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SOLUTION OF THE INVERSE CONTINUOUS CASTING PROBLEM WITH APPLICATION OF THE INVASIVE WEED OPTIMIZATION ALGORITHM

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Abstract

In the paper we propose to apply the Invasive Weed Optimization (IWO) algorithm for solving the inverse problem of continuous casting. The investigated task consists in determination of the heat flux in crystallizer and the heat transfer coefficient in the secondary cooling zone on the way of solving the two-phase Stefan problem, describing the discussed process, and minimizing the appropriate functional by using the IWO algorithm. The applied algorithm belongs to the group of artificial intelligence algorithms and is inspired by the extraordinary ability of the colony of weeds for fast growth, quick reproduction and ease of adapting for environmental conditions.

Key words: Artificial Intelligence, Evolutionary Algorithms, IWO Algorithm, Inverse Continuous Casting Problem

1. INTRODUCTION

Nowadays a number of artificial intelligence optimization algorithms, inspired by the behavior and processes from the real world, have been developed to solve complex optimization problems which are really tough or even impossible to solve. A specific group of artificial intelligence procedures is represented by the swarm intelligence algorithms, the idea of which is based on the collective behavior of self-organized systems of individuals executing the partial tasks and contributing to the success of the entire swarm (Eberhart et al., 2001).

Inspirations for developing these kinds of algorithms were usually taken from the world of animals (Chu et al., 2006; Duran Toksari, 2006; Karaboga & Basturk, 2007) as well as human beings (Geem, 2006) and plants. An example of algorithm imitating the colonizing behavior of plants in their life cycle is the Invasive Weed Optimization (IWO) algorithm. The algorithm has been first introduced by Mehrabian and Lucas (2006) in dynamic and control systems theory and till now it has found a number of practical applications, for example, in tuning of a robust controller (Mehrabian & Lucas, 2006), optimal positioning of piezoelectric actuators (Mehrabian & Yousefi-Koma, 2007), in analysis of electricity markets (Nikoofard et al., 2012), antenna configuration (Mallahzadeh et al., 2008) and others. In this paper the IWO algorithm is intended to be used for solving the inverse problem of continuous casting.

Commonly speaking, a weed is any plant growing where it is not wanted. The main idea of IWO algorithm is based on the competitive behavior of invasive weed colonies. In nature, each weed in the colony produces its seeds. Plants with higher robustness produce more seeds which are next randomly dispersed over the area and grow to new plants. Thus, the descendants of the stronger individuals take control over the region. This process, simulated in the artificial conditions, is applied for solving optimization tasks. Partial solutions of the investigated problem play the role of weeds competing

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with each other so that the artificial weed with a higher fitness produces more solutions in comparison with the weeds of lower adaptation. In this way the search of the algorithm concentrates around the good solutions while the bad solutions are abandoned. This leads to determination of the best available solution.

The authors of the current paper have already used selected algorithms of swarm intelligence for solving the inverse heat conduction problems (Grzymkowski et al., 2012; Hetmaniok et al., 2012a; Hetmaniok et al., 2012b). The inverse problems for the equations of mathematical physics consist in determination of some missing elements which can be, for example, the initial condition, boundary conditions or parameters of material, and the missing part of input information is compensated by the additional information about the consequences resulting from the input conditions (Beck et al., 1985; Binder et al., 1990). Difficulty in solving such problems is caused by the fact that finding their solution in analytical way is almost impossible, moreover the solution, if exists, may be neither unique nor stable. Therefore we say that the inverse problems are the ill-posed problems.

The task considered in this paper is the inverse problem of continuous casting. In this process the molten metal is poured in the controlled way into the crystallizer where it solidifies by taking the appropriate form and then is consecutively moved out (Constalas et al., 2002; Mochnacki & Suchy, 1995; Nowak et al., 2003; Nowak et al., 2011; Santos et al., 2006; Słota, 2009; Słota, 2011a; Słota, 2011b). Our goal will be to determine the cooling conditions represented by the heat flux in the crystallizer and the heat transfer coefficient in the secondary cooling zone. Investigated problem can find an industrial application, for example, in designing the continuous casting systems. By solving the inverse problem of considered kind one can select the boundary conditions such that the solidification could run in the assumed way which gives possibility to control the quality and properties of the final ingots. Expression of the discussed problem will be obtained with the aid of Stefan problem which is a mathematical model serving for description of the temperature distribution in a region with the phase change (Gupta, 2003; Słota, 2011a). The problem will be solved by using the finite difference method with application of the alternating phase truncation method (Mochnacki & Suchy, 1995; Słota, 2011a; Słota, 2011b; Rogers et al., 1979). Functional expressing the error of approximate solution will be minimized with the aid of IWO algorithm. The entire procedure will be investigated with regard to its stability and exactness of obtained approximations of the sought elements.

