NUMERICAL IDENTIFICATION OF MATERIAL PARAMETERS FOR MICROBUCKLING ANALYSIS OF FIBER REINFORCED POLYMER MATRIX COMPOSITES

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Abstract

This research focuses on studying the effect of material parameters upon the compressive response of a unidirectional fiber reinforced polymer matrix composite. To investigate this effect, the finite element method and a periodic unit cell model of unidirectional composite with an initial fiber waviness and inelastic behavior of the matrix were used. The sensitivity of the compressive strength to the hydrostatic pressure, the flow rule and the fiber misalignment angle were presented. The model was verified against an analytical solution and experimental data. The results of this study indicate that a micromechanical model with correctly identified material parameters provides a useful alternative to theoretical models and experimentation.

Key words: polymer-matrix composites, buckling, micro-mechanics, finite element analysis

1. INTRODUCTION

Computational micromechanics is used as a tool to predict the macroscopic properties of composite materials based on the known properties of their constituents. One of the design limiting features of a unidirectional composite lamina at the macroscopic level is the compressive failure in the fiber direction. Experimental results indicate that it is triggered by microbuckling of fibers inside the inelastic matrix. Thus, the key parameters of compressive models are the matrix ductility and the initial waviness of fibers. Following the usual assumption that the induced shear strain γ and the fiber misalignment angle Θ are small, simple equilibrium considerations (Budiansky & Fleck, 1993) relate the applied compressive stress σ to the shear response of the composite $\tau(\gamma)$ by

$$\sigma = \tau(\gamma) / (\gamma + \Theta). \qquad (1)$$

This relationship demonstrates that the shear mode failure will always occur during the buckling process.

The application of a representative unit cell approach in modeling the microbuckling has been reported by Guynn et al. (1992), Morais (1996), Hsu et al. (1999), Pansart et al. (2009) and Gutkin et al. (2010). All the mentioned above papers describe the nonlinear constitutive behavior of matrix by using the von Mises yield criterion. Unfortunately, such an approach does not take into account the pressure sensitivity plastic response of polymer matrices.

The present work attempts to meet this demand. The micromechanical finite element model that incorporates the pressure sensitivity yielding is used to determine the compressive strength of a fiber reinforced polymer composite and to investigate the effect of the above mentioned parameters.

2. PERIODIC UNIT CELL MODEL

2.1. Initial fiber waviness

An initial fiber imperfection is uniformly distributed across the width of the model (figure 1a). For the presentation of the problem, the material coordinate system is introduced so that the x_1 axis is aligned with the fiber direction and the x_2 axis is perpendicular to the fiber direction. In this paper, the imperfection is expressed as a sinusoidal waviness of fibers along the x_1 direction with the amplitude a and the wavelength 2λ

$$\nu_o = a \cos(\pi x_1 / \lambda). \qquad (2)$$

If the sinusoidal imperfection has long wavelength and relatively small amplitude, the fiber misalignment angle can be approximated by

$$\Theta = \pi a / \lambda . \qquad (3)$$
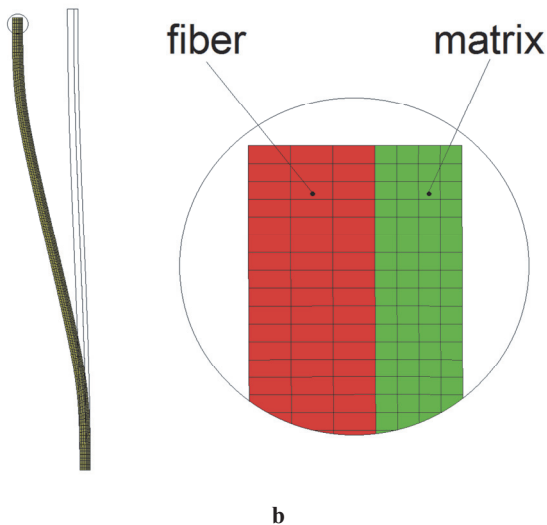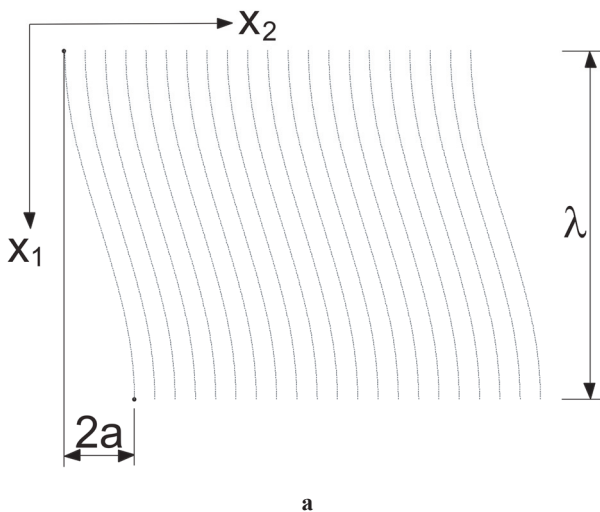

