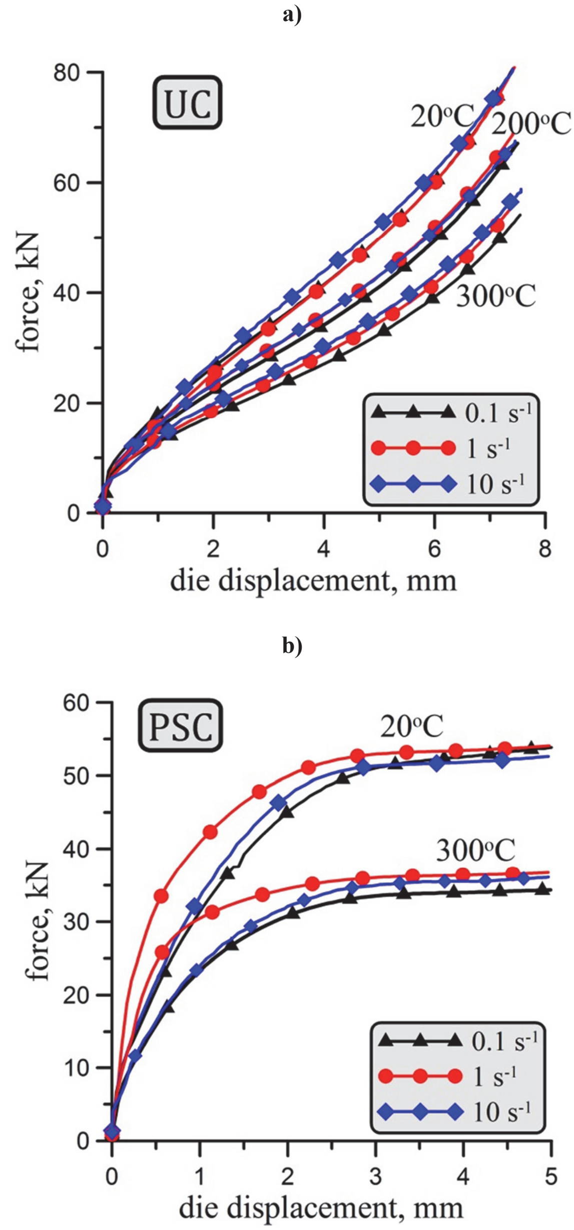


Fig. 3. Results of measurements of forces in the uniaxial compression tests (a) and the plane strain compression tests (b).

For the optimization task, the goal function was defined as the mean square root error between measured and calculated loads:

$$\Phi(\mathbf{x}) = \sqrt{\frac{1}{Nt}\sum_{i=1}^{Nt}\left\{\frac{1}{Ns}\sum_{j=1}^{Ns}\left[\frac{F_{ij}^{c}(\mathbf{x}) - F_{ij}^{m}}{F_{ij}^{m}}\right]^2\right\}} \quad (5)$$

where: $\mathbf{x}$ – vector of the material parameters ($\mathbf{x} = \{A, n, m, Q\}^T$ - parameters of the flow stress equation (3) or $\mathbf{x} = \{A, n, q, m, B\}^T$ - parameters of the flow stress equation (3)), F_m, F_c – measured and calculated loads in the compression test, Ns – number of time intervals in one test, Nt- number of tests (it accounts for all tests performed at various temperatures and various strain rates).

Details of the inverse algorithm used in the present work, are given by Szeliga et al. (2006). Simulations of the plastometric test performed with the FE software described in section 2.2 give accurate results including prediction of inhomogeneities of strain/stress and temperature fields. The inhomogeneities of strains are due to friction conditions in the die–specimen contact surface and due to the effect of the rigid ends (see section 2.1). The inhomogeneities of temperature are caused by the heat generation due to friction and deformation work. On the other hand, such computations are time consuming even when the 2D FE solution is used. The inverse analysis, requiring the iterative recalculations of the FE models, makes the whole identification procedure useless from the practical point of view. Lowering high computing costs is possible to achieve by applying the metamodel defined in section 2.2, instead of the FE model. The idea of the solution is presented in figure 4. This approach was successfully used by Sztangret et al. (2012) for the uniaxial compression tests.

the parameters originate from some laws of physics. Extended Hollomon equation (3) is a kind of phenomenological/approximation equation. Thus the uniqueness of the identification of parameters in this equation is not guaranteed. There could be a lot of local minima with the same value of the goal function and all of them could be applied to the modelling. These problems have been encountered when identification on the basis of uniaxial compression tests was performed by Sztangret et al. (2012). Solving the optimization problems characterized by multi-modal objective function of more than one local minimum is difficult and sometimes fails, when the classical, deterministic gradient or non-gradient methods are applied. Therefore, the effective methods of searching for the global minimum not only within a local region, but also in the whole domain of the objective function, were developed. The probabilistic methods belong to this group of optimization techniques. Due to the random search, which is the base of these methods, the probability of finding a global minimum increases. The algorithms of the

