Publishing House
AKAPIT

COMPUTER METHODS IN MATERIALS SCIENCE
Informatyka w Technologii Materiałów

Vol. 14, 2014, No. 3

SURVEY OF EFFECTIVENESS OF INVERSE ANALYSIS COMPUTATION

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Abstract

The paper presents a survey of inverse analysis focusing on two aspects: computing time and accuracy of the solution. Identification of flow stress model in metal forming processes was considered as the inverse problem. This identification is usually performed by coupling the FE model with optimization techniques which leads to long computing times. Application of the metamodel instead of FE model in the inverse analysis was proposed as a solution of this problem. The second dilemma concerns the choice of the best optimization method. Several bio-inspired optimization algorithms were used in the inverse calculations. Comparison of obtained results is presented in the paper.

Key words: inverse analysis, bio-inspired optimization methods, metamodel

1. INTRODUCTION

Problem of accurate description of boundary conditions and material rheology in metal forming processes still presents difficulties. The coefficients in the rheological, friction and heat transfer models are usually determined from the results of various laboratory tests. All tests involve large inhomogeneities of strains, stresses and temperature, therefore, an interpretation of the results of these tests is a complex problem. Inverse analysis, combined with the Finite Element (FE) solution of the direct problem, has been commonly used to account for all these inhomogeneities. Numerous successful applications of this analysis are described in the literature (Gelin & Ghouati, 1994; Forestier et al., 2002; Szeliga et al, 2006). However, during inverse analysis computation two main problems arise: computing time and accuracy of the solution.

The first problem is caused by coupling the FE model with optimization techniques. FE models provide a good approximation to the simulated processes, but are usually time-consuming. Solution of this problem can be application of metamodel instead of FE model. This approach was proposed e.g. by Sztangret et al. (2012). The fundamental idea of metamodelling relates to an assumption that metamodel (called also surrogate model) approximates the model of analyzed process within the experimental region of interest. Metamodel must clearly and accurately correspond to the model and the metamodel output signal has to be evaluated with a significantly lower computational effort than using the original model. The accuracy of metamodel (usually verified using the methods of statistical evaluation) depends on the used metamodelling technique and number of the measurements data points. Usually, the higher number of points gives the better metamodel accuracy. On the other hand, each data point requires time consuming simulation run. Therefore, it is advisable to maximally reduce the number of necessary data points which can be done using one of common techniques of Design of Experiment (Myers & Montgomery, 1995). More information about metamodelling can be found e.g. in Kusiak et al. (2012). Within this paper Artificial Neural Net-

 ISSN 1641-8581

work (ANN) was used as metamodel of axisymmetrical compression test. The detailed description of ANN can be found in Tadeusiewicz (1993).

Selection of the applied optimization method has influence on both computing time and accuracy of the obtained results. Some advice concerning the accurate choice can be found in Kuś and Mucha (2014) and Sztangret (2014). In the case of metallurgical processes the optimized goal function is usually multimodal therefore, bio-inspired optimization methods are most commonly used. Several bio-inspired algorithms were implemented and used as optimization method in the inverse analysis. The general practical objective of this work was selection of the optimization methods, which will be implemented in the inverse analysis.

2. INVERSE ANALYSIS

Integral or differential equations describing any physical phenomena are set out, in terms of functional analysis, as:

$$K: X \to Y \tag{1}$$

where X and Y are normalized spaces and K is a mapping (linear or nonlinear).

The direct problem is formulated as evaluating $y = K(x) \in Y$ for given $x \in X$ and an operator K that is equivalent to solve a boundary value problem for differential equation or to evaluate an integral. The inverse problem is defined as evaluating the $x \in X$ value for given K and $y \in Y$.

It could be shown that inverse problems described as the integral/differential equations are ill-posed, i.e. at least one of the conditions is not satisfied: a solution exists, it is unique and it continuously depends on input data (stability), Hadamard (1923). Those problems require regularization procedure and one of the solution is transforming them to the following, well-posed, problems:

$$\Phi(\tilde{x}) \coloneqq \|K\tilde{x} - y\|^2 \tag{2}$$

The form (2) leads to minimization with respect to the parameters, which are identified: boundary conditions parameters, material parameters or process parameters. In terms of optimization terminology, the inverse problem is to find the minimum of the objective function (2).

