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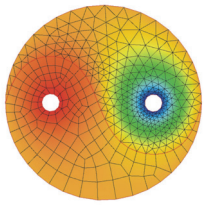

APPLICATION OF *hp*-ADAPTIVE FEM, LOCAL NUMERICAL HOMOGENIZATION AND DISCRETE ELEMENT METHOD TO MODELING OF ASPHALT PAVEMENT STRUCTURES

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Abstract

This paper presents a novel approach to modeling of asphalt pavement structures. Analysed medium exhibits a complex behaviour due to its specific structure. It is a multi-layered domain consisting of both bound asphalt layers and unbound layers of the compacted aggregate (e.g. the base course or the anti-frost layer). The first layer type can be modeled as a continuum, whereas the latter one as a composition of small, separable bodies being in contact. Moreover, the asphalt mixture reveals strong heterogeneity. Its main constituents (the aggregate and the bituminous binder) exhibit completely different response to the applied load. Thus, a specific approach is proposed to consider this complexity. Presented approach enables one to avoid costly laboratory or 'in situ' tests performed to evaluate the response of the adopted pavement structure.

In order to account for the heterogeneity of the asphalt mixture, local numerical homogenization is used. Brief description of this computational homogenization method is presented. The aggregate is assumed to be elastic and the binder is modeled as a Burgers visco-elastic material. The latter model is also briefly described. Numerical modeling of the asphalt layers is performed using hp-adaptive FEM. Its integration with local numerical homogenization is presented in details. Numerical modeling of the unbound layers is done using discrete element method (DEM).

Results of preliminary numerical tests that confirm the efficiency of the proposed approach are presented.

Key words: *hp*-adaptive FEM, local numerical homogenization, discrete element method

1. INTRODUCTION

1.1. Asphalt pavement structures

A typical asphalt pavement structure consists of several asphalt layers as well as a few unbound layers resting upon the improved subgrade. Asphalt layers can be considered as a continuum, whereas unbound layers should be modeled as a discrete domain. As far as the microstructure of a single asphalt layer is concerned, it is made of two main constituents:

- aggregate – usually the crushed stone (90÷97% of the weight ratio),
- bituminous binder (3÷10% of the weight ratio).

Typically, also the filler (up to 1% of the weight ratio) is added in order to improve the adhesion of the binder to the aggregate. Content of the air voids is equal to about 5% of the volumetric ratio in case of standard asphalt mixtures.

Unbound layers consist only of aggregate, which is compacted properly before asphalt layers are laid on. Only the top surface of the unbound layer being in contact with the base course (bottom asphalt layer) is sprinkled with the asphalt emulsion. Bearing capacity of the unbound layers is based on the proper compaction as well as on the wedging of the aggregate seeds of different sizes and shapes.

Proposed numerical model of the whole asphalt pavement structures is an effort to account for the

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presented complexity of the asphalt pavement structures. Numerical aspects of the model are listed below:

- asphalt layers are modeled as a continuum using *hp*-adaptive FEM,
- aggregate (in asphalt layers) is modeled as linear elastic,
- bituminous binder is modeled as a visco-elastic Burgers material,
- local numerical homogenization is used in order to account for the heterogeneity of the asphalt mixture structure,
- unbound layers are modeled using discrete element method (DEM),
- for the sake of simplicity the preliminary numerical tests are performed with the assumption of the constant temperature.

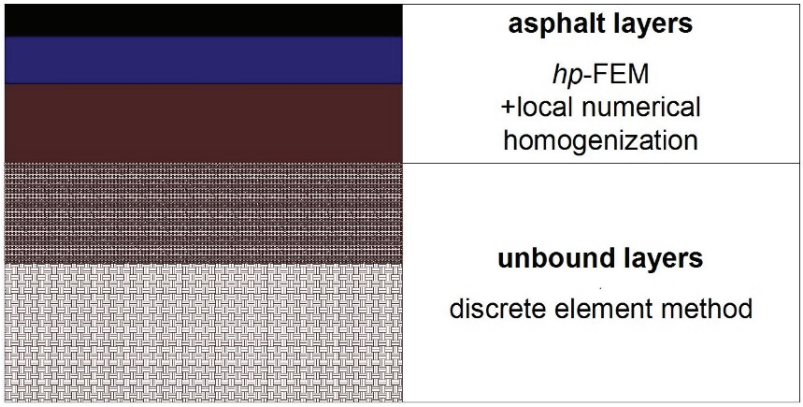

Fig. 1. *Typical asphalt pavement structure with corresponding numerical methods used for modeling of respective type of layers.*

A scheme of a typical asphalt pavement structure with corresponding numerical methods we use in our model is shown in figure 1.

1.2. Burgers model

In this chapter a short description of Burgers model is presented. This is a typical model used in numerical analyses of bituminous materials. Validation of the model has been done by several researchers, e.g. Woldekidan (2011), Collop et al. 2003. Many of augmented versions of the model accounting for temperature, damage effects, etc. can be found in literature. However, for the sake of simplicity, we describe application of its standard version. Simplified mechanical 1D interpretation of Burgers model is shown in figure 2 as a combination of spring and dashpot elements.