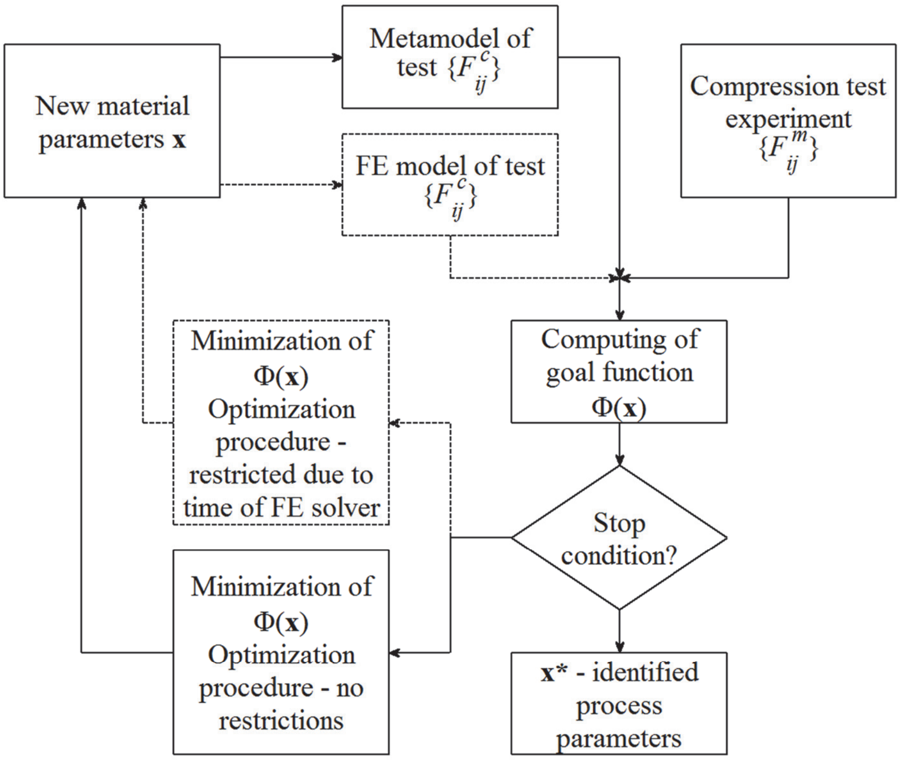


Fig. 4. *Flow chart of the inverse algorithm with metamodel (a - solid line) and FE model (b -dotted line).*

Besides the problem of high computing time of the inverse analysis based on the classical FE approach, there is another problem caused by the fact that the optimized goal function (5) is often multimodal. To avoid that problem the searching space should be narrowed (Kirsh, 1996). This is possible if

probabilistic methods are the results of observation of the nature and its perfect mechanisms: evolution, behaviour of population of individuals, etc. These algorithms can be applied to any kind of the objective function: non-linear, discontinuous or multimodal, where classical algorithms fail. Optimization algorithms inspired by the nature belong to the group of heuristic algorithms, and although they belong to the iterative algorithms, requiring many calculation runs, they can be effectively used here because calculation times of the objective function based on the metamodel are very short. Fast development of these optimization methods has been recently observed, e.g. efficient implementations of evolutionary algorithms (EAs) using graphics processing unit (GPU) were proposed for solving various problems in materials science (Baumes et al., 2011).

In the inverse calculations of material parameters of the plastometric test with the metamodel approach, the following nature inspired algorithms were implemented:

- genetic algorithms,
- evolutionary algorithms,
- particle swarm optimization,
- simulated annealing method,
- modified particle swarm optimization method.

The modification of the particle swarm optimization method is based on the introduction of the sensitivity analysis of the objective function with respect to the optimization variables. Information obtained from sensitivity analysis was used in dynamic control of the particles behaviour in the swarm. It improved the convergence of the optimization procedure (Sztangret et al., 2010).

4. RESULTS OF IDENTIFICATION

In the conventional interpretation of the compression tests stress is calculated as force-to-contact area ratio and strain is calculated as $\ln(h_0/h)$ for the UC test and $(2/\sqrt{3})\ln(h_0/h)$ for the PSC test, where h is the current height of the sample. It means that this interpretation assumes uniform strain and stress distribution, what leads to erroneous results. Figure 5 shows comparison of the flow stress calculated from the UC and PSC tests using conventional interpretation. Of course, methods of correction of this interpretation have been known long time ago but the most accurate results giving the same flow stress from the two tests were obtained when inverse analysis was applied (Szeliga et al., 2002).

4.1. Conventional inverse analysis with FE model

Conventional two-stage inverse analysis (Szeliga et al, 2006) was performed first. Two stages were applied to avoid extreme times of computations. In the first stage each test was investigated separately and the stress-strain curve in a tabular form was obtained. Simplified corrections for variations of the temperature and the strain rate were introduced. These tabular data were approximated next using equation (3) or (4) and coefficients obtained in the approximation $\mathbf{x} = \{A, n, m, Q\}^T$ or $\mathbf{x} = \{A, n, q, m, Q\}^T$ were used as a starting point for the second stage of the inverse analysis. This point was usually close to the minimum of function (5) and the solution could be obtained in a reasonable time, in particular when non-gradient optimization methods were used. Figure 6 shows stress strain curves obtained in a tabular form from the first stage of the analysis. It is seen that the results are consistent.

