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SENSITIVITY ANALYSIS AS A SUPPORT FOR OPTIMIZATION OF INDUSTRIAL PROCESSES

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Abstract

The problem of optimization of industrial processes consisting of a series of forming operations which define the production cycle, is considered in the paper. Optimization of production cycles is a hard task due to the large number of control variables, mutually exclusive or contradictory objective functions formulated at the production stages and numerical complexity of the process models. Application of sensitivity analysis is one of the solution to reduce the problem. Various methods of global sensitivity analysis were presented in the paper. The procedures were applied to the selected industrial processes: the final part of the manufacturing chain for anchors and controlled cooling of rails. The most important parameters of the processes were determined. The sensitivity analysis results were presented and discussed in the paper.

Key words: sensitivity analysis, optimization, simulation of industrial processes

1. INTRODUCTION

Manufacturing of parts is a complex task which consists of several stages. At each stage the process is optimized with respect to defined criterion. The outputs from one stage are the inputs for the next one. The optimal solution in a stage of the manufacturing chain is not equivalent to the optimum of the whole process. It means that the information flow is not only forward but backward, as well. Numerical simulations of the intermediate processes with advanced, multiscale models of the material are necessary to understand the phenomena occurring during forming but, on the other hand, they are time consuming. Optimization procedure calls process simulation many times, thus, classical optimization approach for production chains is often impossible. Following that, the supporting algorithms were looked for to decrease the calculations cost of optimization and the sensitivity analysis techniques were selected for that purpose. Sensitivity analysis was applied by the authors for the individual processes, see, e.g., Myczkowska et al. (2011), Szeliga (2011), Szeliga (2012). A comprehensive study of the sensitivity analysis in metal forming was presented by Szeliga (2013). In this work an application of sensitivity analysis for production chains is presented. Therefore, the main objective of the paper was to support optimization task formulated for manufacturing chains by performing preliminary step of optimization with sensitivity analysis algorithms.

2. SENSITIVITY ANALYSIS

Sensitivity Analysis (SA) investigates the model (mathematical and physical description of the phenomenon under study) behavior for various input data and model parameters. It determines how the variations of input data and parameters are distributed on the variations of model outputs and influence them. A good practice of the numerical modeling is to validate the model and SA provides techniques

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enabling this evaluation. The main goals of SA application are:

- verification whether a model simulates the phenomenon under study in a proper way (e.g. according to physical laws),
- determination of the model parameters which contribute to the model outputs variations, the most,
- identification of the parameters which are not significant for the analyzed model outputs,
- determination of the parameters domain of the highest influence of the model variations,
- estimation of parameters uncertainty,
- verification whether parameters interact with each other,
- for inverse problems:
  - verification whether the norm defined in the output space is proper to solve the defined inverse problem,
  - verification whether the goal function in optimization task includes information allowing to perform the optimization,
  - determination of the parameters identification accuracy,
  - reduction of calculation cost of optimization procedure (decrease of the number of direct problem solver evaluations),
  - as the preliminary step - to select the starting point/the first region of interest or the first population for optimization algorithm,
  - in optimization process - to construct hybrid algorithms (e.g. the combination of an evolutionary algorithm to select local minima and a gradient method to explore those minima) or modified algorithms (e.g. evolutionary procedure enriched with the information about the local sensitivities to increase the efficiency of the procedure.

The SA methods are classified using various criteria. One of the possibility is to group the algorithms with respect to the manner of parameter analysis:

- global methods - they calculate one (global) value expressing the sensitivity of a parameter for the whole parameters domain; these methods are derived from statistics and the probability theory,
- local methods - they calculate the sensitivity of a parameter for a small interval of parameter variation; local sensitivities are defined as the partial derivatives of a model output with respect to the input model parameters.

Below the algorithms of global sensitivity analysis which were applied to the optimization problems of industrial processes are presented. Global SA requires a definition of the following terms:

- expression which characterizes the measure of model output,
- definition of the variation interval for each input parameter,
- selection of the points in the parameters domain,
- sensitivity measure - the sensitivities are estimated based on the model output variations caused by changes in the model parameters.

In that work SA was performed with two algorithms: one-at-a-time method developed by Morris (1991) and Factorial Design (FD) (Saltelli et al., 2000).

Morris Design (MD) belongs to the screening design methods: the global indices of the parameters importance are estimated on information gathered from the whole parameters domain at the "screening" process. In MD algorithm the elementary effect ξ_i is introduced to calculate the sensitivities:

$$\xi_i(\mathbf{x}) := \frac{y(x_1,\ldots,x_{i-1},x_i+\Delta_i,x_{i+1},\ldots,x_k)-y(\mathbf{x})}{\Delta_i} \tag{1}$$

where: $\mathbf{x} \in \Omega \subset \mathrm{P}^k$ - the k-dimensional vector of process parameters x_i, Ω - domain of the analysis mapped to the $[0, 1]^k$ hypercube, Δ_i -increment of the i^{th} parameter.