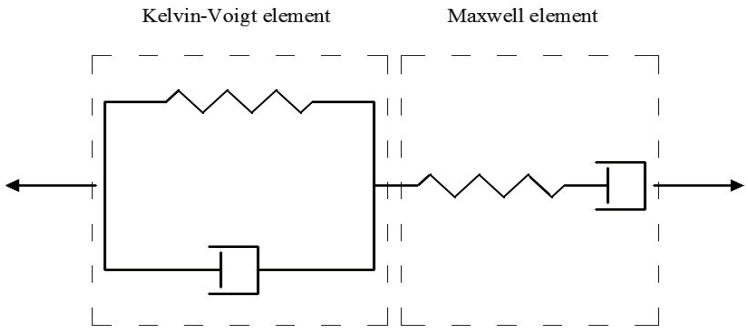

Fig. 2. *Mechanical 1D Burgers model.*

Usually one uses "*N*" Kelvin-Voigt elements joined in series in order to characterize bitumen response in a more efficient way. Such a model is able to capture all of the visco-elastic phenomena in a reliable way. More than ten Kelvin-Voigt elements should be used in order to enhance the efficiency of modeling of bituminous materials (Woldekidan, 2011). Its easy numerical implementation is also an advantage in context of multiscale analyses. More sophisticated versions of the Burgers model will be used in future.

Strong formulation of the problem modeling asphalt layers is presented below.

Find the displacements $\mathbf{u}(\mathbf{x}, t)$ *such that:*

$$\begin{cases} div\dot{\boldsymbol{\sigma}} + \dot{\boldsymbol{X}} = \mathbf{0} & \forall t, \boldsymbol{x} \in \omega_i \subset \Omega \\ \dot{\boldsymbol{\sigma}} = \boldsymbol{C}[\dot{\boldsymbol{\varepsilon}}(\dot{\boldsymbol{u}}) - \dot{\boldsymbol{\varepsilon}}^*] & \forall t, \boldsymbol{x} \in \omega_i \subset \Omega \\ \dot{\boldsymbol{\varepsilon}} = \frac{1}{2}[\boldsymbol{\nabla}\dot{\boldsymbol{u}} + (\boldsymbol{\nabla}\dot{\boldsymbol{u}})^T] & \forall t, \boldsymbol{x} \in \omega_i \subset \Omega \\ \dot{\boldsymbol{\varepsilon}}^* = f(\boldsymbol{\sigma}, \chi, \ldots) & \forall t, \boldsymbol{x} \in \omega_i \subset \Omega \\ + initial, boundary \\ and\ continuity/debonding\ conditions \end{cases} \quad (1)$$

where dot above a symbol denotes differentiation with respect to time and:

$\boldsymbol{\sigma}$ – stress tensor
$\boldsymbol{X}$ – body forces
$\boldsymbol{C}$ – tensor of material parameters
χ – internal variable (inelastic strains in the case of the Burgers model)
$\boldsymbol{\varepsilon}$ –strain tensor
$\boldsymbol{\varepsilon}^*$ –inelastic strain defined by Burgers model

The corresponding weak formulation of problem (1) is as follows.

Find the displacements $\boldsymbol{u}(\boldsymbol{x},t) \in H_0^1(\Omega) + \widehat{\boldsymbol{u}}$ *such that:*

$$\int_\Omega [\dot{\boldsymbol{\varepsilon}} - \dot{\boldsymbol{\varepsilon}}^*] : \dot{\boldsymbol{\sigma}} d\omega = \int_\Omega \boldsymbol{v} \cdot \dot{\boldsymbol{X}} d\omega + \int_{S_\sigma} \boldsymbol{v} \cdot \dot{\boldsymbol{t}}^0 ds \ \forall t, \forall \boldsymbol{v} \in H_0^1(\Omega) \quad (2)$$

where:
$\boldsymbol{v}$ – test functions
$\boldsymbol{t}^0$ – tractions
H_0^1 – the Sobolev space of the first order of functions satisfying homogeneous Dirichlet boundary conditions

Due to the visco-elastic character of the material response, transient analysis is necessary. Therefore incremental form of (2) is used

$$\int_\Omega \Delta\boldsymbol{\varepsilon} : {}^{t-\Delta t}\dot{\boldsymbol{\sigma}} d\omega = \int_\Omega \boldsymbol{v} \cdot \Delta\boldsymbol{X} d\omega + \int_{S_\sigma} \boldsymbol{v} \cdot \Delta\boldsymbol{t}^0 ds + \int_\Omega \Delta\boldsymbol{\varepsilon}^* : {}^{t-\Delta t}\dot{\boldsymbol{\sigma}} d\omega \quad (3)$$