2. PROBLEM DEFINITION

Continuous casting is a process of controlled pouring of the molten metal into the crystallizer for solidifying and taking the appropriate form. After that the solidified ingot is rolled out from the crystallizer through the secondary cooling zone. The scheme of a device for continuous casting of pure metals working in an undisturbed cycle is presented in figure 1. We assume that the cooling conditions change with reference to the direction of the forming ingot but are identical in the entire perimeter of the ingot. We also assume that the dimensions of the ingot cross section satisfy the condition saying that the ingot thickness is much smaller than its width (denoted by b). Additionally, we estimate that the heat flows only in the direction perpendicular to the ingot axis. Such assumption results from the fact that the amount of directed towards the ingot move in comparison with the amount of heat carried in the direction perpendicular to the ingot axis is negligible (Mochnacki & Suchy, 1995). We discuss the apparently steady field of temperature generated in the course of undisturbed working cycle of the continuous casting device.

Taking into account the heat symmetry and the above assumptions we consider the ingot region as a two-dimensional region divided into two subregions: Ω_1 taken by the liquid phase and Ω_2 occupied by the solid phase, separated by the freezing front Γ_g (described by means of function $r = \xi(t)$). In these subregions, with the space orientation taken as in figure 1, the heat transfer process, including the apparently steady field of temperature and location of the freezing front, can be described by means of the two-phase Stefan problem (Mochnacki & Suchy, 1995).

Boundary of region $\Omega = [0,b]\times[0,z]$ is divided into four parts, as it can be seen in figure 1, where the boundary conditions are defined:

$$\Gamma_0 = \{(x,0) : x \in [0,b]\}, \tag{1}$$

$$\Gamma_1 = \{(0,z) : z \in (0,z]\}, \tag{2}$$

$$\Gamma_2 = \{(b,z) : z \in (0,z_c]\}, \tag{3}$$

$$\Gamma_3 = \{(b,z) : z \in (z_c,z]\}. \tag{4}$$

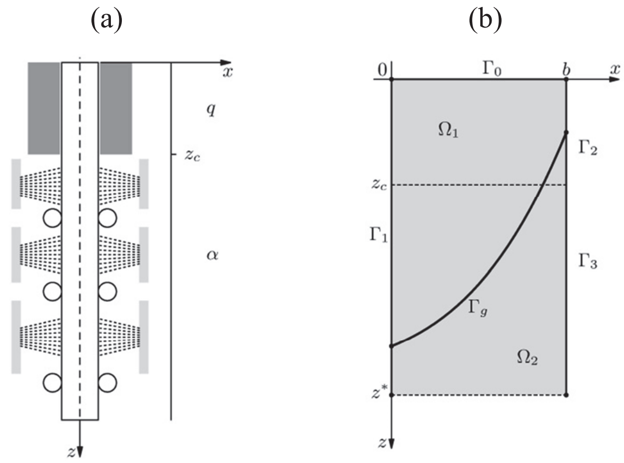

Fig. 1. *Scheme of the continuous casting device (figure (a)) and domain of the two-dimensional problem (figure (b))*

In the investigated problem we intend to determine the cooling conditions for the ingot in such a way that the temperature in selected points of the solid phase would take the given values:

$$T_2(x_i, z_j) = U_{ij}, \quad i = 1,...,N_1, \quad j = 1,...,N_2, \tag{5}$$

For $(x_i,z_j) \in \Omega_2$, where N_1 denotes the number of sensors and N_2 means the number of measurements taken from each sensor. Other elements which require to be determined are: function ξ denoting the freezing front location and functions T_k describing the temperature distribution in regions Ω_k (k = 1,2). Functions of temperature within regions Ω_k (for k = 1,2) satisfy the following heat conduction equation

$$c_k \ \rho_k w \frac{\partial T_k}{\partial z}(x,z) = \lambda_k \frac{\partial^2 T_k}{\partial x^2}(x,z), \tag{6}$$

where c_k, ρ_k and λ_k denote, respectively, the specific heat, mass density and thermal conductivity in liquid phase (k = 1) and solid phase (k = 2), w is the velocity of continuous casting, and finely x and z refer to the spatial variables.