A finite distribution F_i of elementary effects ξ_i calculated for the i^{th} factor is determined by sampling $\mathbf{x}$ in Ω. Expected value μ_i and standard deviation σ_i for i^{th} model parameter are estimated based on the distribution F_i, and using the classic estimators for independent random samples. Since the calculations of elementary effects ξ_i are sensitive to the parameter increment Δ_i, finite distribution F_i is determined with various values of Δ_i. To compare means μ_i and standard deviations σ_i estimations calculated for various Δ_i, the indices are normalized:

$$\bar{\mu}_i = \frac{\mu_i}{\|\boldsymbol{\mu}\|} \qquad \bar{\sigma}_i = \frac{\sigma_i}{\|\boldsymbol{\sigma}\|} \tag{2}$$

The higher expected value μ_i of the parameter elementary effect is, the higher sensitivity of the model output with respect to that parameter is observed. The high value of the standard deviation σ_i means that either the parameter impact on the model output is nonlinear or the parameter interacts with other model parameters.

Another approach is formulated in the two-level Factorial Design (FD). In the method, for each parameter the upper limit (marked as "+") and the lower limit (marked as "-") is specified and they define two-levels of the parameter. The points in the algorithm are generated starting with all low levels and ending levels. It means that for k parameters, model is run 2^k times. The average response from high level runs is contrasted with the average response from the low runs to determine the effect:

$$Effect_{FD} = \frac{\sum y^+}{n^+} - \frac{\sum y^-}{n^-} \quad (3)$$

where: y is the model output, "+", "-" is the upper/lower limit of the parameter range, respectively; n – the number of model simulations at each level.

Comparing MD and two-level FD methods of estimation the parameter effect to model output, one can observe, that FD is computationally cheaper than MD: for three parameters, FD requires $2^3 = 8$ runs of the model while MD – 16 runs to obtained equivalent results, see figure 1.

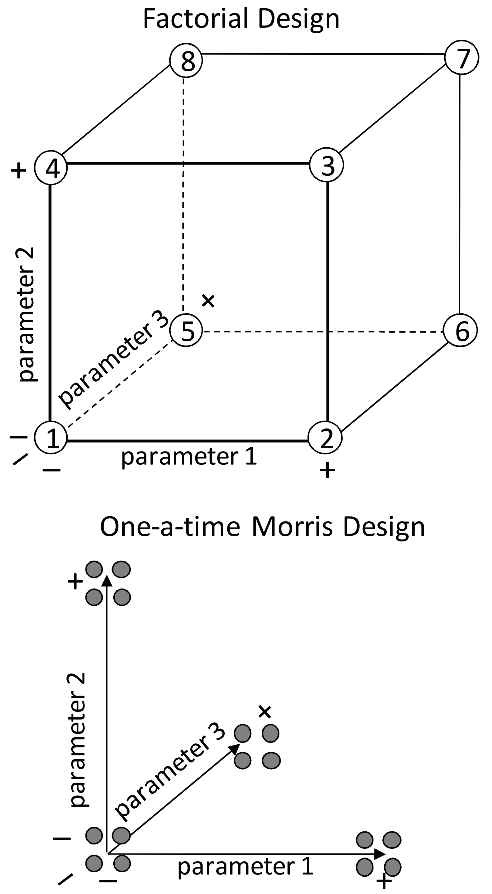

Fig. 1. *Two-level Factorial Design versus one-at-a-time Morris Design.*

3. APPLICATION OF SA TO THE SELECTED INDUSTRIAL PROCESSES

SA methods were applied to two industrial chains to support the optimization of those processes. The first task was the problem of one of stages of manufacturing chain for fasteners. The optimization of the whole manufacturing chain for fasteners is possible but is very complex and would require long computing times. Only selected parts of this chain were optimized, see results for hot part of the chain (Legwand et al., 2014) and variant optimization for cold forging of screws (Skóra & Pietrzyk, 2014). Detailed description of the process can be found in these papers.

In the present paper the final part of the manufacturing chain was selected as an example of application of the sensitivity analysis to improve efficiency of the optimization. The sensitivity of the product exploitation properties, represented by the maximum load of the anchor-concrete joint, was determined. Work necessary to pull out the anchor from the concrete plate was the output of the model. To determine this work strength tests for the joint were simulated using finite element method. The work was calculated as an integral of the force with respect to the displacement of the grip. Due to high computational cost of the model, the lowest possible number of simulations was the criterion for the selection of the SA method. Thus, Factorial Design was applied. The shape of the anchor was represented by the angle of the cone (α), angle of the expansion sleeve (β), and thickness of the expansion sleeve (a), see figure 2. The material properties of the sleeve was defined as the yield stress R_e. The results of the FD analysis are presented in figure 3. It is observed that the strength of the anchor-concrete joint described as the work to the maximum force, is the most sensitive to the thickness of the expansion sleeve a, and next to the properties of the sleeve material R_e.