where ${}^{t-\Delta t}\dot{\boldsymbol{\sigma}}$ denotes stress rate field at time instant $(t-\Delta t)$
The total strain increment is decomposed into visco-elastic ($\Delta\boldsymbol{\varepsilon}_{ve}$), elastic ($\Delta\boldsymbol{\varepsilon}_{el}$) and viscous ($\Delta\boldsymbol{\varepsilon}_{v}$) terms

$$\Delta\boldsymbol{\varepsilon} = \Delta\boldsymbol{\varepsilon}_{el} + \Delta\boldsymbol{\varepsilon}_{ve} + \Delta\boldsymbol{\varepsilon}_{v} \quad (4)$$

and after papers by Collop et al. (2003), Woldekidan (2011), Woldekidan et al. (2012)

$$\Delta\boldsymbol{\varepsilon}_{el} = \boldsymbol{C}\Delta\boldsymbol{\sigma} \quad (5)$$

$$\Delta\boldsymbol{\varepsilon}_{ve} = \sum_{i=1}^{N} \left\{ {}^{t-\Delta t}\boldsymbol{\varepsilon}_{ve}^i \left(e^{-\frac{\Delta t}{\tau_i}} - 1 \right) + \Delta t e^{-\frac{\Delta t}{2\tau_i}} \boldsymbol{C}_{ve}^i \left({}^{t-\Delta t}\boldsymbol{\sigma} + \frac{\Delta\boldsymbol{\sigma}}{2} \right) \right\} \quad (6)$$

$$\Delta\boldsymbol{\varepsilon}_{v} = \Delta t \boldsymbol{C}_{vp} \left({}^{t-\Delta t}\boldsymbol{\sigma} + \frac{\Delta\boldsymbol{\sigma}}{2} \right) \quad (7)$$

where:
$\Delta\boldsymbol{\sigma}$ – stress increment
$\boldsymbol{C}$

$$= \frac{1}{E} \begin{bmatrix} 1 & -\nu & -\nu & 0 & 0 & 0 \\ -\nu & 1 & -\nu & 0 & 0 & 0 \\ -\nu & -\nu & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2(1+\nu) & 0 & 0 \\ 0 & 0 & 0 & 0 & 2(1+\nu) & 0 \\ 0 & 0 & 0 & 0 & 0 & 2(1+\nu) \end{bmatrix}$$

i – i-th Kelvin-Voigt element (i=1,..,N)
η_{ve}^i – viscosity of i-th Kelvin-Voigt element
τ^i – retardation time of i-th Kelvin-Voigt element
$\boldsymbol{C}_{ve}^i, \boldsymbol{C}_{vp}$ –like $\boldsymbol{C}$ but with ν replaced by ν_{ve}^i and ν_{vp} respectively and with E replaced by η_{ve}^i and η_v respectively
${}^{t-\Delta t}\boldsymbol{\varepsilon}_{ve}^i$ – visco-elastic strain of i-th Kelvin-Voigt element at time t-Δt

The general algorithm for time integration according to Burgers model is presented in figure 3. Calculation of inelastic strain increments is performed in a loop over all Gauss points. If an integration point is located within the aggregate, inelastic strain increments are equal to zero. It is due to the elastic material model used for aggregate seeds.

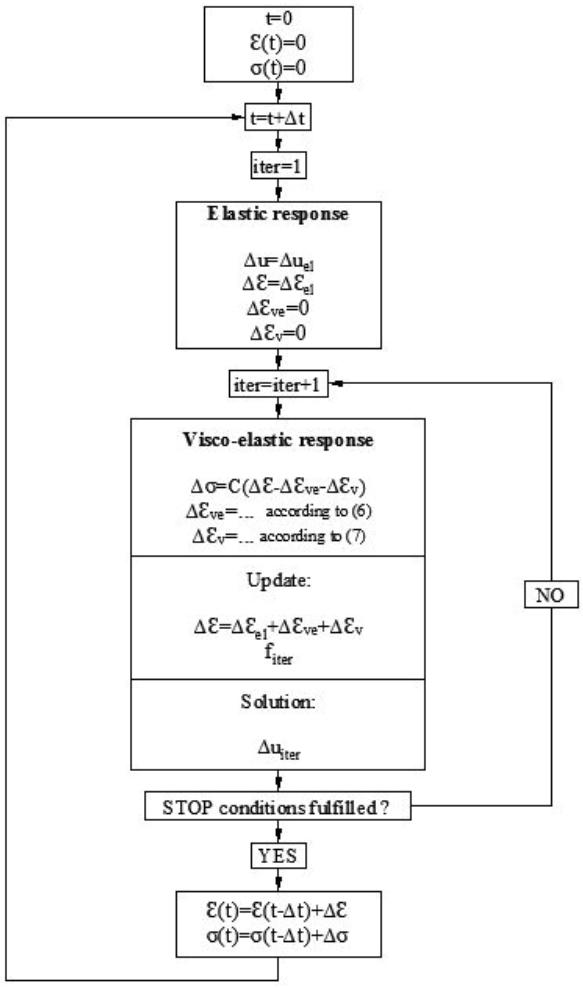

Fig. 3. *Algorithm of time integration according to Burgers model.*

The first step at each time instant is to solve the elastic problem. Subsequently, inelastic strain increments are updated according to Burgers constitutive model. Their occurrence implies updating of the load vector $\boldsymbol{f}$ then. Meeting the STOP conditions allows one to proceed with the next time step. Strain and stress increments from the last iteration are considered as the final ones at the analysed time instant.