On the distinguished above boundary parts, the appropriate boundary conditions must be satisfied. Thus, on boundary Γ_0 the boundary condition of the first kind with the given pouring temperature is defined ($T_p > T$):

$$T_1(x,0) = T_p, \tag{7}$$

on boundary Γ_1 the homogeneous boundary condition of the second kind is given

$$\frac{\partial T_k}{\partial x}(x,z) = 0, \tag{8}$$

on boundary Γ_2 (crystallizer) the boundary condition of the second kind is set

$$-\lambda_k \frac{\partial T_k}{\partial x}(x,z) = q, \tag{9}$$

on boundary Γ_3 (secondary cooling zone) the boundary condition of the third kind is given

$$-\lambda_k \frac{\partial T_k}{\partial x}(x,z) = \alpha\,(T_k(x,z) - T_\infty) \tag{10}$$

and finally on interface Γ_g the condition of temperature continuity and the Stefan condition must be fulfilled

$$T_1(\xi(z),z) = T_2(\xi(z),z) = T \ , \tag{11}$$

$$L\rho_2 \ w \frac{d\xi(z)}{\partial z} = -\lambda_1 \frac{\partial T_1(x,z)}{\partial \ x}\Big|_{x=\xi(t)} + \lambda_2 \frac{\partial T_2(x,z)}{\partial \ x}\Big|_{x=\xi(t)} . \tag{12}$$

In the above equations α describes the heat transfer coefficient, q denotes the heat flux, T_p is the pouring temperature, T_∞ describes the ambient temperature, T^* denotes the solidification temperature and L describes the latent heat of fusion.

The investigated task focuses on determination of the following function

$$f(z) = \begin{cases} q & \text{for } z \le z_c, \\ \alpha & \text{for } z > z_c, \end{cases} \tag{13}$$

defining the sought elements which are the heat flux in the crystallizer and the heat transfer coefficient in the secondary cooling zone.

The suggested approach applied in the research is as follows: for the fixed form of function f problem (6)-(12) turns into the direct Stefan problem, solving of which enables to find the courses of temperature $T_2(x_i,z_j) = T_{ij}$ corresponding with function f. By using the calculated temperatures T_{ij} and the given temperatures U_{ij} the following functional is constructed

$$J(f) = \sum_{i=1}^{N_1} \sum_{j=1}^{N_2} (T_{ij} - U_{ij})^2, \tag{14}$$

defining the error of approximate solution. Thus, by minimizing functional (14) we can find such form of function f that the reconstructed temperatures will be as close as possible to their measurement values which is the goal of our approach. Minimization of functional (14) will be realized with the use of the Invasive Weed Optimization Algorithm and for solving the direct Stefan problem, associated with the investigated task, we apply the finite difference method combined with the alternating phase truncation method. Let us notice that each running of the procedure requires the direct Stefan problem to be solved in course of multiple repetitions.

3. SOLUTION OF DIRECT PROBLEM

To solve the direct Stefan problem we used the finite difference method completed with the alternating phase truncation method (Mochnacki & Suchy, 1995; Słota, 2011a; Słota, 2011b; Rogers et al., 1979). In this method in place of temperature T, appearing in the Stefan problem (2)-(12), we introduce the enthalpy related to the volume unit

$$H(T) = \begin{cases} \int_0^T c(u)\,\rho(u)\,du & \text{for} \quad T \le T^*, \\ \int_0^T c(u)\,\rho(u)\,du + L\,\rho_2 & \text{for} \quad T > T^*. \end{cases} \tag{15}$$

Function $H(T)$ is discontinuous in the point given by the temperature of phase change T^*. Its left-hand and right-hand limits at this point will be denoted as H_s and H_l:

$$H_s = \lim_{T \to T^*_-} H(T) = \int_0^{T^*} c(u)\,\rho(u)\,du, \tag{16}$$

$$H_l = \lim_{T \to T^*_+} H(T) = H_s + L\,\rho_2. \tag{17}$$

If we use equation (15) in the Stefan problem, we will obtain in both phases a heat conduction equation where the temperature is be replaced with enthalpy.