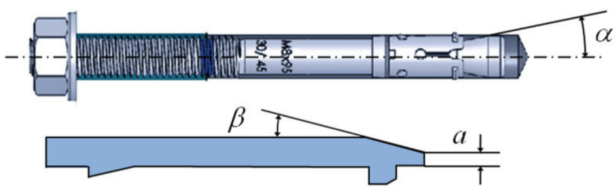

Fig. 2. *View of the anchor and its characteristic dimensions.*

Based on the results of FD trials and errors optimization combined with an expert knowledge was performed and the optimum of the process parameters: $\alpha = 11^{\circ}$, $\beta = 9^{\circ}$, a = 1.35 mm and R_e = 200 MPa was confirmed (Skóra & Pietrzyk, 2014).

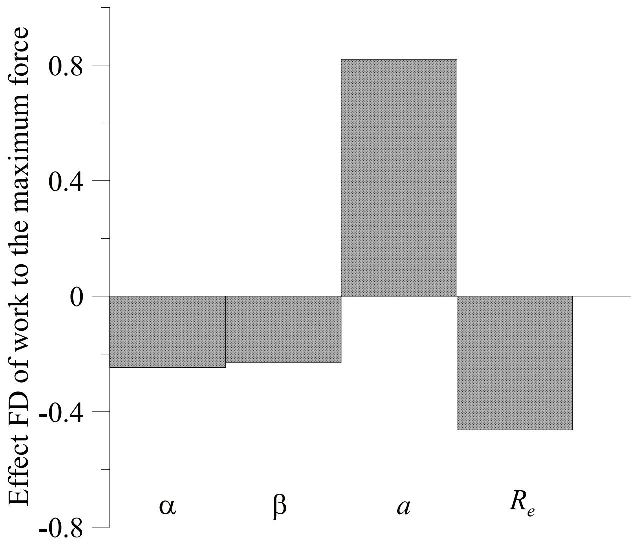

Fig. 3. FD effects of the strength of the anchor-concrete joint with respect to the shape parameters and properties of the expansion sleeve material.

The second industrial case study investigated in the paper was the process of rails cooling. Optimization of the controlled cooling of rails is an important industrial problem. The cooling system developed in the IMŻ Gliwice and described by Kuziak et al. (2014) was considered in the present paper. The sensitivity of the product microstructure and properties with respect to the cooling parameters was determined. The cooling process was described with Avrami equation for phase transformations combined with finite element solution of the heat transfer equation. Equations describing microstructure and properties of product were also implemented in the FE code. Comparing to the anchor model, the rails cooling model was not so computationally expensive, therefore, Morris Design method was used to estimate the sensitivities.

The nine input parameters for MD were selected: $\mathbf{x} = \left\{h_{tr}, t_i, d_j\right\}_{i=1,\ldots,5;\, j=1,3,5}$, where $h_{tr} \in [1700, 2000]$, W/m^{2}K, the heat transfer coefficient represented in the analysis by its maximum value; $t_i \in [5, 40]$, s, if i = 1, 3, 5 then t_i is the time of accelerated cooling (rail head immersed in the solution, if i = 2, 4 then t_i is the time of air cooling; $d_j \in [25, 35]$, mm, j = 1, 3, 5, depth of the immersion of the head in the solution at j stage of the process. The intervals of the parameters existence cover the technological conditions of the cooling process.