The STOP conditions are as follows:

$$\frac{\|\Delta\boldsymbol{\varepsilon}_{iter}-\Delta\boldsymbol{\varepsilon}_{iter-1}\|}{\|\Delta\boldsymbol{\varepsilon}_{iter}\|} < TOL \text{ and}$$
$$\frac{\|\Delta\boldsymbol{u}_{iter}-\Delta\boldsymbol{u}_{iter-1}\|}{\|\Delta\boldsymbol{u}_{iter}\|} < TOL \qquad (8)$$

where:

$\Delta\boldsymbol{x}_{iter}$ – increment of value $\boldsymbol{x}$ at current iteration

$\Delta\boldsymbol{x}_{iter-1}$ – increment of value $\boldsymbol{x}$ at previous iteration

TOL – value of admissible error

2. LOCAL NUMERICAL HOMOGENIZATION

2.1. General idea

Local numerical homogenization (LNH) was proposed by Jhurani and Demkowicz (2009). It is a method of computational homogenization, which we use in order to avoid time consuming so-called "brute force" analysis. We apply this method to modeling of asphalt layers because of their specific structure. Top layers of the asphalt pavement structure (wearing course, binder course) are relatively thin with respect to the aggregate dimensions, which they are made of. Thus, separation of scale condition cannot be met. That is the main assumption of all RVE-based approaches to multiscale modeling (e.g. Geers et al., 2003). Local numerical homogenization is free of this limitation. Moreover, it can be easily used for modeling of heterogeneous materials with non-periodic microstructure, which one deals with analysing asphalt mixture.

The main concept of the method is to reduce the number of finite elements in a specific manner. One needs to generate a coarse mesh. Then, within each coarse mesh element, it should be refined in order to capture all heterogeneities of the microstructure. The crucial thing is that we do not solve the problem on the fine mesh, however. What we do instead is evaluation of effective coarse mesh element stiffness matrix on the basis of known fine mesh element stiffness matrices. Thus, the whole problem can be solved using the coarse mesh. Details of the local numerical homogenization are presented in Jhurani's dissertation (2009). Preliminary tests performed in order to validate the method are presented in papers by Klimczak and Cecot (2011, 2013). The general algorithm of local numerical homogenization in a form we use it is presented in figure 4.

Evaluation of the effective stiffness matrices for coarse mesh elements is the key-point of the whole algorithm. Our task is to find such an effective stiffness matrix for every coarse mesh element that the

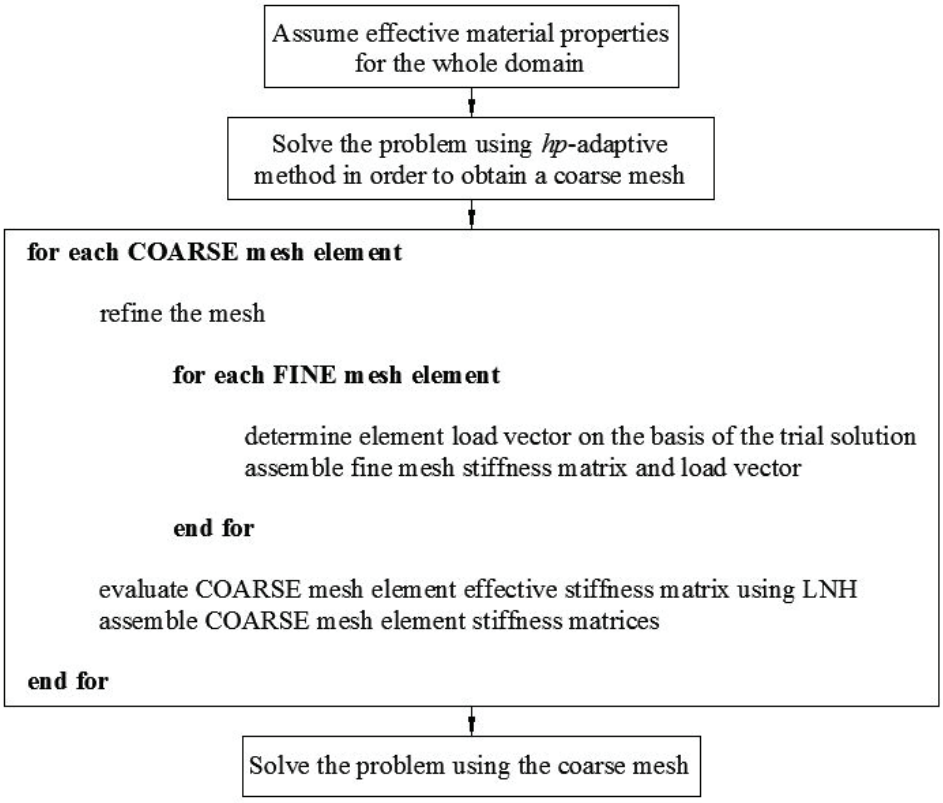

Fig.4. *Algorithm of local numerical homogenization*

norm of the difference of two solutions is minimum. These two solutions are:

– solution obtained using one coarse mesh element,
– solution obtained using the fine mesh in the subdomain of the considered coarse element.