Algorithm of the phase truncation method for each time step consists of two stages. In the first stage the entire region is reduced to the liquid phase, it means that to these points in which the value of enthalpy is lower than H_l some amount of heat is delivered (contractually) such that the enthalpy takes the value equal to H_l. The heat conduction problem in the one-phase region, obtained in this way, is solved by using one of the well known methods (for example, the finite difference method or the finite element method) by receiving the approximate distribution of enthaply. Next, in points to which some amount of heat was artficially delivered, the same amount of heat must be removed. Distribution of enthalpy obtained after this operation is taken as the initial distribution for the second stage of computations.

In the second stage the entire region is reduced to the solid phase, it means from these points of the region in which the value of enthalpy is higher than H_s some amount of heat is removed (contractually) such that the enthalpy takes the value equal to H_s. Similarly as in the first stage, the approximate distribution of enthalpy is found and must be corrected. In points from which some amount of heat was artficially removed, the same amount of heat must be added. This completes the second stage and, at the same time, one step of the calculations (transfer from time t_i to time t_{i+1}) of the method.

In the alternating phase truncation method for each time step the heat conduction equation must be solved twice. In this connection one must take care of the appropriate consideration of the boundary conditions such that they would affect the discussed system just for time Δt instead of $2\Delta t$. In the first stage of the method the real boundary conditions are taken into considerations only on these parts of boundary where the contact of liquid phase with environment takes place. The other parts of boundary are isolated. Whereas in the second stage the real boundary conditions are taken into considerations only on these parts of boundary where the contact of solid phase with environment takes place.

4. INVASIVE WEED OPTIMIZATION ALGORITHM

The IWO algorithm simulates the colonizing behavior of invasive weeds, in particular its tactics for finding a suitable place for spreading over and growth. The adaptation abilities of the weed colony are imitated in the following way: particular solutions of considered act as the single weeds. Adaptation of each individual is measured with the aid of fitness function depending on the solved problem. Weed characterized by the best value of fitness function creates the biggest number of seeds which disseminate some distance away giving the champions the best possibility to find better place with better adaptation. The algorithm presented is realized in the following steps (Mehrabian & Lucas, 2006):

1. Random generation of n individuals composing the initial population. For each individual the value of fitness function is calculated by formula

$$fit(y) = \frac{1}{1 + J(y)}, \tag{18}$$

where J denotes the minimized function.

2. Determination of the number of seeds for each individual. The number of seeds S_i of a given individual means its chances for reproduction. Therefore it is supposed that the greater it is the better the adaption of a considered individual is. Thus, the number of seeds is expressed in the following way

$$S_i = S_{\min} + \left\lfloor fit_i - fit_{min}) \frac{S_{\max} - S_{\min}}{fit_{max} - fit_{min}} \right\rfloor, \qquad (19)$$

where fit_i, $fit_{\max}$ and $fit_{\min}$ are the values of fitness function for a given individual, the best and the worst population member, respectively, $S_{\max}$ and $S_{\min}$ describe the assumed admissible maximal and minimal number of seeds and $\lfloor \ \rfloor$ denotes the integer part.

3. Dispersion of seeds over the region and creation of new individuals. New individual $\mathbf{y} = [y_1,\ldots y_n]$ is created by the random generation of y_i, but in such a way that the seed "falls on the ground" at the determined distance from the senior individual. The distance of the admissible seeds' flight is described by the normal distribution with mean value equal to zero and standard deviation decreasing in each iteration according to the following formula

$$\sigma_i = \left(\frac{I_{\max} - i}{I_{\max}} \right)^3 (\sigma_{init} - \sigma_{fin}) + \sigma_{fin}, \qquad (20)$$

where $I_{\max}$ represents the maximal number of iterations, i is the number of current iteration, whereas σ_{init} and σ_{fin} denote the assumed initial and final values of standard deviation.