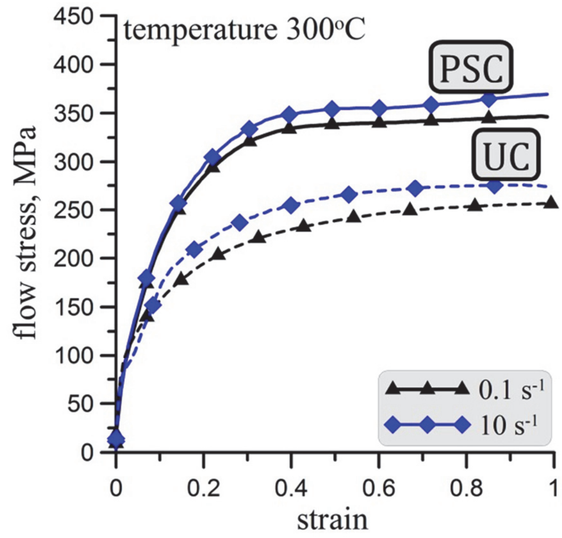


Fig. 5. *Flow stress calculated as force-to-contact area ratio presented as a function of the uniform strain for the temperature of 300°C.*

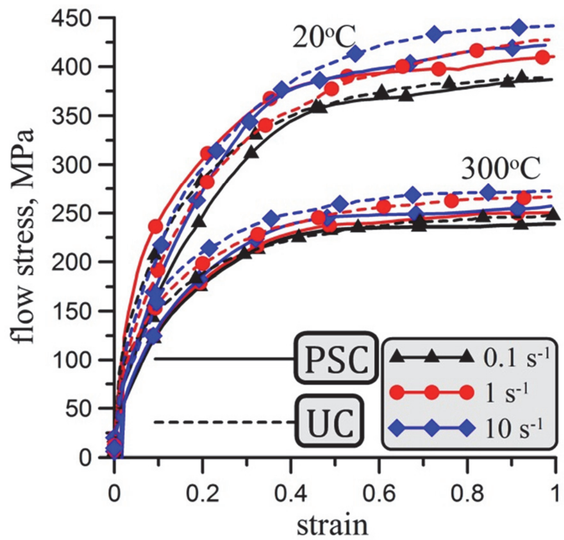


Fig. 6. *Flow stress calculated in a tabular form from the first stage of the inverse analysis.*

Coefficients in equation (3) obtained from the second stage of the conventional inverse analysis are given in table 1. Coefficients in equation (4) obtained from the second stage of the conventional inverse analysis are given in table 2. The final value of the objective function (5), which represents the accuracy of the solution, is given in the last column of these tables.

Table 1. *Coefficients in equation (3) obtained using various inverse algorithms for various tests.*

test	*A*, MPa	*n*	*m*	*Q*, J/mol	Φ
1. UC inverse + FE	193.2	0.301	0.02	2000	0.0983
2. UC/PSC inverse + FE	177.4	0.298	0.016	2167	0.112
3. UC inverse + metamodel	197.6	0.264	0.016	1975	0.0827
4. PSC inverse + metamodel	204.2	0.291	0.032	1468.6	0.133
5. UC/PSC inverse + metamodel	201.0	0.275	0.024	1718.6	-

Table 2. *Coefficients in equation (4) obtained using various inverse algorithms for various tests.*

test	*A*, MPa	*n*	*q*	*m*	*Q*, J/mol	Φ
1. UC inverse + FE	742	0.492	0.5928	0.0178	0.00148	0.0519
2. UC/PSC inverse + FE	740.8	0.4968	0.5793	0.0175	0.00157	0.0476
3. UC inverse + metamodel	698.7	0.444	0.556	0.022	0.00138	0.0349
4. PSC inverse + metamodel	772.5	0.55	0.539	0.0183	0.00143	0.114
5. UC/PSC inverse + metamodel	735.6	0.497	0.547	0.0201	0.0014	-

4.2. Metamodel of the experiment

The metamodels of the PSC and UC tests (in fact four different metamodels - two for equation (3) and two for equation (4) for each of the tests) were built. Each metamodel predicts load values in ten specific deformation time steps, therefore it consists of ten different ANNs (as described in section 2.3). The input signals of each network were: temperature, effective strain rate and coefficients in flow stress equation (four in case of equation (3) and five in case of equation (4)). The compression force F for the specified time interval was the ANN output signal. The ANNs were designed with typical MLP neural networks. The number of records used to train and to test each metamodel are presented in table 3.

Table 3. *Number of records used to train and to test metamodels.*

Metamodel		Training set	Testing set
Equation (3)	UC	10742	1194
	PSC	2007	224
Equation (4)	UC	1062	118
	PSC	643	72

Each record consisted of six or seven input signals and one output signal. A root mean square error was used as a measure of the accuracy of the neural network metamodel. Several tests were performed to adjust optimal topologies of the networks used in metamodel. The topologies and errors are presented in table 4. The errors were relatively low and metamodel can be used in the inverse analysis instead of the FE model.

4.3. Inverse analysis with metamodel

Inverse analysis with the metamodel used as a direct problem model (figure 4a) was performed for both UC and PSC tests. Coefficients in equation (3) obtained from this analysis are given in table 1 in rows 3 and 4. Average coefficients for both tests are given in row 5 of this table. These coefficients were obtained by averaging flow curves obtained from the two tests, therefore, the final value of the objection function is not given in this case. Coefficients in equation (4) obtained from the inverse analysis with the metamodel are given in table 2 in rows 3 and 4. Average coefficients for both tests are given in row 5 of this table. As previously, these coefficients were obtained by averaging flow curves obtained from the two tests.

Analysis of the results shows that conventional inverse analysis gave similar flow curves for both UC and PSC tests, therefore, flow stress determined on the basis of both tests (row 5 in table 1 and table 2) was used in all comparisons below. Contrary, inverse analysis with the metamodel gave slightly different results for the UC test and PSC test. Figure 7 shows comparisons of the flow stress calculated from equation (3) and from equation (4) with coefficients determined on the basis of both UC and PSC tests using conventional inverse analysis with the FE model and using inverse analysis with the metamodel. It is seen that reasonably good agreement between the two inverse approaches was obtained and that this agreement is better for equation (4).