In fact, we do not need to solve both of the problems at this level. We operate on would-be solutions. As it was shown in Jhurani's dissertation (2009), this local homogenization problem reduces to minimization of the following formula:

$$E(\widehat{\boldsymbol{K}}^{\dagger}) = \frac{1}{2}\left\|\left(\boldsymbol{K}^{\dagger} - \boldsymbol{A}\widehat{\boldsymbol{K}}^{\dagger}\boldsymbol{A}^{T}\right)\boldsymbol{f}\right\|_{\boldsymbol{B}}^{2} + \frac{\varepsilon}{2}\left\|\boldsymbol{K}^{\dagger} - \boldsymbol{A}\widehat{\boldsymbol{K}}^{\dagger}\boldsymbol{A}^{T}\right\|_{F,\boldsymbol{B}}^{2}\|\boldsymbol{f}\|_{2}^{2} \quad (9)$$

where:

$\boldsymbol{A}$ – interpolation operator that maps coarse element d. o. f. to fine mesh d. o. f.

$\boldsymbol{K}$ – assembled fine mesh stiffness matrix (only in the subdomain occupied by the coarse element)

$\widehat{\boldsymbol{K}}$ – effective coarse element stiffness matrix

$\widehat{\boldsymbol{K}}^{\dagger}$ – Moore-Penrose pseudoinverse of matrix $\widehat{K}$

$\boldsymbol{f}$ – assembled fine mesh load vector (only in the subdomain occupied by the coarse element)

ε – dimensionless positive parameter (used for regularization)

$\|\boldsymbol{x}\|_{\boldsymbol{B}} = \sqrt{trace(\boldsymbol{x}^{T}\boldsymbol{B}\boldsymbol{x})}$ – Euclidean norm weighted with matrix $\boldsymbol{B}$

$\|\boldsymbol{X}\|_{\boldsymbol{F}} = \sqrt{trace(\boldsymbol{X}^{T}\boldsymbol{X})}$ – Frobenius norm

$\|\boldsymbol{X}\|_{\boldsymbol{F},\boldsymbol{B}} = \sqrt{trace(\boldsymbol{X}^{T}\boldsymbol{B}\boldsymbol{X})}$ – Frobenius norm weighted with matrix $\boldsymbol{B}$

The first term measures the error introduced by the local numerical homogenization, whereas the second one was added for regularization purposes. Details of the minimization of (9) are presented in Jhurani's dissertation (2009). One should note that after this operation we obtain $\widehat{\boldsymbol{K}}^{\dagger}$ in fact. Thus, $\widehat{\boldsymbol{K}} = \left(\widehat{\boldsymbol{K}}^{\dagger}\right)^{\dagger}$ finally.

Fine mesh load vector usually is not known explicitly. In our numerical tests we assess the fine mesh load vector on the basis of the trial solution obtained in the second step of the algorithm presented in figure 4.

It should be pointed out that the fine mesh solution is not computed in this algorithm even locally.

2.2. Integration of local numerical homogenization with visco-elastic Burgers model

Integration of local numerical homogenization with Burgers model requires several modifications of the above presented algorithm. Elastic solution serves not only as a trial solution for further visco-elastic analysis but it is also used to determine boundary conditions for the coarse mesh element. For each time step effective stiffness matrix for coarse mesh element is evaluated on the basis of known fine mesh element stiffness matrices and updated fine mesh load vector. Further details of the whole procedure are presented in paper by Klimczak and Cecot (2013).

2.3. Numerical results

In this chapter numerical results of the proposed approach are presented. The homogenized solutions were obtained without mesh adaptation. Only uniform, thus not optimal, refinements were used to capture all details of the microstructure.

For test purposes a plane strain state in a rectangular domain (4m x 2m) with boundary conditions and distribution of inclusions presented in figure 5 was analysed. Fine mesh used for LNH purposes is also presented in this figure. At three edges of the rectangular domain no-penetration Dirichlet boundary conditions are assumed. A constant distributed load of 1.53kN/m^2 is applied at the top edge. Poisson ratio for both materials is equal to 0.3. Young modulus for white elements is equal to 2.04MPa, whereas for black ones it equals one tenth of this value. For several seconds the load value is maintained at the same level, then it is removed.