4. Calculation of the fitness function values for new individuals created in the previous step.

5. Selection of n best adapted individuals from among the new individuals and members of the former population. The n selected best individuals create the new population.

6. Steps 2-5 are repeated it times.

5. NUMERICAL VERIFICATION

For verifying the proposed approach let us consider the continuous casting of aluminium described by the following parameters (Słota, 2011 a): $\lambda_1 = 104$ [W/(m·K)], $\lambda_2 = 204$ [W/(m·K)], $c_1 = 1290$ [J/(kg·K)], $c_2 = 1000$ [J/(kg·K)], $\rho_1 = 2380$ [kg/m^3], $\rho_2 = 2679$ [kg/m^3], $L = 390000$ [J/kg], velocity of casting $w = 0.002$ [m/s], solidification temperature $T^* = 930$ [K], ambient temperature $T_\infty = 298$ [K], pouring temperature $T_p = 1013$ [K] and $b = 0.1$ [m].

Values of the heat flux and the heat transfer coefficient, which we intend to reconstruct, are known to be as follows

$$q = 400000 \text{ [W/m}^2\text{]},$$

$$\alpha = 4000 \text{ [W/(m}^2\text{·K)]},$$

which makes it possible for us to evaluate the quality of reconstructed elements.

In the investigated region two thermocouples ($N_1 = 2$) are located in the distance of 0.01 and 0.02 m away from the boundary. 100 measurements of temperature ($N_2 = 100$) were read from each thermocouple. The distance along the Oz axis between the successive measurements was equal to 0.002 m. In calculations we used the exact values of temperature and values disturbed by the 1, 2 and 5% random error of normal distribution simulating the measurement values.

For minimizing functional (14) we applied the IWO algorithm executed for individual of form $\mathbf{y} = [q,\alpha]$ and for the following values of parameters selected in result of several testing calculations: maximal number of iterations $it = 1000$, number of individuals in one population $n = 40$, maximal and minimal number of seeds $S_{\max} = 5$, $S_{\min} = 1$, initial and final standard deviation $\sigma_{init} = 5$, $\sigma_{fin} = 0.1$. Selection of parameters has been made on the basis of solution of some other inverse problems (with and without the phase changes - see for example (Hetmaniok, 2014)). We have chosen these values of parameters which ensured the most exact reconstructions of sought coefficients and enabled to obtain them with the lowest calculation costs. Because of the big difference between the expected values of reconstructed elements the initial weeds populations were randomly selected from different intervals (for reconstructing q from interval [250000, 500000] and for reconstructing α from interval [1000, 5000]). Specific quality of the IWO algorithm is its heuristic nature, meaning that each execution of the procedure can give slightly different results. Therefore, to assure the best results we evaluated the calculations for 10 times in each considered case and the best of the received results were taken as the reconstructed elements. Each execution of the procedure requires to solve the appropriately formulated direct Stefan problem which was realized by using the finite difference method with application of the alternating phase truncation method for the mesh with steps equal to $\Delta x = b/500$ and $\Delta z = 0.001$.