Table 4. *Topologies and errors of used metamodels.*

Force	Equation (3)				Equation (4)			
	UC		PSC		UC		PSC	
	Topology	Error [%]	Topology	Error [%]	Topology	Error [%]	Topology	Error [%]
F1	6-23-6-1	0.0103	6-18-4-1	0.5863	7-21-9-1	0.6025	7-24-1	2.0554
F2	6-18-5-1	0.0098	6-20-1	0.7684	7-14-9-1	0.4158	7-25-1	1.9954
F3	6-28-1	0.0095	6-16-1	0.9477	7-28-2-1	0.4411	7-30-4-1	1.8496
F4	6-24-1	0.0094	6-25-6-1	0.9325	7-29-7-1	0.4126	7-17-1	2.7726
F5	6-15-4-1	0.0095	6-22-1	1.3063	7-16-6-1	0.4086	7-24-1	2.9332
F6	6-30-1	0.0095	6-15-1	1.4829	7-12-1	0.5206	7-26-1	2.8582
F7	6-11-5-1	0.0088	6-28-1	2.9868	7-18-9-1	0.4964	7-18-1	3.2004
F8	6-24-9-1	0.0082	6-15-1	3.0892	7-21-1	0.6007	7-26-1	3.3412
F9	6-21-10-1	0.0079	6-28-1	2.8627	7-24-5-1	0.7588	7-11-1	3.0462
F10	6-24-1	0.0077	6-12-1	3.0233	7-18-1	1.3943	7-16-1	4.4424

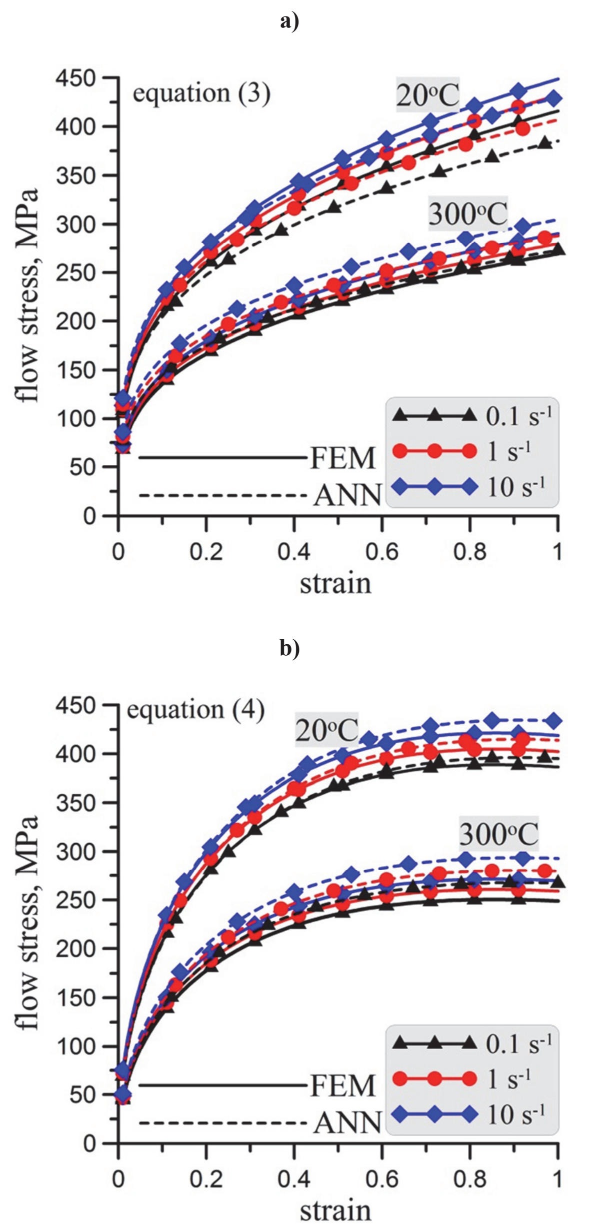


Fig. 7. *Flow stress calculated from equation (3) (a) and from equation (4) (b) with coefficients determined on the basis of both UC and PSC tests using conventional inverse analysis with the FE model and using inverse analysis with the metamodel.*

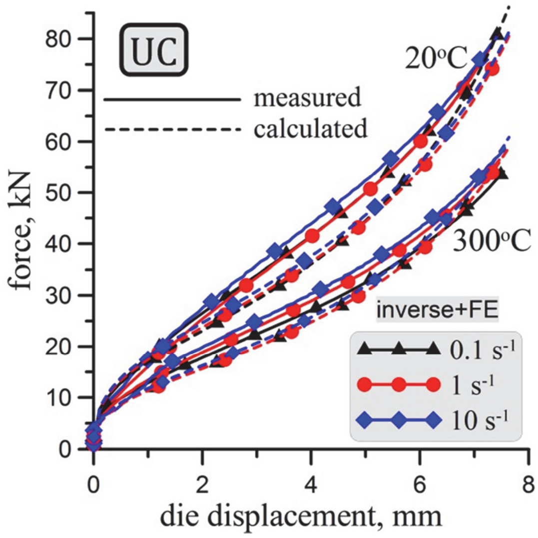
5. RESULTS OF VALIDATION OF MODELS AND DISCUSSION

The objective of the paper was evaluation of possibility of making the inverse analysis more efficient without losing accuracy of the solution. Results of validation of subsequent approaches are presented below. It was shown earlier that very good agreement was obtained for the first stage of the conventional inverse analysis, what is shown in figure 6. Accuracy of the identification of the coefficients in the model depends, to a large extend, on the flexibility of the model and its capability to describe response of the material in a wide range of strain rates and temperatures.