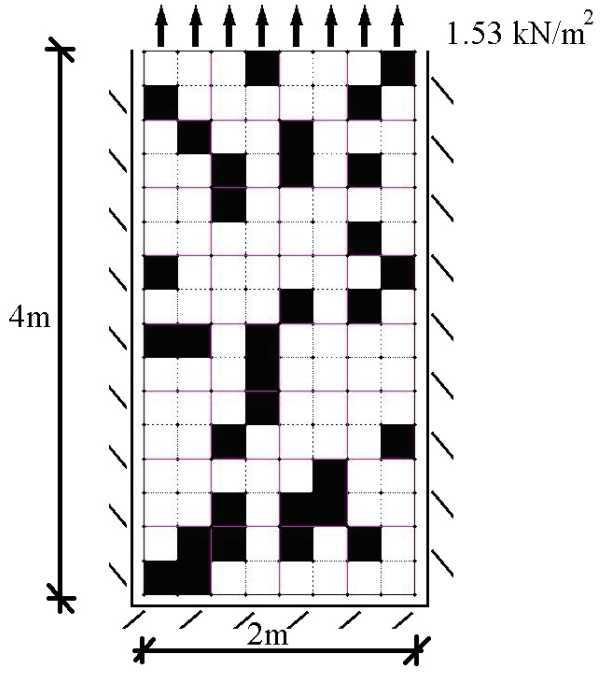

Fig. 5. *Test example 1. Analysed domain with boundary conditions.*

Results obtained using proposed approach are compared with the "brute force" solution ("exact"). Vertical displacements along the upper edge at the time instant preceding the load removal are presented in figure 6.

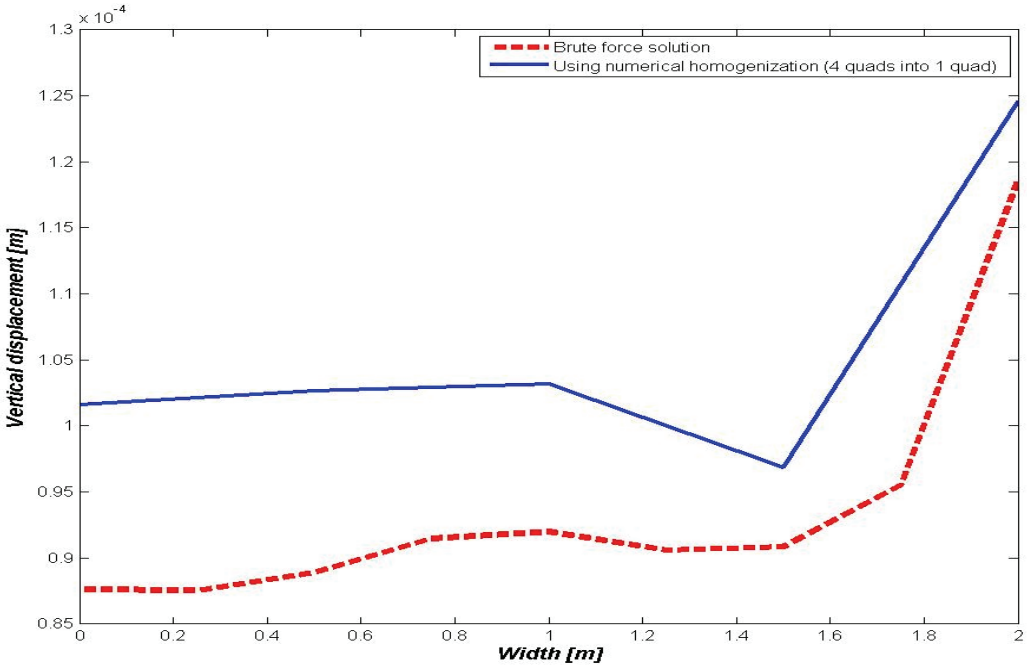

Fig. 6. *Test example 1. Vertical displacements along the upper edge with and without homogenization.*

The maximum difference of the results is of order of about 15%. Both "coarse" and "fine" meshes were rather rough (306 *dofs* – fine mesh, 90 *dofs* – coarse mesh), so these results are the preliminary ones, nevertheless promising since the number of degrees of freedom was reduced significantly without introducing large additional modeling error.

3. *hp*-ADAPTIVE FEM

In the proposed approach to modeling of heterogeneous visco-elastic materials we integrate the automatic *hp*-adaptive FEM (Demkowicz, 2006; Demkowicz et al., 2008) with local numerical homogenization (Jhurani and Demkowicz, 2009).

3.1. General idea of the *hp*-adaptive FEM

There are three main types of the adaptive FEM. In *h*-adaptive version size of finite elements is reduced in the areas with large approximation error, while in the *p* approach the error is reduced by increasing order of approximation in selected subdomains. Appropriate combination of anizotropic *h* and *p* refinements is called *hp*-adaptive method. The main advantages of the adaptive methods are fast convergence and error controlled solution

In our research we take advantage of automatic *hp*-adaptive version of FEM developed by Demkowicz and his collaborators (Demkowicz, 2006; Demkowicz et al., 2008). We make use of their two FEM codes: *hp2D* and *hp3D*.