As it was mentioned above, in many practical application partial differential equations describe a physical process. As an example, a hot and cold compression tests for plastic deformation are considered in this work as a direct problem. It is assumed that the material is a rigid-plastic body and two models: a mechanical one and a thermal one define the deformation process. The inverse problem, called an identification task, is to determine the parameters of a rheological model of a material.

Then the objective function (2) is of the form (Szeliga, 2013):

$$\Phi(\mathbf{x}) = \sqrt{\sum_{i=1}^{n} \left[\frac{F_i^c(\mathbf{x},\mathbf{p}) - F_i^m}{F_i^m}\right]^2} \tag{3}$$

where: F^c, F^m – calculated and measured loads during the compression process, respectively, n - number of sampling points, $\mathbf{x}$ – vector of parameters of a rheological model, $\mathbf{p}$ – vector of process variables (e.g. temperatures, strain rates).

In the case considered in this work, two rheological models of a deformed material are analysed. First model includes softening which occurs when deformation is conducted at higher temperature, and is given by following formula:

$$\sigma_P = A\varepsilon^n e^{-q\varepsilon} \dot{\varepsilon}^m e^{-\beta T} \tag{4}$$

where: ε is the strain; $\dot{\varepsilon}$ is the strain rate; T is the temperature; and A, n, q, m, β are parameters which have to be evaluated in the inverse analysis.

The second model is accurate in the case of deformation performed at low temperature:

$$\sigma_P = A\varepsilon^n \dot{\varepsilon}^m e^{\left(Q/_{RT}\right)} \tag{5}$$

where: ε is the strain; $\dot{\varepsilon}$ is the strain rate; T is the temperature (in Kelvin); R is the gas constant; and A, n, m, Q are parameters which have to be evaluated in the inverse analysis.

The robust and efficient optimisation of the goal function (3) is crucial to solve an inverse problem due to multimodal character of the objective function. Following that a wide range of the optimisation procedures was analysed and applied. They are presented in the next section.

3. BIO-INSPIRED OPTIMIZATION ALGORITHMS

Biology gives inspiration to optimization algorithms, which can be used to enhance the search efficiency of the global optimum of considered problem (Sztangret et al., 2009). The optimization methods employed in determination of the coefficients of the equations (4) and (5) are briefly described below.

3.1. Genetic Algorithm

Genetic Algorithm (GA) is based on the evolution theory (Arabas, 2001; Cytowski, 1996; Deb 2001; Goldberg, 1995; Schwefel, 1995). It uses binary coded population of individuals. Each individual represents one solution of the problem. GA uses three operators, which are executed iteratively: reproduction, crossover and mutation. The reproduction operator chooses and copy individuals to new population according to their fitness value (fitness value is linked to the optimization objective function). The higher value of the fitness function, the higher probability has an individual to be chosen for reproduction. The crossover operator takes two steps. In the first step two members of new population are randomly chosen. In the second step, parts of genotypes are exchanged between each other. The mutation operator is a random chromosome's bit inversion. The GA stops when the given maximal number of iterations is attained or there is no significant change of the fitness value.

3.2. Evolutionary Algorithms

Evolutionary Algorithms (EAs) are very similar to GA (Arabas, 2001; Cytowski, 1996; Schwefel, 1995). They are based on the same assumptions. The difference lies in the fact that individuals are not coded. There are several versions of the EAs and following three of them were considered in the work: (1+1), (μ+λ) and (μ,λ) strategies. In the first one the population consists of one individual only. There is no crossover operator. Reproduction is confined to choose individual with higher fitness value. In the (μ+λ) strategy the current population consists of μ individuals and all three operators (same as in GA) are used. The new population of λ individuals is created. Population in the next generation contains μ individuals chosen from the association of the current and new populations of the higher objective function values. The only difference between strategies (μ+λ) and (μ,λ) is that current population in successive generation is created by choosing μ individuals from new population.

3.3. Simulated Annealing Method

The Simulated Annealing (SA) method shows the analogy to the cooling of metals (Cytowski, 1996). At high temperatures molecules move unconstrained, because they have high energy. If the metal is cooled slowly, thermal mobility decreases and in consequence, the energy of molecules drops. Eventually the system reaches minimum of its energy. Algorithm based on SA works on only one solution. Primary operation of the algorithm is to generate next approximation of the best solution. This operation is very similar to the mutation operator of the Evolutionary Algorithms.