a

b

Fig. 1. Periodic unit cell model. (a) geometry; (b) finite element mesh.

As can be found in the literature (see for example Yurgatis, 1987; Fleck, 1997), the values of the initial fiber misalignment Θ are usually 2°–3°. Unit cell models with the uniformly distributed imperfection characterized by $\Theta = 1.71^o$, 2.40^o and 3.42^o were analyzed.

2.2. Finite element formulation

Finite element models are constructed by using ANSYS finite element code. Two-dimensional finite element meshes made of plane strain, biquadratic elements with eight nodes (Plane183) are generated for the appropriate geometries of micromechanical models (figure 1b). The fiber and matrix layers have 1575 and 2100 elements each, respectively. The width of matrix region was chosen so as to accommodate the fiber diameter $d_f = 7$ μm, and respect the fiber volume content $V_f = 60$ %. The length of models corresponds to the half wavelength, $\lambda = 75 d_f$.

The longitudinal compression is imposed by means of a vertical pressure prescribed at the top side of the unit cell in such a way that the top side remains plane. The bottom side of the unit cell is fixed in the vertical direction. One node is also fixed in the direction perpendicular to the compressive load to prevent rigid body motion.

The periodicity conditions on the right and left sides of the unit cell models are defined to represent a periodic microstructure consisting of an infinitely large array of repeated unit cells. The displacements of nodes on the right side of the unit cell are related to the displacements of their counterparts on the left side as follows (Pansart et al., 2009)

$$u_i^{n2} - u_i^{ref2} = u_i^{n1} - u_i^{ref1} \qquad \text{for } i = 1, 2. \qquad (4)$$

where u_i is the displacement in the i-axis direction, n_1 and n_2 is a pair of nodes facing each other on edge 1 and edge 2, respectively, ref_1 and ref_2 is a pair of reference nodes on edge 1 and edge 2.

In order to handle the potential snap-through associated with the post-buckling response effectively, the arc-length method (Riks, 1979) is adopted. A minimum arc-length radius was defined by using the following parameters: the minimum multiplier of reference arc-length radius was $r_{min} = 10^{-8}$ and the number of substeps was $n = 2500$. A residual forces tolerance of $tol_F = 0.001$ was selected to control the convergence of the iterative procedure. The above-mentioned parameters gave a converged solution for the problem.

**Table 1.** *Mechanical properties of the AS4/PEEK composite and its constituents (Kyriakides et al., 1995).*

AS4 fiber		PEEK matrix			AS4/PEEK composite		
E_f (GPa)	v_f	E_m (GPa)	v_m	k (MPa)	E_{11} (GPa)	σ_c^- (GPa)	V_f (%)
214	0.263	6.14	0.356	40	128	1.21	60