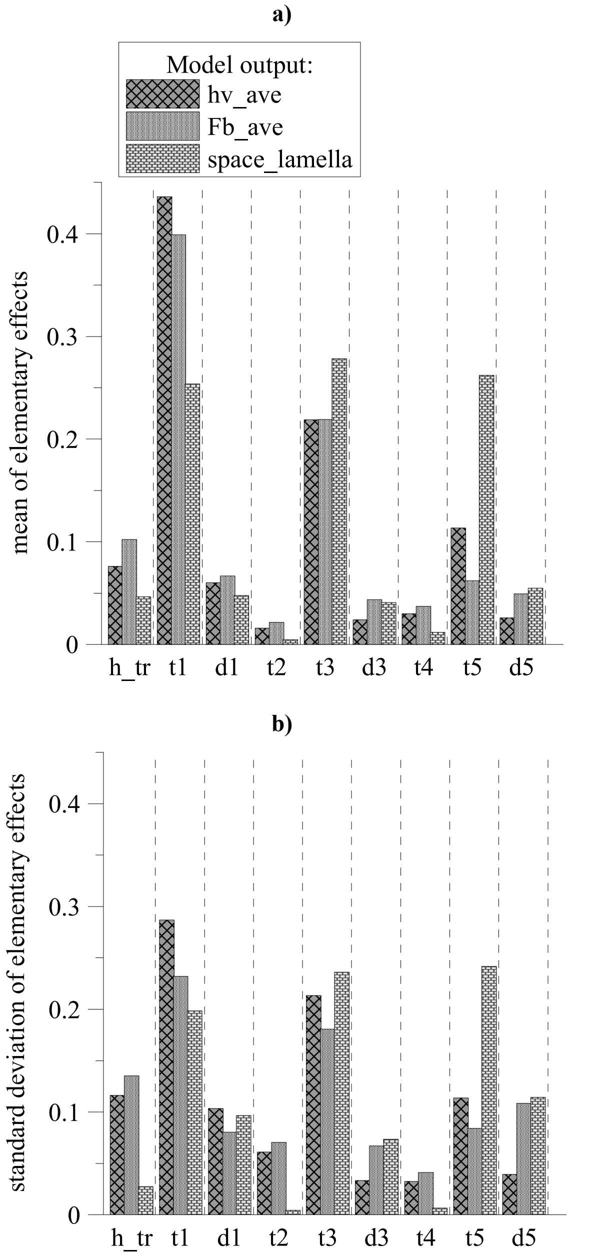

Fig. 4. MD sensitivity indices calculated for three model outputs: average hardness in the rail head, volume fraction of bainite and interlamellar spacing, with respect to the cooling process parameters: means (a), standard deviations (b).

The cooling model output vector **y** for MD was composed of the parameters, which should be objectives of optimization in the design of the cooling schedule: $\mathbf{y} = \left\{S_0, F_b, \Delta HB\right\}$, where S_0 – the average interlamellar spacing, F_b – the average volume fraction of bainite and ΔHB – the homogeneity parameter for the hardness:

$$\Delta HB = \sqrt{\sum_{i=1}^{n_{gp}} \left(\frac{HB_i - HB_{ave}}{HB_i} \right)^2} \tag{4}$$

where H_{ave} – the average hardness at the rail head cross section, HB_i – the hardness in the i^{th} Gauss point, n_{gp} – the number of the Gauss points at the rail head cross section.

All three model outputs were calculated for the rail head. The results of calculations of sensitivity indices $\underline{\mu}_i$ and $\underline{\sigma}_i$ are shown in figure 4. It is seen that volume fraction of bainite is sensitive to the duration of all immersion times of the rail head. Standard deviations for these variables are large, what means that relation of the bainite volume fraction on the optimization variables is nonlinear. Similarly high sensitivities were obtained for the interlamellar spacing and the average hardness. It can be concluded that all times of immersion will be effective in elimination of the bainite from the microstructure and decreasing the interlamellar spacing. The influence of the remaining variables was much smaller.

4. CONCLUSIONS

Presented in the paper global SA algorithms were applied to the selected industrial processes and the most important process parameters were determined. The results of SA were the input information for the optimization of the processes. The parameters of the highest effects have the highest influence on the process control. In other words, if the model outputs are strongly sensitive to the parameter, this parameter is estimated with higher accuracy in the optimization than a parameter for which the outputs are weakly sensitive. The SA is also helpful in the optimization performed using trials and errors method combined with an expert knowledge, by reducing the number of model simulations. Such optimization approach is just applied for manufacturing chains consisting of several stages.

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ANALIZA WRAŻLIWOŚCI JAKO NARZĘDZIE WSPOMAGAJĄCE OPTYMALIZACJĘ PROCESÓW PRZEMYSŁOWYCH

Streszczenie

W pracy podjęto tematykę związaną z optymalizacją procesów przemysłowych składających się z szeregu procesów pośrednich tworzących cykl produkcyjny. Optymalizacja cykli jest zadaniem trudnym ze względu na dużą liczbę zmiennych sterujących, często przeciwstawne lub wzajemnie wykluczające się funkcje celu poszczególnych etapów, złożoność modeli numerycznych symulujących procesy. Zastosowanie analizy wrażliwości jako kroku wstępnego poprzedzającego właściwą optymalizację, może znacznie zredukować zadanie optymalizacji. W pracy omówiono różne metody globalnej analizy wrażliwości. Wymienione procedury zastosowano dla przykładowych procesów: wytwarzania kotwi oraz kontrolowanego chłodzenia szyn, wyznaczając najbardziej istotne parametry procesów. W pracy przedstawiono i omówiono wyniki obliczeń przeprowadzonej analizy wrażliwości.

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