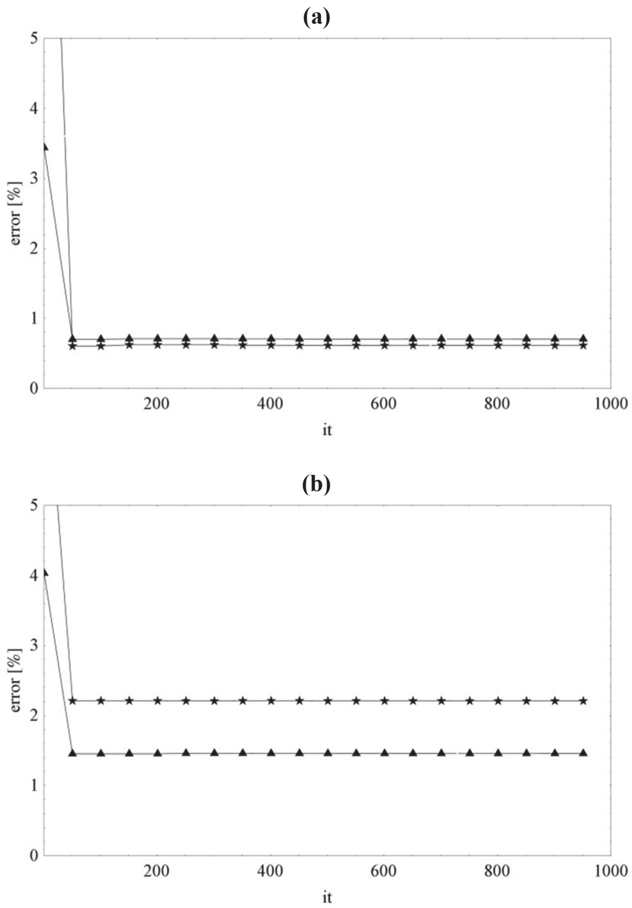

Fig. 2. Relative errors of parameter f reconstruction for the successive iterations (∗ – for q, ▲ – for α) obtained for 2% (figure (a)) and 5% (figure (b)) noise of input data

In figure 2 the relative errors of the heat flux q and the heat transfer coefficient α identification in dependence on the number of iterations obtained in one execution of the procedure, for input data disturbed by 2% and 5% error, respectively, are displayed. We can see that in both cases after about 50 iterations we are able to obtain very good reconstructions of the cooling conditions and the further iterations do not significantly improve the results. Some other, more sophisticated techniques, which are planned to be investigated in the future, might probably be more effective to improve the results. However, at the moment we find the obtained reconstructions satisfying. For the exact input data the reconstruction errors converge very quickly to zero which confirms the stability of the procedure used. Averaged results of the cooling conditions identification received in 10 executions of the procedure for the exact input data, as well as for all considered perturbations are collected in table 1.

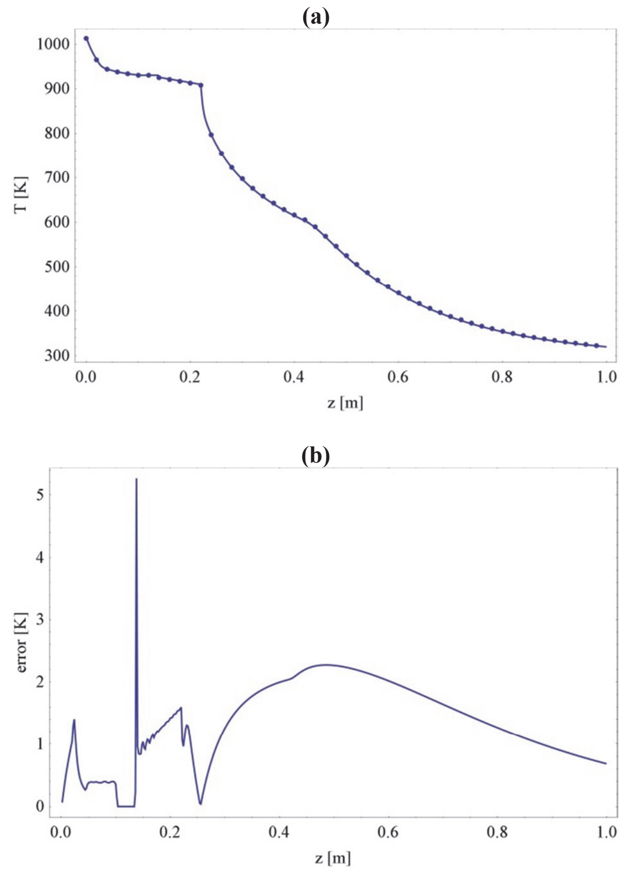

Fig. 3. Exact (solid line) and reconstructed (dots) distributions of temperature (figure (a)) in control point located 0.01m away from boundary b obtained for 5% noise of input data and absolute error of this reconstruction (figure (b))

In figures 3 and 4 the distributions of temperature reconstructed in measurement points located 0.01m and 0.02 m away from boundary of the region respectively, are compared with the known exact distributions. The results were obtained for input data disturbed by 5% error and they show that even in this worst case the reconstructed and known courses of temperature agree. The absolute errors of these reconstructions are at the level of few Kelvins which additionally confirms the almost perfect reconstruction of temperature. Maximal relative errors of temperature reconstructions obtained in 10 executions of the procedure for each considered input data perturbation are presented in the last column of table 1.