2.3. Constitutive model of matrix

Simulations were carried out for an example case of the AS4/PEEK composite system with the fiber volume content of 60 %. The properties of this material and its constituents are listed in table 1. For the present problem, the fibers are assumed to be isotropic and linearly elastic solids which are perfectly bonded to the matrix. The matrix is modeled within the framework of the finite deformations as an elasto-plastic solid which hardens isotropically.
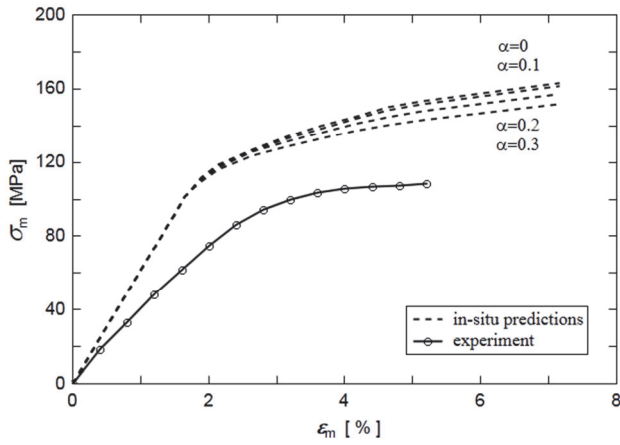
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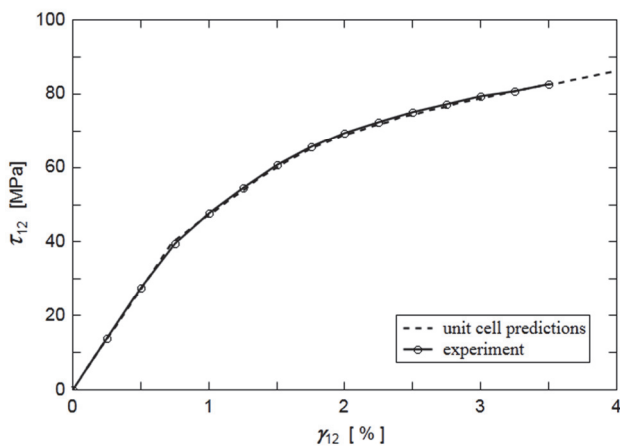
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Fig. 2. *Identification of the plasticity model. (a) comparison of the in-situ axial responses of PEEK matrix with test data (Kyriakides et al., 1995); (b) adjusting experimental shear response of the AS4/PEEK composite (Kyriakides et al., 1995) with model responses.*

The deformation of polymeric materials is highly sensitive to the hydrostatic pressure and plastic flow of these materials can exhibit plastic dilatancy. To address this requirement, the Drucker-Prager plasticity model (Drucker & Prager, 1952), which incorporates the linear dependence on the hydrostatic stress, is used. In terms of the first invariant of stress I_1 and the second invariant of the deviatoric part of stress J_2, the yield function is given as

$$f = \alpha I_1 / 3 + \text{sqrt}(J_2) - k , \qquad (5)$$

where α is the pressure sensitivity factor, k is the flow stress of the material under pure shear. Experiments showed that the pressure-sensitivity factor α ranges from 0.10 to 0.25 for polymers (Kinloch & Young, 1983; Quinson et al., 1997). Note, that if $\alpha = 0$, eq. (5) reduces to the von Mises yield function. The plastic strain increment $d\varepsilon^p$ is specified through the flow rule

$$d\varepsilon^p = d\lambda(\partial g/\partial \sigma), \qquad (6)$$

where $d\lambda$ is a positive scalar of proportionality which determines the amount of plastic strain and g is a plastic potential which in turn determines the direction of plastic strain. If $g = f$ then plastic flow is associative and, in turn, the pressure sensitivity and plastic dilatancy are described by the same parameter α. If $g \neq f$ then plastic flow is non–associative and the plastic volume change is controlled by choosing an appropriate parameter β in the plastic potential function, i.e. $g = \beta I_1 / 3 + \text{sqrt}(J_2)$.