Computing with automatic *hp*-adaptive codes is performed in several steps. First, an initial mesh is generated. Then, on the basis of reference solution obtained on uniformly refined mesh with approximation order increased by one, an optimal mesh is constructed. The projection based interpolation error estimate is used to decide, whether the initial mesh is to be refined in order to obtain the a priori assumed error level (e.g. 0.1%).

Our first research effort was to implement visco-elastic Burgers material model into both of the mentioned codes.

3.2. Integration of *hp*-adaptive FEM with local numerical homogenization

We make use of *h*p-adaptive FEM at two stages of LNH. Firstly, it is used to solve the problem with assumed effective material parameters. Coarse mesh for LNH purposes is generated in such a manner.

Secondly, we apply the *hp*-adaptive FEM to discretize the local problems for each coarse mesh element. This is the key point of the LNH routine.

In order to reduce the computational cost we perform *hp*-adaptation only at selected (arbitrarily, so far) time instants. One of our future research efforts is to make use of the approach called 'homogenization in time'. Further reduction of the computational time without losing of the time integration accuracy is possible.

Only assembling of stiffness matrices for such local problems is needed. Thus, the mesh adaptation is performed on the basis of the material distribution to provide compliance of the mesh with the heterogeneities of the composite structure, see e.g. Serafin (2012).

The *hp*-adaptive FEM was already used for inelastic problems (Serafin, 2012). The numerical results for elastic-plastic Mises model confirmed that the fully automatic algorithm is efficient in such nonlinear cases. Since the Burgers model is of the same type (inelastic one) the fully automatic hp-adaptation will also deliver a fast convergence for problems with these constitutive equations.

4. DISCRETE ELEMENT METHOD

4.1. General idea

Discrete element method (DEM) is widely used in the field of granular flows, rock and powder mechanics. Its efficiency in context of asphalt pavement structures was discussed several times in the literature (e.g. Abbas et al., 2007). DEM holds the position of one of the leading methods in this field. Our goal is to incorporate this method into the proposed numerical model of asphalt pavement structures. Bottom layers of the pavement structure (e.g. a base course, an anti-frost layer, an improving layer) as well as the subgrade itself can be modeled efficiently using DEM. Fundamentals and applications of this method are presented e.g. in papers by Rojek (2007a, 2007b).

Within discrete element method it is assumed that the whole domain is made of rigid bodies of arbitrary shape interacting with one another. Typically, for the sake of computational time, spheres (in 3D) and cylinders (in 2D) are used. Moreover, different kinds of interaction forces can be accounted for depending on the analysis level - from the van der Waals forces, contact forces, up to the gravity force. Analysis in time is carried out by a time step method. At each of the time steps following phases can be distinguish for every particle:

- detection of possible collisions with neighbouring particles,
- calculation of resultant forces acting on the particle,
- calculation of the rotational and translational displacements.

These three stages are performed in a loop iteratively until the equilibrium is reached. Then the next time step is analysed.

Integration of DEM with FEM was described e. g. by Rojek (2007b). Introducing of additional constraints on the interface between two subdomains is necessary. In our case it is the lower straight line shown in figure 7. Application of these constraints is usually performed using Lagrangian multipliers or penalty function.

4.2. Numerical results

In our research we use *Yade* code – an open source framework for discrete element analysis. Its documentation, describing both DEM fundamentals and its implementation in *Yade*, was provided by Šmilauer and Chareyre (2010).

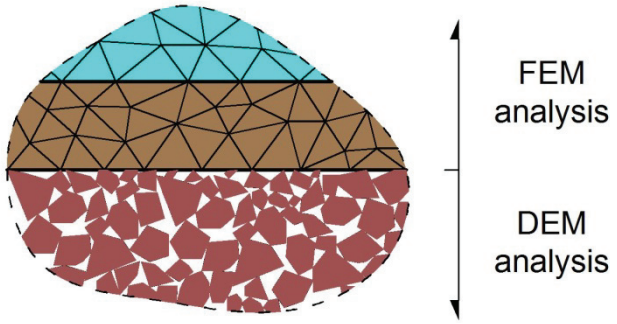

Fig. 7. *Interface between subdomains analysed by FEM (asphalt layers) and DEM (unbound layers).*

In figure 8a a set of 400 spherical particles at the beginning of the analysis is presented. No binder was assumed between particles. The boundary conditions are as follows:

- the bottom wall is fixed, only the side walls can change their position orthogonally under the pressure caused by the particles,
- load is subjected to the upper wall.

In figure 8b the final equilibrium of the analysed domain is presented. Both of the figures are screenshots from *Yade* framework.