In general, table 1 contains the statistical elaboration of the results obtained in 10 executions of the procedure for various noises of input data. Relative errors of the heat flux and the heat transfer coefficient reconstruction for exact input data are smaller than 0.1%. The errors increase obviously with the increasing value of input data perturbation but still in each case are much smaller than the input data

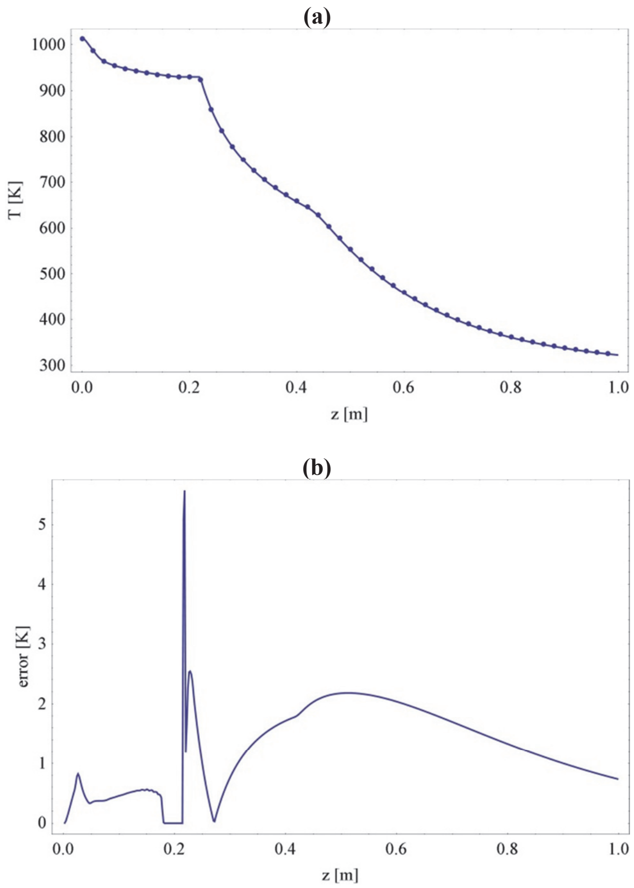

Fig. 4. Exact (solid line) and reconstructed (dots) distributions of temperature (figure (a)) in control point located 0.02m away from boundary b obtained for 5% noise of input data and absolute error of this reconstruction (figure (b))

Table 1. Reconstructed values of the cooling conditions (parameters f), relative errors (δ), standard deviations (σ) of these reconstructions, standard deviations (σ^p) expressed as a percent of mean values of parameters f reconstruction and the maximal relative errors (δ_T^{max}) of temperature reconstruction obtained for various noises of input data

noise	f	δ [%]	σ	σ^p [%]	δ_T^{max} [%]
0%	400380.26	0.0951	0.0071	$1.767\cdot10^{-6}$	0.0261
	4001.56	0.0390	$2.917\cdot10^{-5}$	$7.293\cdot10^{-7}$	
1%	403226.46	0.8066	$1.498\cdot10^{-3}$	$3.747\cdot10^{-7}$	0.5158
	3975.56	0.6110	$5.998\cdot10^{-5}$	$1.500\cdot10^{-6}$	
2%	397543.07	0.6142	$9.663\cdot10^{-3}$	$2.413\cdot10^{-6}$	0.5146
	4028.20	0.7051	$4.590\cdot10^{-5}$	$1.148\cdot10^{-6}$	
5%	408832.55	2.2081	$5.056\cdot10^{-3}$	$1.264\cdot10^{-6}$	0.5992
	3941.61	1.4597	$3.687\cdot10^{-4}$	$9.217\cdot10^{-6}$	