3.4. Particle Swarm Optimization

Particle Swarm Optimization method (PSO) is inspired by the behaviour of swarms of birds, insects or fish shoal looking for food or shelter (Foryś, 2007; Kennedy & Eberhart, 1995). Every member of the swarm searches its neighbourhood but also follow the others, usually the best member of the swarm. In the algorithm based on this behaviour, the swarm is considered as particles representing single solutions. Each particle is characterized by its own position and the velocity. Particles move through decision space and the best position they ever had is remembered and returned as the optimal solution.

4. INVERSE ANALYSIS RESULTS

The aim of inverse analysis was obtaining the coefficients of constitutive models based on plastometric tests for DP600 steel and CuCr alloy. The chemical compositions of investigated materials are given in tables 1 and 2.

Table 1. *Chemical composition of DP600 steel [%].*

C	Mn	Si	Cr	Mo	V	Ti	P	S
0.071	1.45	0.25	0.55	0.03	0.005	0.002	0.01	0.006

Table 2. *Chemical composition of CuCr alloy [%].*

Cu	Cr	Ni	Si	Fe	As	Bi
balance	0.81	<0.001	<0.001	0.026	<0.001	<5 ppm

In the case of DP600 steel plastometric tests were conducted at high temperature (above 800°C). At that temperature it is important to take into account the softening phenomenon. Therefore, the constitutive model took the form according to the equation (4). Plastometric tests for CuCr alloy were performed at low temperature (below 300°C). In this case, the constitutive model was described by equation (5).

Inverse calculations were performed by coupling FE model with simplex optimization method and metamodel with bio-inspired optimization algorithms. Two metamodels of the hot and cold axisymmetrical compression test were built using ANN technique. Both metamodels consist of nine different ANNs (each for different tool displacement). As an input temperature, effective strain rate and coefficients in constitutive model were selected. The compression force F for the specified tool displacement was the ANN output signal. The ANN was designed with typical MLP neural network. A root mean square error (RMS) was used as a measure of the accuracy of the neural network model. Several tests were performed to adjust optimal topologies of the networks used in metamodel. The error was calculated as:

$$e = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\frac{F_i^c - F_i^m}{F_{i\,\max}^m - F_{i\,\min}^m}\right)^2} \qquad (6)$$

where: n – number of tests, F_i^c, F_i^m – force calculated by the ANN and measured, respectively, $F_{i\,\max}^c, F_{i\,\min}^m$ – maximum and minimum force in the experiment, respectively.

The errors of each network are presented in table 3.

Table 3. *The errors of neural networks.*

Force	e [%]	
	Hot compression test	Cold compression test
F1	0.1697	0.0248
F2	0.1611	0.0254
F3	0.1910	0.0262
F4	0.2466	0.0274
F5	0.2236	0.0302
F6	0.2195	0.0349
F7	0.3126	0.0385
F8	0.2679	0.0426
F9	0.3797	0.05

All errors are relatively low and metamodel can be used in the inverse analysis instead of the FE model.The optimization methods described in chapter 3 have been implemented and used to determine the coefficients of equation (4) and (5). The objective function was given by equation (3). Identification was performed using FE model and ANN based metamodel. Obtained coefficients in equations (4) and (5), as well as the values of the objective function (3) are presented in tables 4 and 5 respectively.

Table 4. *Coefficients in equation (4) obtained using inverse analysis with FE model and ANN based metamodel.*

	A	N	q	m	β	Φ
Simplex + FE	6038.8	0.376	0.52	0.105	0.003368	0.1305
(1+1) + ANN	3938.1	0.3653	0.6145	0.113	0.0028844	0.1232
(μ,λ) + ANN	4753.3	0.4128	0.7466	0.1122	0.0029655	0.1241
(μ+λ) + ANN	3859.7	0.3373	0.5238	0.11	0.00293	0.1236
GA + ANN	5957.7	0.4565	0.8095	0.1069	0.003046	0.1276
PSO + ANN	3473.2	0.2747	0.3069	0.1143	0.003	0.1269
SA + ANN	2677.5	0.2288	0.1184	0.1259	0.00289	0.134

Table 5. *Coefficients in equation (5) obtained using inverse analysis with FE model and ANN based metamodel.*