2.4. Calibration of the matrix plasticity model

In this paper, the hardening curve of matrix is back-calculated from the shear stress-strain curve of the AS4/PEEK composite by using numerical homogenization. To simulate in-plane shear loading, the same two-dimensional unit cell model as the one described in section 2.2 was used. Comparisons of the in-situ axial response of matrix and the predicted shear response of composite with corresponding test data (Kyriakides et al., 1995) are shown in figures 2a and 2b. It can be seen from figure 2b that the axial stress-strain curves of the matrix shown in figure 2a produce an accurate solution for the AS4/PEEK composite under in-plane shear loading.
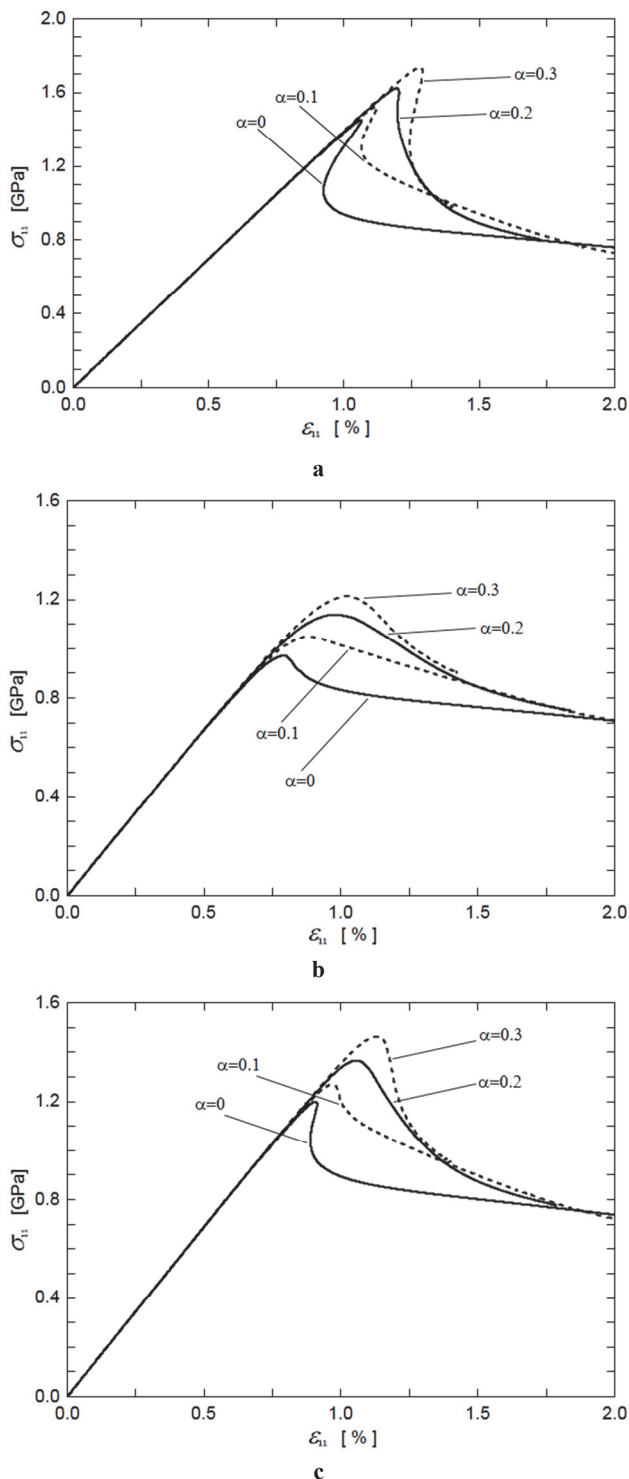
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Fig. 3. *Prediction of the compressive stress–strain curves for various values of the pressure sensitivity factor α. Responses for (a) Θ = 1.71º; (b) Θ = 2.40º; (c) Θ = 3.42º.*

2.5. Numerical homogenization

The compressive responses of the periodic unit cell models are discussed in terms of macroscopic stress and strain. According to the homogenization theory, the macroscopic compressive stress σ_{11} is equal to the average normal traction at the bottom edge of the model (obtained as the sum of reactions

divided by the actual cross-section). The corresponding average strain ε_{11} was computed from logarithmic formula, namely $\varepsilon = \ln(1 + \delta / \lambda)$, where δ is the displacement of nodes at the top edge of the model.
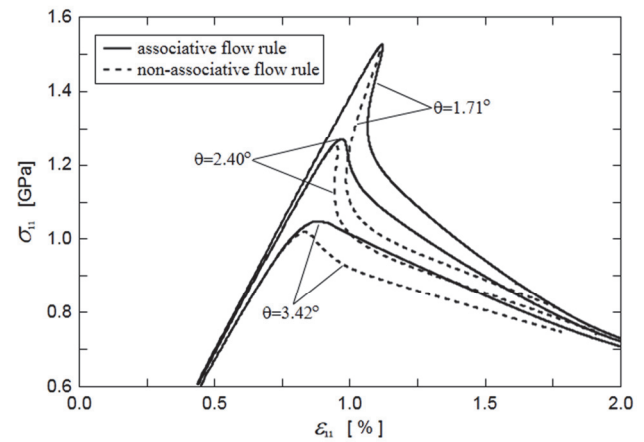

