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NUMERICAL SIMULATIONS OF STRAIN LOCALIZATION FOR LARGE STRAIN DAMAGE-PLASTICITY MODEL

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Abstract

This paper deals with the phenomenon of strain localization in nonlinear and nonlocal material models. Particularly, in the description of the material not only nonlinear constitutive relations (damage, plasticity) are included, but also geometrical nonlinearities (large strains) are taken into account. The strain localization in the analysed model has a twofold source: geometrical effects (necking) and softening due to damage of the material. To avoid pathological mesh sensitivity of numerical test results the gradient averaging is applied in the damage model. The material description is implemented within the finite element method and numerical simulations are performed for a uniaxial tensile bar benchmark. Selected results are presented for the standard and regularized continuum.

Key words: strain localization, large strain, damage, plasticity, gradient-enhancement, AceGen package

1. INTRODUCTION

One of the features of materials with microstructure (e.g. concrete or composites) is strain localization which is closely related to the softening of the material. The localization means that from some point the whole deformation concentrates in a narrow zone while a major part of the structure experiences unloading. The strain localization has a twofold source: geometrical effects (e.g. necking of metallic bars) or material instabilities (e.g. microcracking or nonassociated plastic flow). Although the softening and the localized deformations are visible in the macroscopic material response, they have the physical origin in the evolution of the microstructure.

The application of standard continuum models to these problems fails to provide an objective description of the phenomena. For the descending stress-strain relationship (e.g. due to damage of the material) the equilibrium equations lose their ellipticity in the post-peak regime. This leads to an ill-posed boundary value problem that entails a pathological mesh-sensitivity in the numerical solution. The reason for the discretization-dependence in the computational tests is that the localization is simulated in the possibly smallest material volume which depends on the assumed mesh.

The localization phenomenon associated with the softening response can be properly reproduced using enhanced continuum theories which have a non-local character and take into account higher deformation gradients in the constitutive description (Peerlings et al., 1996). The gradient averaging involves an internal length scale which is an additional material parameter coming from the microstructure. The parameter is usually associated with the width of the localization band and is determined for instance by an average grain size. The paper includes the description of the material model which incorpo-

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rates hyperelasticity, plasticity (with or without hardening) based on Auricchio and Taylor (1999) and gradient enhanced damage. The analysis is performed with the assumption of large deformations and isothermal conditions. The implicit gradient model is incorporated, which is reflected in an additional partial differential equation to be solved. The paper presents the results of a computational test of localization in a tensile bar. The simulations are performed using Mathematica–based package AceGen/AceFEM (Korelc, 2009). The application of the automatic code generator AceGen significantly simplifies implementation of the elaborated models and due to automatic computation of derivatives allows one to avoid an explicit derivation of the tangent matrix for the Newton-Raphson procedure.

2. SHORT PRESENTATION OF MATERIAL MODEL

The following simulations are performed for a material model which involves hyperelasticity, plasticity and damage and takes into account large strains. However, damage is not directly coupled with plasticity thus depending on the assumed material parameters the model can also reproduce hyperelasticity-plasticity or hyperelasticity-damage. The material description is developed with the assumption of isotropy and isothermal conditions and is based on a classical multiplicative split of the deformation gradient into its elastic and plastic parts: $\mathbf{F} = \mathbf{F}^e\mathbf{F}^p$ (Simo and Hughes, 1998). The free energy function in the presented model is assumed to be an isotropic function of the elastic left Cauchy-Green tensor $\mathbf{b}^e = \mathbf{F}^e\mathbf{F}^{eT}$, a scalar measure of plastic flow γ and a scalar damage parameter ω:

$$\psi = (1-\omega)\psi^e(\mathbf{b}^e) + \psi^p(\gamma) \tag{1}$$

The constitutive relations of hyperelasticity are expressed through the elastic part of the free energy function which is assumed in the following form (Simo and Hughes, 1998):

$$\psi^e = \frac{\kappa}{2}\left[\frac{1}{2}(J_{be}-1) - \frac{1}{2}\ln(J_{be})\right] + \frac{\mu}{2}\left(\mathrm{tr}(J_{be}^{-1/3}\mathbf{b}^e) - 3\right) \tag{2}$$

where J_{be} is the determinant of the elastic left Cauchy-Green tensor and κ and μ are material parameters. The Kirchhoff stress tensor $\boldsymbol{\tau}$ is related to the elastic left Cauchy-Green tensor $\mathbf{b}^e$ with the formula:

$$\boldsymbol{\tau} = 2\frac{\partial\psi^e}{\partial\mathbf{b}^e}\mathbf{b}^e \tag{3}$$

Damage is understood in the described model as a degradation of the elastic free energy function in the form:

$$\psi^{e,d} = (1-\omega)\psi^e \tag{4}$$

where ω is a scalar damage variable which grows from zero for the intact material to one for a complete material destruction, and which is computed from the damage growth function. In the following numerical simulations the exponential model adopted from Mazars and Pijaudier-Cabot (1989) is applied:

$$\omega(\kappa) = 1 - \frac{\kappa_0}{\kappa}\left(1-\alpha+\alpha\exp(-\beta(\kappa-\kappa_0)\right) \tag{5}$$

where α and β are model parameters, κ is a history variable calculated as $\max(\tilde{\varepsilon}, \kappa_0)$, where $\tilde{\varepsilon}$ = $\det(\mathbf{F})$ - 1 is a deformation measure which governs damage and κ_0 is a damage threshold.