error, whereas the maximal relative error of temperature reconstruction in each discussed case is insignificant. In case of the exact input data the maximal relative error of temperature reconstruction in the control point does not exceed the value of 0.03%, whereas in case of the perturbed input data it is always lower than 0.6% (see the last column in table 1). Stability of the procedure is indicated by the standard deviations of the sought parameters reconstruction which in all considered cases are very close to zero. The results compiled in table 1 were obtained for the entire procedure, it means for 1000 iterations. However, as we have noticed while commenting figure 2, we received satisfying results of reconstruction after executing much less iterations. Standard deviations σ^p of parameters f reconstruction calculated in 250 iterations for 0,1,2 and 5% perturbation of input data, expressed as a percent of mean values of identified elements, are equal to 0.0404, 0.0089, 0.0069 and 0.0099 (in reconstructing q) and 0.0112, 0.0040, 0.0033 and 0.0030 (in reconstructing α), respectively. These values are very small which means that the procedure stabilizes much earlier.

Considering time of the process, the computations needed for executing 50 iterations of the procedure, which was enough to obtain satisfying results, took about 30 minutes. Currently we investigate the same procedure but with application of the parallel computations. It reduces the time to about 5 minutes.

6. CONCLUSIONS

The aim of this paper was to propose the procedure for solving the inverse problem of continuous casting. The problem consisted in identification of the cooling conditions so that the reconstructed values of temperature are as close as possible to the measurement values taken in the selected points of considered region. Moreover, the important part of the proposed procedure lied in minimization of the appropriate functional which was realized with the aid of IWO algorithm belonging to the group of swarm intelligence algorithms. The presented results of experimental verification show that the elaborated method ensures the approximate solution rapidly convergent to the exact solution and perturbed by the error much smaller than the error of input data, in identifying both cooling conditions as well as in reconstructing the temperature distribution. Standard deviations of reconstructed elements in each considered case were very low which indicate stability of the procedure. Advantages of using the approach with the IWO algorithm are its relative easiness in application and no particular assumptions needed to be satisfied by minimized functional. The weakness of the procedure lies in solving the direct problem associated with the considered inverse problem. Therefore if we are only able to solve the direct problem, which is supposed to be a much easier task, the appropriate solution of the inverse continuous casting problem can be found as well.

Considered problem of continuous casting was also solved by the authors by applying some other biologically inspired optimization algorithms, like Ant Colony Optimization algorithm, Artificial Bee Colony algorithm or immune algorithm (see for example (Hetmaniok et al., 2013)). Comparing the results one can observe that the smallest number of iterations, and in consequence the smallest number of the objective function runs, was in case of the Artificial Bee Colony algorithm as well as the Ant Colony Optimization algorithm, whereas the Invasive Weed Optimization algorithm appeared to be the most stable. However, differences in the running time are at the level of several minutes, therefore we may state that the efficiencies of bio-inspired algorithms, tested by us in solving the discussed problem, are quite similar.

In the earlier works we used for solving the discussed problems the deterministic algorithms as well. However they appeared to be sensitive to the starting point selection. For the starting points located quite close to the sought solution we were obtaining very good results. For the starting points located farther, the exactness of received results was not so good, especially in the case of disturbed input data. Application of the IWO algorithm gave us the possibility to become independent on selection of the starting point and to receive the very good results in each case.

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ROZWIĄZANIE ODWROTNEGO ZAGADNIENIA ODLEWANIA CIĄGŁEGO PRZY ZASTOSOWANIU OPTYMALIZACYJNEGO ALGORYTMU INWAZJI CHWASTÓW

Streszczenie

W niniejszej pracy przedstawiamy zastosowanie algorytmu inwazji chwastów IWO (Invasive Weed Optimization algorithm) do rozwiązania odwrotnego zagadnienia odlewania ciągłego. Badany problem polega na wyznaczeniu strumienia ciepła w krystalizatorze oraz współczynnika wnikania ciepła w strefie chłodzenia wtórnego poprzez rozwiązanie dwufazowego zagadnienia Stefana opisującego dyskutowany proces oraz poprzez minimalizację odpowiedniego funkcjonału przy użyciu algorytmu IWO. Zastosowany algorytm należy do grupy algorytmów sztucznej inteligencji i zainspirowany został niezwykłą zdolnością kolonii chwastów do szybkiego wzrostu, reprodukcji oraz łatwości w dostosowaniu się do warunków środowiskowych.

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