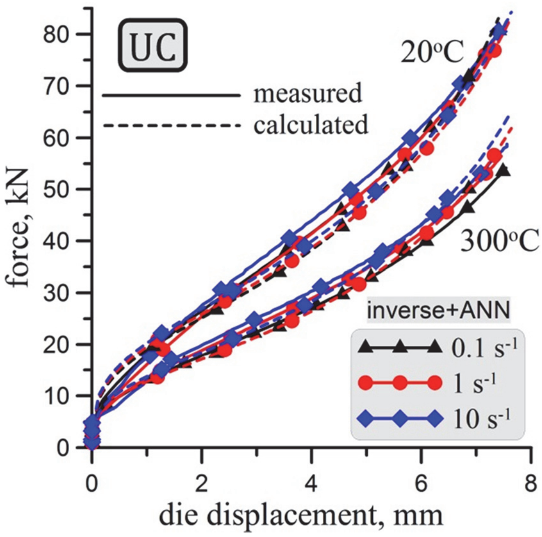
Verification of the models was performed by comparison between forces measured in the tests and calculated by the FE code with the considered model introduced in the constitutive law (1). Identification of the coefficients in equation (3) was considered first. Figure 8 shows comparison of forces measured in the uniaxial compression tests and calculated by the FE code with equation (3) for the following three sets of coefficients:

- Determined by the conventional inverse analysis with the FE model for both UC and PSC tests (row 2 in table 1) – notice that there was no difference between flow stress determined from the UC test and PSC test separately.
- Determined by the inverse analysis with metatamodel for UC tests (row 3 in table 1).
- Determined by the inverse analysis with metatamodel for both UC and PSC tests (row 5 in table 1).

a)



b)



c)

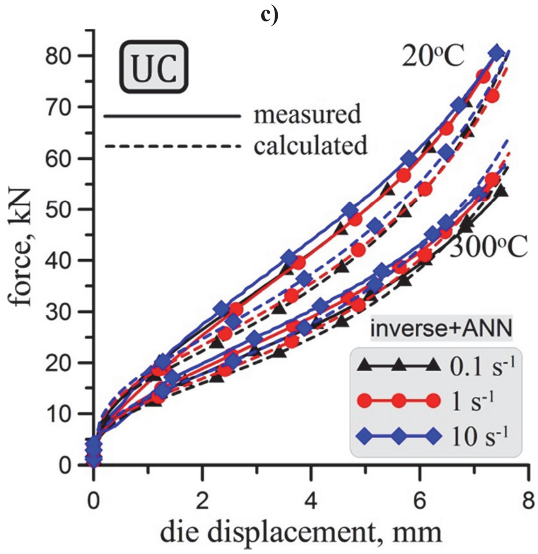
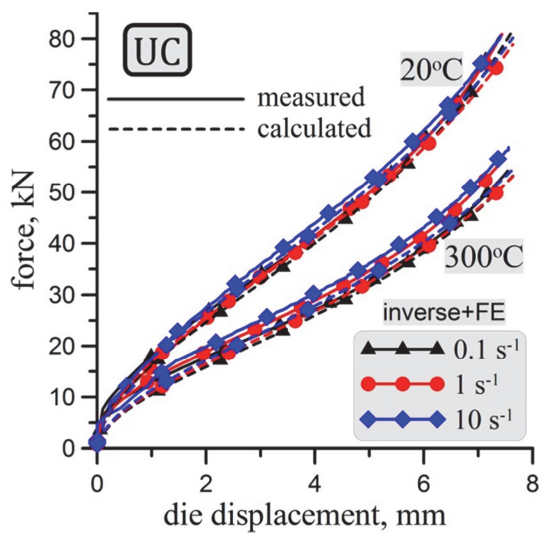


Fig. 8. *Comparison of forces measured in the uniaxial compression tests and calculated by the FE code with equation (3) and coefficients in row 2 (a), row 3 (b) and row 5 (c) in table 1.*

a)



b)

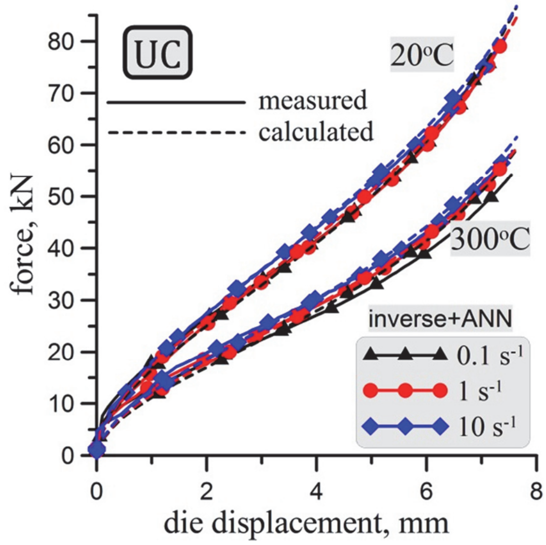
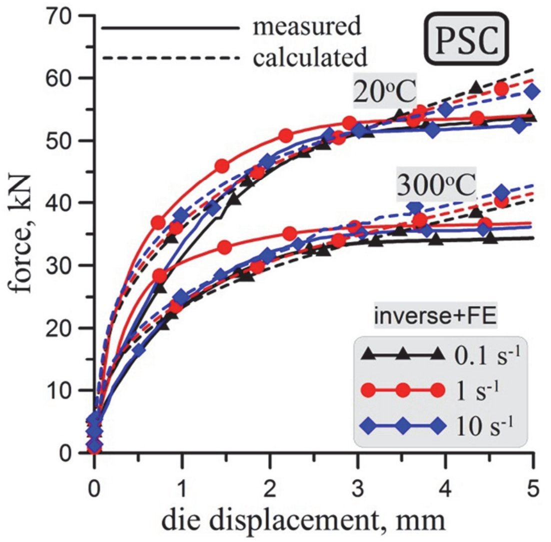