Fig. 4. *Comparison between the associative (α = β = 0.1) and non–associative (α = 0.1, β = 0) flow rule for various values of the fiber misalignment angle Θ.*

3. RESULTS AND DISCUSSION

The combination of nonlinear matrix behavior and initial fiber waviness into one micromechanical model enables the determination of an overall response of composites subjected to compressive loading. The macroscopic stress-strain curves calculated for three values of the initial fiber misalignment $\Theta = 1.71º$, $2.40º$ and $3.42º$ are shown in figures 3a–3c. The nonlinear responses of the AS4/PEEK composite under longitudinal compression for each angle Θ are predicted for four values of the pressure sensitivity factor $\alpha = 0$, 0.1, 0.2 and 0.3. Generally, the main features of the predicted response are similar to those obtained experimentally (Kyriakides et al., 1995; Moran et al., 1995). It can be seen that the initial deformation of the composite is linear elastic. The limit load is reached shortly after non-linearity is first noted. After the maximum of the compressive stress has been attained, geometric softening associated with the fiber bending outweighs the strain hardening of the matrix, and the load begins to drop. As expected, the critical stress predicted by the periodic unit cell model is sensitive to the fiber misalignment angle Θ. An increase of the value of Θ reduces the compressive strength of the composite as well as the snap-through instability beyond the limit load. On the other hand, an increase of the value of α improves the resistance of the composite to compressive failure. This finding clearly indicates that the application of a matrix with high internal friction during composite processing should provide better

failure characteristics. It is interesting to observe that the use of the von Mises plasticity model ($\alpha = 0$) leads to lower compressive strengths compared to the Drucker-Prager plasticity model ($\alpha > 0$).

In order to assess the effect of the flow rule on the compressive response of the AS4/PEEK composite, the macroscopic stress-strain curves calculated for one value of the pressure sensitivity factor $\alpha = 0.1$ through the model in which the matrix followed the associative flow rule were compared to the corresponding curves obtained from the non–associative flow rule. The results of this comparison are summarized in figure 4 for three values of the initial fiber misalignment Θ. The non–associative curves relate to the limiting case, $\beta = 0$ in which no volume change during plastic deformation takes place. It can be seen that the flow rule has negligible influence on the onset of buckling. A noticeable difference was only observed beyond the limit load in the post-buckling stage.
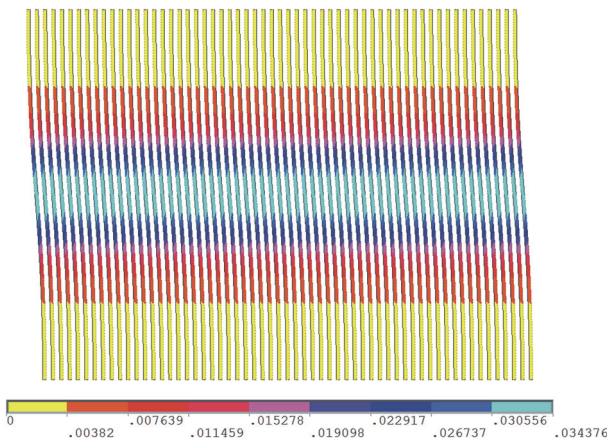