The damage condition takes the form:

$$F_d(\tilde{\varepsilon}, \kappa) = \tilde{\varepsilon} - \kappa \leq 0 \tag{6}$$

For $F_d < 0$ there is no growth of damage.

The plastic part of the presented model is described in the effective stress space which means that it governs the behaviour of the undamaged skeleton of the material. Thus the following formulation takes into account the effective Kirchhoff stress tensor which is computed as: $\hat{\boldsymbol{\tau}} = \boldsymbol{\tau}/(1-\omega)$. The plastic regime is defined through the yield function F_p which is an isotropic function of the effective Kirchhoff stress tensor $\hat{\boldsymbol{\tau}}$ and the plastic multiplier γ.

$$F_p(\hat{\boldsymbol{\tau}}, \gamma) = f(\hat{\boldsymbol{\tau}}) - \sqrt{2/3}\,(\sigma_y - q(\gamma)) \leq 0 \tag{7}$$

The function $f(\hat{\boldsymbol{\tau}})$is assumed to be the Huber-Mises-Hencky equivalent stress $f = \sqrt{2J_2}$, which depends on the second invariant of the deviatoric part of the effective Kirchhoff stress tensor $\mathbf{t}$: $J_2 = \frac{1}{2}\mathbf{t}^2$. The function q represents the isotropic linear hardening as: $q(\gamma) = -h\gamma$, where h is a hardening modulus.

The associative flow rule is assumed in the form:

$$-\frac{1}{2}\mathcal{L}_v\mathbf{b}^e = \dot{\gamma}\,\mathbf{N}\mathbf{b}^e \tag{8}$$

Where $\mathcal{L}_v$ is the Lie derivative of $\mathbf{b}^e$ (Bonet and Wood, 2008) and $\mathbf{N}$ is a normal to the yield hypersurface.

3. GRADIENT-TYPE REGULARIZATION

A variety of approaches can be applied to preserve numerical results from the pathological mesh-sensitivity observed for a standard continuum model reproducing the behaviour of materials exhibiting softening, see e.g. Areias et al. (2003). In this paper a gradient regularization is applied, which is not only a computationally convenient approach but it is also motivated by micro-defect interactions. The introduction of the gradient enhancement into the material description requires the choice of a non-local parameter and the formulation of a corresponding averaging equation. In the literature different variables to be averaged are taken into account, e.g. the stored energy function (Steinmann, 1999) or the plastic strain measure (Żebro et al., 2008). In the described model, the local strain measure governing damage $\tilde{\varepsilon}$ is replaced with its non-local counterpart $\tilde{\varepsilon}$ which is specified by the averaging equation:

$$\tilde{\varepsilon} - l^2\nabla^2\tilde{\varepsilon} = \tilde{\varepsilon} \tag{9}$$

with homogeneous natural boundary conditions. The parameter l appearing in equation (9) is a material-dependent length parameter called the internal length scale. The application of the gradient averaging to the material model including large strains is additionally difficult due to the distinction of the undeformed and deformed configuration. Thus, the averaging equation and the internal scale l can be specified in the initial or current configuration. Based on the results obtained by Steinmann (1999) and Wcisło et al. (2012) which show that the spatial averaging does not fully preserve the numerical results from the dependence on the discretization, the material averaging is chosen for the following simulations.

4. NUMERICAL SIMULATIONS OF STRAIN LOCALIZATION

In this section the numerical examples of strain localization in hyperelastic-plastic-damage model are presented. Firstly, the results for material softening due to damage are presented for standard and regularized continuum. In the second subsection the results for strain localization due to geometrical softening in plasticity are discussed. Finally, the complex model of hyperelasticity-plasticity coupled with gradient damage is considered. All model variants have been implemented in the Mathematica-based packages AceGen and AceFEM (Korelc, 2009). The former is an automatic code generator used for the preparation of finite element code whereas the latter is a FEM engine.

4.1. Hyperelasticity coupled with damage

The simulations of the material model including hyperelasticity coupled with local damage are performed for a tensile bar with imperfection presented in figure 1. The enforced displacement and the boundary conditions preserve the uniaxial stress state. The material parameters applied in the simulations are: $E = 200$ GPa, $\nu = 0.3$, $\kappa_0 = 0.011$, $\alpha = 0.99$, $\beta = 1$. In the central zone the damage threshold is assumed to be $\kappa_0 = 0.01$.