Fig. 9. *Comparison of forces measured in the uniaxial compression tests and calculated by the FE code with equation (4) with of coefficients obtained from the conventional inverse analysis for both UC and PSC tests (row 2 in table 2) and obtained from the inverse analysis with the metamodel also for both UC and PSC tests (row 5 in table 2).*

It is seen in figure 8 that good result was obtained for conventional inverse approach independently whether only data from the UC test were used (these results are not presented) or both tests were used in the inverse analysis (figure 8a). Very good results were obtained from the inverse analysis with metamodel when UC data only were used (figure 8b). Identification of coefficients of equation (3) using the inverse analysis with the metamodel for both tests gave worse results. The general conclusion from the results in figure 8 is that equation (3) has no capability to describe properly behaviour of the material in a wide range of temperatures and strain rates.

Identification of coefficients in equation (4) was considered next. Figure 9 shows comparison of forces measured in the uniaxial compression tests and calculated by the FE code with equation (4) for the two sets of coefficients obtained from the conventional inverse analysis for both UC and PSC tests (row 2 in table 2) and obtained from the inverse analysis with the metamodel also for both UC and PSC tests (row 5 in table 2).

a)



b)

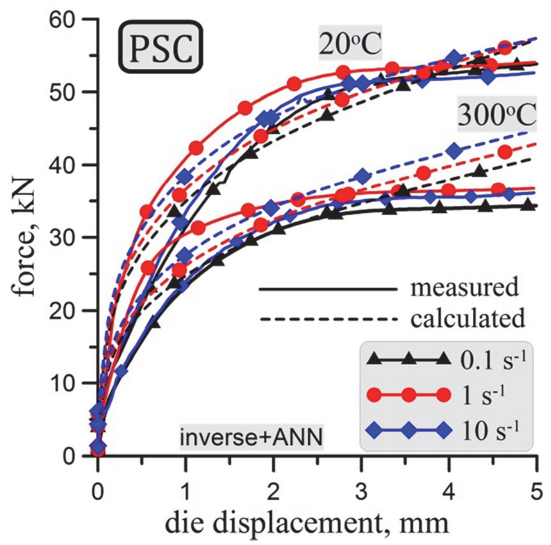
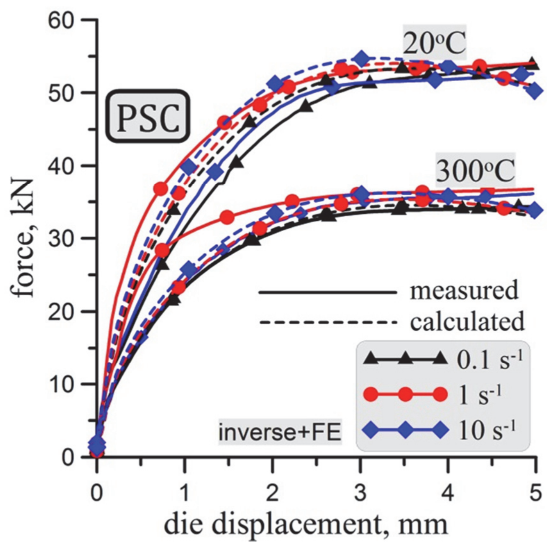


Fig. 10. *Comparison of forces measured in the uniaxial compression tests and calculated by the FE code with equation (3) with coefficients determined by the conventional inverse analysis for both UC and PSC tests (row 2 in table 1) (a) and with coefficients determined by the inverse analysis with the metamodel again for both UC and PSC tests (row 5 in table 1) (b).*

Similar analysis was performed for the PSC tests. Identification of coefficients in equation (3) was considered first. Figure 10a shows comparison of forces measured in the plane strain compression tests and calculated by the FE code with equation (3) with coefficients determined by the conventional inverse analysis for both UC and PSC tests (row 2 in table 1). Figure 10b shows comparison of forces measured in the plane strain compression tests and calculated by the FE code with equation (3) with coefficients determined by the inverse analysis with the metamodel for both UC and PSC tests (row 5 in table 1). It means that in both cases the coefficients in the material model were determined using inverse analysis for both UC and PSC tests.

a)



b)