Fig. 5. *Contour plot of the effective plastic strain in matrix at the stage of deformation corresponding to the limit load.*
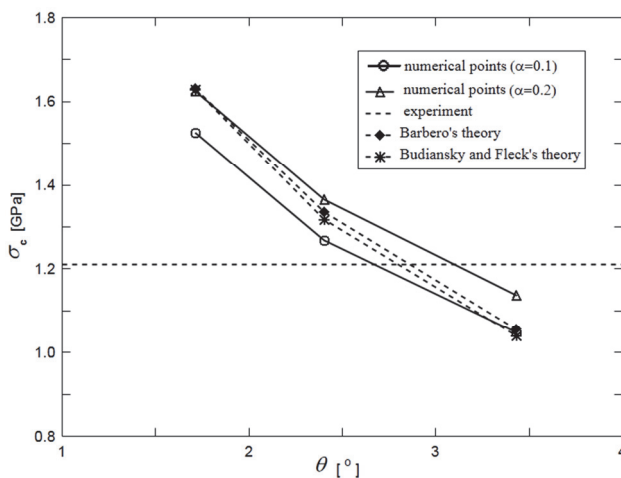

Fig. 6. *Comparison of the calculated critical stress with experimental data (Kyriakides et al., 1995) and theoretical prediction (Budiansky & Fleck, 1993; Barbero, 1998).*

Contour plot of the effective plastic strain in matrix at the stage of deformation corresponding to the limit load is shown in figure 5. Due to the periodicity in the transverse direction, the solution is mirrored infinitely. As can be seen in the graph above, the most intense plastic deformation is localized in a narrow band crossing the unit cell model at the mid-length.

In order to determine material parameters for microbuckling analysis of the AS4/PEEK composite, the limit stresses obtained through the periodic unit cell models were compared to experimental data reported by Kyriakides et al. (1995) as well as to predictions made with theories proposed by Budiansky and Fleck (1993) and Barbero (1998). The former analytical formulation is based on the physics of kink–band formation whereas the latter one on the stability analysis. Both theories take into account the matrix physical non-linearity and the presence of initial fiber misalignment. The Budiansky and Fleck theory suggests that the longitudinal compressive strength σ_c can be derived as

$$\sigma_c = G \ [1+(\sigma_{ty}/\tau_y)^2 \tan^2\beta] \ / $$
$$[1+n(3/7)^{1/n}[\Theta / (n\text{-}1) \ \gamma_*]^{n\text{-}1/n}] , \quad (7)$$

where $\gamma_* = \gamma_y / [1+(\sigma_{ty}/\tau_y)^2 \tan^2\beta]^{1/2}$. In this formula, the ratio of the transverse yield stress to the shear yield stress, $(\sigma_{ty}/\tau_y) = 2$, γ_y is the shear yield strain, G is the elastic shear modulus of the composite, n denotes the strain hardening coefficient, Θ is the initial misalignment angle and β is the kink-band angle. In the Barbero formulation, the longitudinal compressive strength σ_c is predicted to be

$$\sigma_c = G + (16C_2\Theta / 3\pi) - $$
$$4\text{sqrt}(2C_2\Theta(8C_2\Theta +3\pi G)) / 3\pi , \quad (8)$$

where $C_2 = G^2/4\tau_c$ and τ_c is the in-plane shear strength. The compressive strengths of the AS4/PEEK composite predicted from the analytical formulations were plotted in figure 6. The predictions are made for the following material constants taken from the shear stress-strain response shown in figure 2b: G = 5.79 GPa, $\gamma_y = 1.03 \%$, $n = 4.7$, $\beta = 15^{\circ}$, $\tau_c = 80.25$ MPa. It is interesting to note that the analytical predictions lie between the numerical values of the limit stresses obtained for $\alpha = 0.1$ and 0.2. This finding supports the general idea that the von Mises plasticity model ($\alpha = 0$) is unable to appropriately reproduce the deformation of polymeric matrices. Thus, the Drucker-Prager plasticity models with the pressure sensitivity factor α within the

range of 0.1 – 0.2 are more reliable. By comparing the numerical values of the limit stresses obtained for $\alpha = 0.1$ and 0.2 with the mean value of experimental data, it can be concluded that the models with the imperfection parameter Θ within the range of $2.50^{\circ} – 3.25^{\circ}$ guarantee agreement with experiments.

4. SUMMARY

A micromechanical finite element analysis has been conducted to investigate the effect of the matrix constitutive behavior and initial fiber waviness on the compressive strength of the unidirectional AS4/PEEK composite. It was shown that the kind of plasticity model significantly affects the limit load whereas the kind of flow rule is negligible. Furthermore, it was shown that a micromechanical model with correctly identified material parameters provides a useful alternative to theoretical models and experimentation.