	A	n	m	Q	Φ
Simplex + FE	193.23	0.3	0.02	2000.4	0.0983
(1+1) + ANN	197.38	0.2596	0.0149	1955.3	0.0825
(μ,λ) + ANN	197.33	0.2574	0.0147	1942.8	0.0825
(μ+λ) + ANN	199.97	0.2604	0.0135	1920.3	0.0826
GA + ANN	199.88	0.2587	0.0128	1915	0.0827
PSO + ANN	196.74	0.259	0.0167	1963.4	0.0826
SA + ANN	161.4	0.2924	0.0209	2813.5	0.1097

It was assumed that inverse + FE approach is an accurate method of identification of the rheological model on the basis of compression tests, what was confirmed in numerous publications (e.g. Szeliga & Pietrzyk, 2007; 2010). Therefore, FE simulations of the compression tests were performed to validate approach with metamodel. Equations (4) and (5) with coefficients given in tables 4 and 5 respectively, were introduced in the constitutive law in the FE model. Figure 1 shows comparison between force calculated based on sets of obtained coefficients. Black lines represent the force obtained using FE model coupled with the coefficients obtained as the results of the optimization performed using a simplex method. All forces acquired using bio-inspired optimization methods coupled with metamodel lie in the gray area. Despite some differences in values of coefficients, results (the obtained values of the objective function (3)) are very similar, which proves that objective function (3) is multimodal.

Greater differences occurred in the number of objective function calls. In the inverse analysis conducted for DP600 steel the best algorithm seems to be evolutionary strategy (1+1). However, in the case of CuCr alloy genetic algorithm turned out to be much better. Yet for DP600 steel GA ranked in a second last place. These differences are caused by stochastic rules applied in all bio-inspired algorithms. Therefore it is difficult to indicate the best method and sometimes it leads to inacceptable solutions (see SA results for CuCr alloy – figure 3). All the obtained results are equally possible due to no physical constraints of the parameter subsets defined, although the interval of variation is selected for each parameter separately. To increase the efficiency of the optimization process, the hybrid methods, combining bio-inspired algorithms and, e.g., gradient optimization procedures to explore local minima, can be applied.