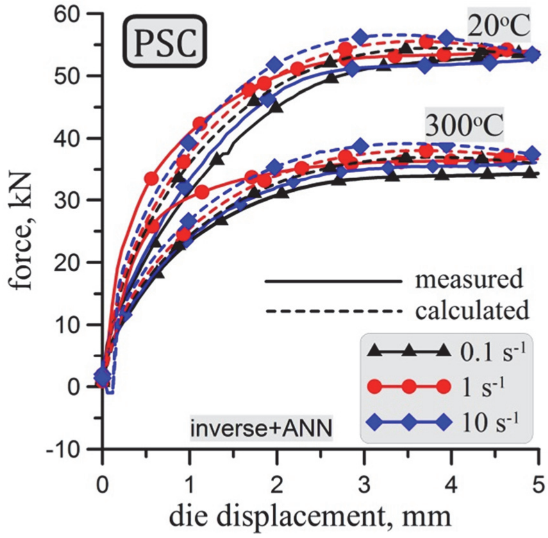


Fig. 11. *Comparison of forces measured in the uniaxial compression tests and calculated by the FE code with equation (4) with coefficients determined by the conventional inverse analysis for both UC and PSC tests (row 2 in table 2) (a) and with coefficients determined by the inverse analysis with the metamodel also for both UC and PSC tests (row 5 in table 2) (b).*

The general conclusion from this part of the analysis is that function (3) is not flexible enough to describe properly material response in the investigated range of temperatures and strain rates. Identification of coefficients in equation (4) was considered next. Figure 11a shows comparison of forces measured in the plane strain compression tests and calculated by the FE code with equation (4) with coefficients determined by the conventional inverse analysis for both UC and PSC tests (row 2 in table 2). Figure 11b shows comparison of forces measured in the plane strain compression tests and calculated by the FE code with equation (4) with coefficients determined by the inverse analysis with the metamodel for both UC and PSC tests (row 5 in table 2). It means that again in both cases the coefficients in the material model were determined using inverse analysis for both UC and PSC tests.

It is seen in figures 9 and 10 that equation (3) is not capable to describe behaviour of the material in the plane strain compression test in the investigated range of temperatures and strain rates while quite good results were obtained for equation (4) for both methods of identification of the material model.

6. CONCLUSIONS

Evaluation of the possibility of identification of the flow stress model on the basis of the inverse analysis with metamodel for the PSC tests was the objective of the paper. Numerical tests allowed drawing the following conclusions:

- Identification of the material model based on the inverse analysis for the PSC tests is much more difficult and time consuming than for the UC tests.
- Conventional inverse analysis performed for the uniaxial compression and plane strain compression gave similar flow stress models. The model determined on the basis of both tests gave very good prediction of forces in the UC and PSC tests.
- Inverse analysis with the metamodel gave very good results for the uniaxial compression test. Slightly worse results were obtained when flow stress was determined on the basis of the inverse analysis for the two tests.
- Although the values of coefficients obtained from various tests may differ, the agreement between measured and calculated forces is good. It means that there is no unique solution of the problem, but the accuracy of the obtained solution is satisfactory.
- Inverse analysis with the metamodel is few orders of magnitudes faster than the conventional approach with the FE model. Accuracy of the former solution is quite good and it should be recommended for practical applications of the inverse analysis.

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REFERENCES

Davenport, S.B., Silk, N.J., Sparks, C.N., Sellars, C.M., 1999, Development of constitutive equations for the modelling of hot rolling, *Materials Science and Technology*, 16, 1-8.

Finno, R.J., Harris, W.W., Mooney, M.A., Viggiani, G., 1997, Shear bands in plane strain compression of loose sand, *Geotechnique*, 47, 149-165.

Forestier, R., Massoni, E., Chastel, Y., 2002, Estimation of constitutive parameters using an inverse method coupled to a 3D finite element software, *Journal of Materials Processing Technology*, 125, 594-601.

Gelin, J.C., Ghouati, O., 1994, An inverse method for determining viscoplastic properties of aluminium alloys, *Journal of Materials Processing Technology*, 45, 435-440.

Hand, R.J., Foster, S.R., Sellars, C.M., 2000, Temperature changes during hot plane strain compression testing, *Materials Science and Technology*, 16, 442-450.

Haykin, S., 1999, *Neural Networks: A Comprehensive Foundation*, Prentice Hall.

Hosford, W.P., 1966, Plane-strain compression of aluminum crystals, *Acta Metallurgica*, 14, 1085-1094.

Kirsch, A., 1996, *An Introduction to the Mathematical Theory of Inverse Problems*, Springer.

Kowalski, B., Sellars, C.M., Pietrzyk, M., 2000, Development of a computer code for the interpretation of results of hot plane strain compression tests, *ISIJ International*, 40, 1230-1236.

Kowalski, B., Wajda, W., Pietrzyk, M., Sellars, C.M., 2001, Sensitivity of constitutive equations on strain and strain rate inhomogeneity determined for FE modeling of plane strain compression tests, *Proc. 4th Conf. ESAFORM*, ed., Habraken, A.-M., Liege, 561-564.

Kowalski, B., Sellars, C.M., Pietrzyk, M., 2006, Identification of rheological parameters on the basis of plane strain compression tests on specimens of various initial dimensions, *Computational Materials Science*, 35, 92-97.

Kusiak, J., Danielewska–Tulecka, A., Oprocha, P., 2009, *Optymalizacja: wybrane metody z przykładami zastosowań*, PWN, Warszawa (in Polish).

Lenard, J.G., Pietrzyk, M., Cser, L., 1999, *Mathematical and physical simulation of the properties of hot rolled products*, Elsevier, Amsterdam.