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REFERENCES

ANSYS, 2012, *Theory reference for the mechanical APDL and mechanical applications, release 14.1.*, ANSYS Inc., Canonsburg PA.

Barbero, E.J., 1998, Prediction of compression strength of unidirectional polymer matrix composites, *J. Compos. Mater.*, 32, 483-502.

Budiansky, B., Fleck, N.A., 1993, Compressive failure of fibre composites, *J. Mech. Phys. Solids*, 41, 183-211.

Drucker, D.C., Prager, W., 1952, Soil mechanics and plastic analysis for limit design, *Q. Appl. Math.*, 10, 157-165.

Fleck, N.A., 1997, Compressive failure of fiber composites, *Adv. Appl. Mech.*, 33, 43-117.

Gutkin, R., Pinho, S.T., Robinson, P., Curtis, P.T., 2010, Micromechanical modeling of shear-driven fibre compressive failure and of fibre kinking for failure envelope generation in CFRP laminates, *Compos. Sci. Technol.*, 70, 1214-1222.

Guynn, E.G., Ochoa, O.O., Bradley, W.L., 1992, A parametric study of variables that affect fiber microbuckling initiation in composite laminates: Part I analyses, *J. Comp. Mater.*, 26, 1594-1611.

Hsu, S.Y., Vogler, T.J., Kyriakides, S., 1999, On the axial propagation of kink bands in fiber composites: Part II analysis, *Int. J. Solids Structures*, 36, 575-595.

Jelf, P.M., Fleck, N.A., 1994, The failure of composite tubes due to combined compression and torsion, *J. Mater. Sci.*, 29, 3080-3084.

Kinloch, A.J., Young, R.J., 1983, *Fracture behavior of polymers*, Elsevier, Netherlands.

Kyriakides, S., Arseculeratne, R., Perry, E.J., Liechti, K.M., 1995, On the compressive failure of fiber reinforced composites, *Int. J. Solids Structures*, 32, 689-738.

Morais, A.B., 1996, Modelling lamina longitudinal compression strength of carbon fibre composite laminates, *J. Compos. Mater.*, 30, 1115-1131.

Moran, P.M., Liu, X.H., Shih, C.F., 1995, Kink band formation and band broadening in fiber composites under compressive loading, *Acta Metall. Mater.*, 43, 2943-2958.

Pansart, S., Sinapius, M., Gabbert, U., 2009, A comprehensive explanation of compression strength differences between various CFRP materials: Micro-meso model, predictions, parameter studies, *Compos. Part A*, 40, 376-387.

Quinson, R., Perez, J., Rink, M., 1997, Yield criteria for amorphous glassy polymers, *J. Mater. Sci.*, 32, 1371-1379.

Riks, E., 1979, An incremental approach to the solution of snapping and buckling problems, *Int. J. Solids Struct.*, 15, 529-551.

Yurgatis, S.W., 1987, Measurement of small angle misalignments in continuous fibre composites, *Compos. Sci. Technol.*, 30, 279-293.

NUMERYCZNA IDENTYFIKACJA PARAMETRÓW MATERIAŁOWYCH W ANALIZIE MIKROWYBOCZENIA KOMPOZYTÓW POLIMEROWYCH WZMOCNIONYCH CIĄGŁYM WŁÓKNEM

Streszczenie

W pracy badano wpływ parametrów materiałowych na wytrzymałość na wzdłużne ściskanie kompozytu jednokierunkowo wzmocnionego. W tym celu wykorzystano metodę elementów skończonych oraz komórkę elementarną kompozytu z początkową imperfekcją włókien i plastyczną matrycą. Analizowano zmianę krzywej ściskania kompozytu w wyniku zmiany tarcia wewnętrznego matrycy, prawa plastycznego płynięcia matrycy oraz początkowej imperfekcji włókien. W celu identyfikacji wymienionych parametrów materiałowych wykorzystano badania doświadczalne oraz istniejące modele analityczne. Wyniki badań pokazują, że model numeryczny jest skuteczną alternatywą w analizie mikrowyboczenia kompozytów polimerowych wzmocnionych ciągłym włóknem.

COMPUTER METHODS IN MATERIALS SCIENCE