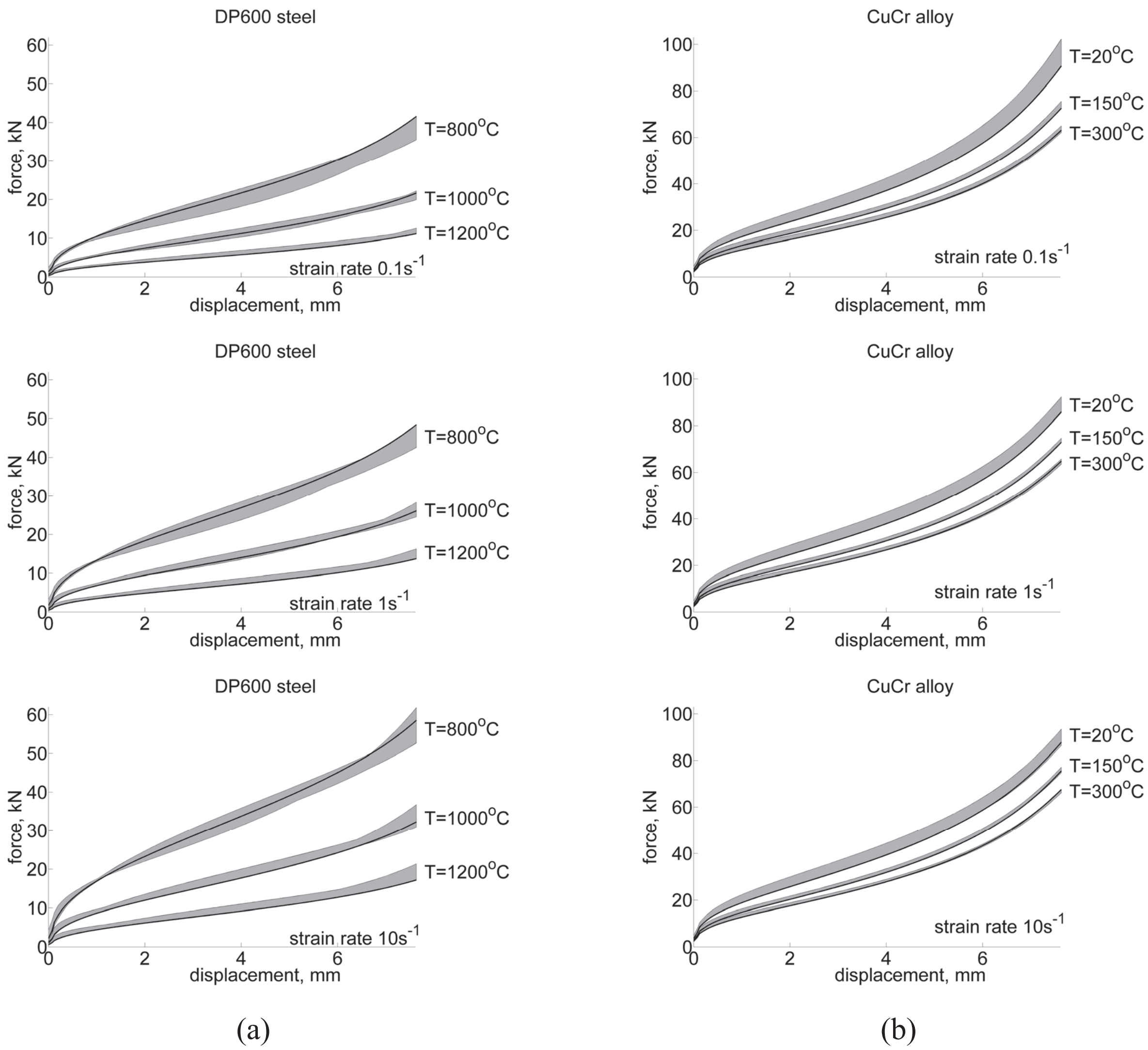

Fig. 1. Comparison of force calculated by FE model with flow stress determined by inverse technique with FE model and metamodel for DP600 steel (a) and CuCr alloy (b); black line – force calculated by FE model; grey area covers forces calculated by bio-inspired algorithms with metamodel.

In order to compare the effectiveness of optimization algorithms, the optimization was performed ten times for each method. Moreover, effectiveness factor was introduced as a product of average optimization error (6) by average number of fitness function calls. The lower value of this factor means, that the algorithm is better. Obtained results are presented in figures 2 and 3.

The analysis of different optimization algorithms focused on two main features: optimization error value and number of objective function calls. Comparing the obtained values, it is difficult to suggest the "best/optimal" optimization method. The optimization errors were similar for most of algorithms.

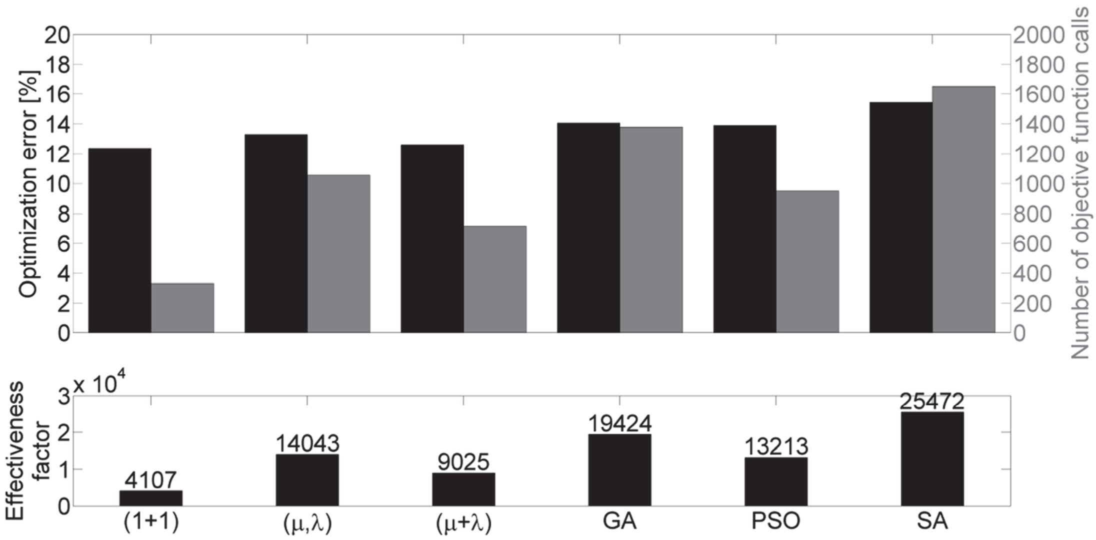

Fig. 2. *The average values of the optimization error, number of objective function calls and effectiveness factor for DP600 steel.*