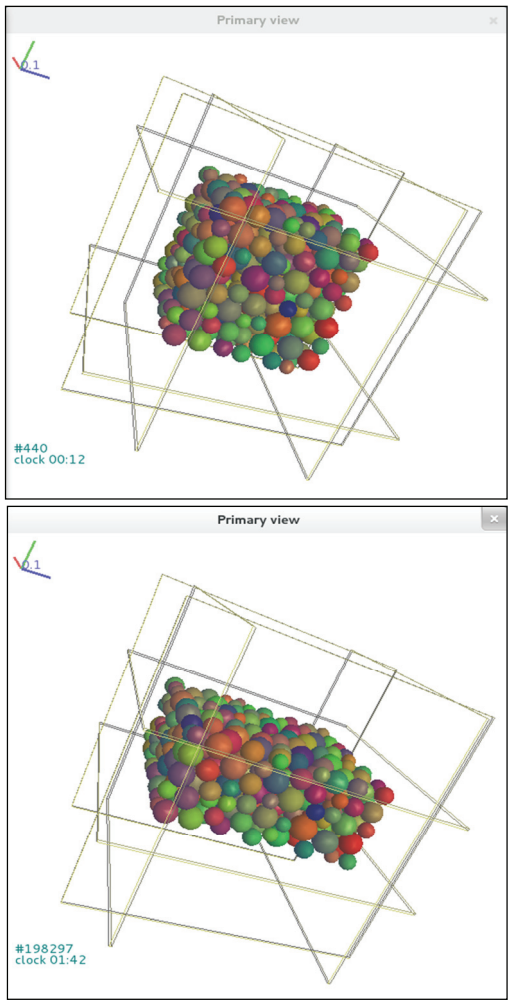

Fig. 8. *a) Beginning of the analysis, b) Final result of the analysis.*

These preliminary results confirm the possibility of the DEM application to modeling of unbound pavement layers.

5. CONCLUSIONS

In this paper a complex numerical model of the whole asphalt pavement structure was presented. Results of preliminary tests performed for benchmark problems are promising:

- local numerical homogenization does not introduce large additional error to the solution,
- local numerical homogenization integrated with Burgers material model is a reliable tool for modeling of heterogeneous visco-elastic materials,
- *hp*-adaptive FEM makes the whole LNH routine more efficient,
- discrete element method is a natural choice for reliable modeling of unbound pavement structure layers.

Further research effort is to:

- make integration of *hp*-adaptive FEM and local numerical homogenization automatic,
- integrate discrete element analysis with the remaining part of the whole model,
- irregularly shaped aggregate seeds need to be implemented in order to make this approach reliable in context of above layers.

Obtained numerical results are going to be experimentally validated as well.

ACKNOWLEDGEMENTS

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ZASTOSOWANIE HP-ADAPTACYJNEJ MES, LOKALNEJ HOMOGENIZACJI NUMERYCZNEJ ORAZ METODY ELEMENTÓW DYSKRETNYCH DO MODELOWANIA KONSTRUKCJI NAWIERZCHNI ASFALTOWYCH

Streszczenie

Artykuł przedstawia nowe podejście do modelowania konstrukcji nawierzchni asfaltowych. Analizowany ośrodek wykazuje złożone zachowanie ze względu na swą specyficzną budowę. Jest to ośrodek wielowarstwowy składający się z zarówno ze związanych asfaltem warstw nawierzchni, jak i niezwiązanych warstw z zagęszczonego kruszywa (np. podbudowa lub warstwa mrozoochronna). Pierwszy z wymienionych rodzajów warstw może być modelowany jako continuum, natomiast drugi z nich powinien być modelowany jako ośrodek dyskretny. Ponadto, mieszanka asfaltowa wykazuje silną niejednorodność budowy. Jej główne składniki (kruszywo i lepiszcze asfaltowe) charakteryzują się zupełnie odmienną odpowiedzią na działające obciążenie. Uwzględnienie całej opisanej wyżej złożoności problemu wymaga specjalnego podejścia. Zaproponowano sposób analizy, który pozwala na uniknięcie kosztownych badań laboratoryjnych lub polowych wykonywanych w celu przewidzenia zachowania przyjętej konstrukcji nawierzchni.

W celu uwzględnienia niejednorodnej budowy mieszanki mineralno-asfaltowej wykorzystano lokalną homogenizację numeryczną. Przedstawiono krótki opis tej metody komputerowej homogenizacji. Kruszywo zamodelowane zostało jako materiał sprężysty, natomiast dla lepiszcza asfaltowego założono lepkosprężysty model Burgersa. Został on krótko opisany. Dotychczas wszystkie testy numeryczne zostały wykonane przy założeniu o stałej temperaturze. Do numerycznego modelowania warstw asfaltowych zastosowano hp-adaptacyjną MES. Jej integracja z lokalną homogenizacją numeryczną została szczegółowo przedstawiona. Do numerycznego modelowania warstw niezwiązanych spoiwami wykorzystano metodę elementów dyskretnych (DEM). Przedstawiony został krótki opis tej metody.

W artykule zaproponowano podejście służące wiarygodnemu modelowaniu konstrukcji nawierzchni asfaltowych. Przedstawione zostały wyniki wstępnych testów numerycznych. Potwierdzają one zasadność proponowanego podejścia. Dalsze prace badawcze poświęcone będą uwzględnieniu wpływu temperatury na zachowanie mieszanki mineralno-asfaltowej, aby uczynić model jeszcze lepiej odwzorowującym rzeczywistość.

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