Loveday, M.S., Mahon, G.J., Roebuck, B., Lacey, A.J., Palmiere, E.J., Sellars, C.M., van der Winden M.R., 2006,

Measurement of flow stress in hot plane strain compression tests, *Materials at High Temperatures*, 23, 85-118.
Madej, Ł., Hodgson, P.D., Pietrzyk, M., 2007, Multi-scale rheological model for discontinuous phenomena in materials under deformation conditions, *Computational Materials Science*, 38, 685-691.
Mirza, M.S., Sellars, C.M., 2001, Modelling the hot plane strain compression test Part 1 – Effect of specimen geometry, strain rate, and friction on deformation, *Materials Science and Technology*, 17, 1133-1141.
Nave, M.D., Barnett, M.R., 2004, Microstructures and textures of pure magnesium deformed in plane-strain compression, *Scripta Materialia*, 51, 881-885.
Pietrzyk, M., Lenard, J.G., Dalton, G.M., 1993, A study of the plane strain compression test, *Annals of the CIRP*, 42, 331-334.
Pietrzyk, M., Tibbals, J.E., 1995, Application of the finite element technique to the interpreta-tion of the plane strain compression test for aluminum, *Proc. Conf. COMPLAS 4*, (eds), Owen, D.R.J., Onate, E., Hinton, E., Pineridge Press, Barcelona, 889-900.
Pietrzyk, M., 2001, Identification of parameters in the history dependent constitutive model for steels, *CIRP Annals - Manufacturing Technology*, 50, 161-164.
Pietrzyk, M., Kuziak, R., Pidvysots'kyy, V., Nowak, J., Węglarczyk, S., Drozdowski, K., 2013, Computer-aided design of manufacturing chain based on closed die forging for hardly deformable Cu-based alloys, *Metallurgical and Materials Transactions A*, 44A, 3281-3302.
Sellars, C.M., 1979, Physical metallurgy of hot working, *Hot working and forming processes*, (ed.), Sellars C.M., Davies G.J., The Metals Soc., London, 3-15.
Silk, N.J., van der Winden, M.R., 1999, Interpretation of hot plane strain compression testing of aluminium specimens, *Materials Science and Technology*, 15, 295-300.
Szeliga D., 2013, *Metal forming identification problems - Comprehensive study*, Wydawnictwa AGH, Kraków.
Szeliga, D., Gawąd, J., Pietrzyk, M., 2006, Inverse analysis for identification of rheological and friction models in metal forming, *Computer Methods in Applied Mechanics and Engineering*, 195, 6778-6798.
Szeliga D., Matuszyk P., Kuziak R., Pietrzyk M., 2002, Identification of rheological parameters on the basis of various types of plastometric tests, *Journal of Materials Processing Technology*, 125-126, 150-154.
Szeliga, D., Pietrzyk, M., 2010, Identification of rheological models and boundary conditions in metal forming, *International Journal of Materials and Product Technology*, 39, 388-405.
Sztangret, Ł., Szeliga, D., Kusiak, J., Pietrzyk, M., 2012, Application of the inverse analysis with metamodelling for the identification of the metal flow stress, *Canadian Metallurgical Quarterly*, 51, 440-446.
Sztangret, Ł., Szeliga, D., Kusiak, J., 2010, Analiza wrażliwości jako metoda wspomagająca optymalizację parametrów procesów metalurgicznych, *Hutnik-Wiadomości Hutnicze*, 76, 721-725 (in Polish).
Sztangret, M., Pietrzyk, M., 2012, Próba ściskania próbek płaskich jako metoda symulacji fizycznej procesów przeróbki plastycznej oraz narzędzie do identyfikacji modeli reologicznych materiałów, *Hutnik-Wiadomości Hutnicze*, 79, 35-40 (in Polish).
Tadeusiewicz, R., 1993, *Sieci neuronowe*, Akademicka Oficyna Wydawnicza, Warszawa (in Polish).
Tatsuoka, F, Sakamoto, M., Kawamura, T., Fukushima, S., 1986, Strength and deformation characteristics of sand in plane strain compression at extremely low pressures, *Soils and Foundations*, 126, 65-84.
Timothy, S.P., Yiu, H.L., Fine, J.M., Ricks, R.A., 1991, Simulation of single pass of hot rolling deformation of aluminium alloy by plane strain compression, *Materials Science and Technology*, 7, 255-263.

METAMODEL JAKO ZAMIENNIK MODELU MES PRÓBY SPĘCZANIA W PŁASKIM STANIE ODKSZTAŁCENIA W ANALIZIE ODWROTNEJ

Streszczenie

Artykuł przedstawia zastosowanie próby spęczania w płaskim stanie odkształcenia w identyfikacji parametrów modelu materiału. Sprawdzono również możliwość zastąpienia modelu MES przez metamodel w analizie odwrotnej. Istnieje kilka prób plastometrycznych wykorzystywanych w celu wyznaczenia własności materiału. Pomimo, że próba PSC jest bardzo dokładna, najczęściej stosowana jest próba spęczania osiowosymetrycznego. Powodem tego są trudności w interpretacji wyników próby PSC. Rozwiązaniem wspomnianego problemu jest zastosowanie analizy odwrotnej. Wymaga ona jednak wyników symulacji zastosowanej próby. Symulacje te najczęściej wykorzystują model MES. Jednak ze względu na gęstość siatki MES w obszarach, w których wartość gradientu odkształcenia jest największa, czas symulacji staje się nieakceptowalnie długi. Z tego powodu sztuczne sieci neuronowe zostały wykorzystane jako metamodel próby PSC i zastosowane w analizie odwrotnej mającej na celu wyznaczenie parametrów materiału dla stopów miedzi. Porównując wyniki otrzymane z wykorzystaniem próby PSC i próby spęczania osiowosymetrycznego otrzymano dobrą zgodność.

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