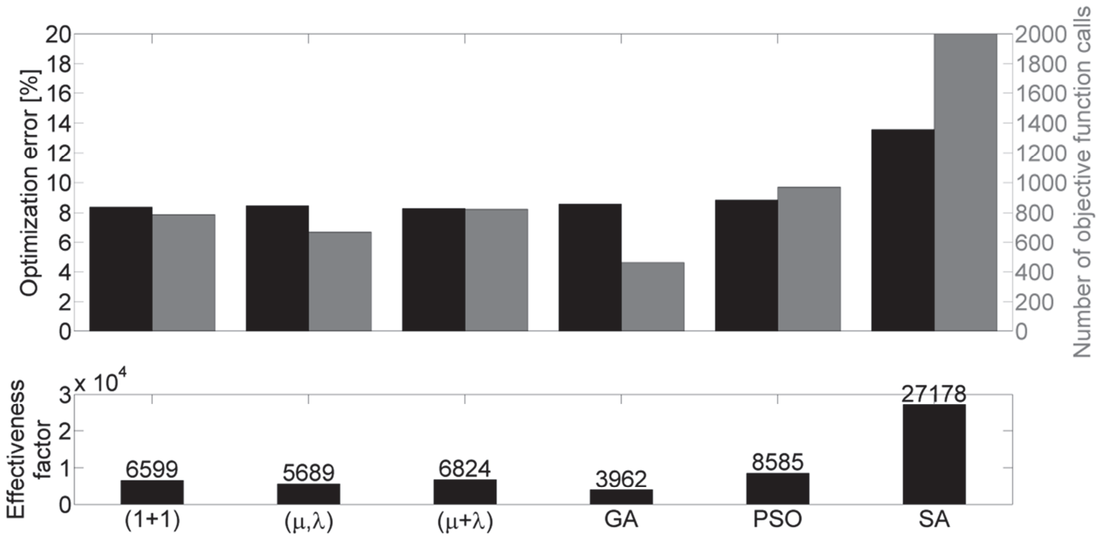

Fig. 3. *The average values of the optimization error, number of objective function calls and effectiveness factor for CuCr alloy.*

5. SUMMARY

The main goal of the work was study over effectiveness of inverse analysis computation. Two features were taken into account: computing time and accuracy of the solution. Computing time was significantly reduced by applying metamodel based on the ANN technique replacing the FE model. The metamodelling error was smaller than 0.4% what confirmed good accuracy. The multimodality of the objective function confirms the rightness of application of bio-inspired optimization methods. All investigated methods proved to be accurate and efficient in terms of the computation time when metamodels were used as direct problem models. It was not possible to indicate the best method for all investigated applications. It means that combination of various methods should be advised and experience of the user of the considered inverse problem is important in selection of the optimization strategy.

ACKNOWLEDGEMENTS

Financial assistance of the NCN, project N N508 629 740, is acknowledged.

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EFEKTYWNOŚĆ OBLICZENIOWA W ANALIZIE ODWROTNEJ

Streszczenie

Artykuł przedstawia badania nad efektywnością obliczeniową w analizie odwrotnej pod kątem czasu obliczeń oraz dokładności otrzymywanych rozwiązań. Jako problem odwrotny rozważono identyfikację parametrów modelu reologicznego. Identyfikacja jest zazwyczaj przeprowadzana poprzez zastosowanie metody optymalizacji w połączeniu z modelem MES co prowadzi do długich czasów obliczeń. Zastosowanie metamodelu w analizie odwrotnej zostało zaproponowane jako rozwiązanie tego problemu. Drugą poruszoną kwestią jest wybór odpowiedniej metody optymalizacji. Kilka algorytmów inspirowanych naturą zostało użytych w obliczeniach odwrotnych. W pracy przedstawiono otrzymane wyniki.

Received: July 23, 2014
Received in a revised form: September 29, 2014
Accepted: November 14, 2014