The results of the computational test performed for two FE discretizations with linear interpolation 20x2x2 and 40x4x4 are depicted in the first graph of figure 2. The computations for the finest mesh 80x8x8 fail for the displacement control (snap-back occurs). We can observe in figures 2 and 3 that the results significantly differ for each discretization and that the zone of strain localization covers only the middle rows of elements.

The next test is performed for the same sample but the hyperelastic model is coupled with gradient damage. The internal length parameter is assumed to be $l = 3$ mm. The reactions sum diagram is presented in the second graph of figure 2. It can be noticed that diagrams are close to one another, especially for the medium and the fine mesh. The deformed mesh with the damage variable distribution is presented in figure 4. The simulation confirms that for the gradient model the behaviour of the sample does not depend on the adopted mesh. The width of the damage zone which is related to the internal length parameter is similar for each discretization.

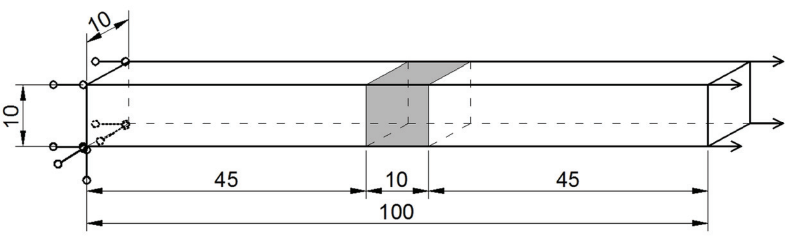

Fig. 1. Geometry and boundary conditions of a bar with imperfection.

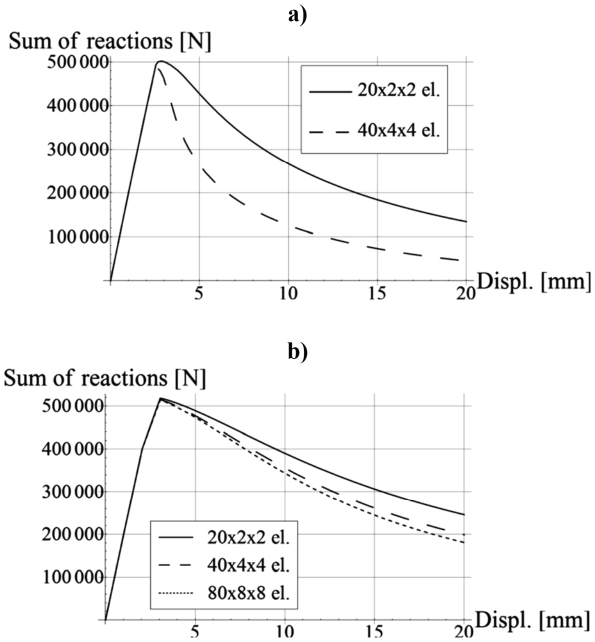

Fig. 2. Sum of reactions vs. displacement for a) hyperelasticity–damage model and b) hyperelasticity–gradient–damage model.

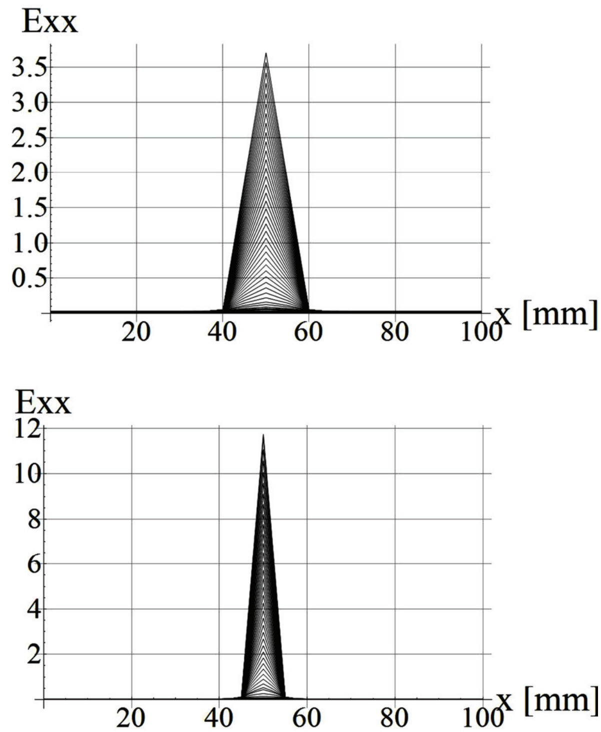

Fig. 3. Evolution of Green strain E_{xx} for meshes 20x2x2 and 40x4x4 (hyperelastic–damage model).

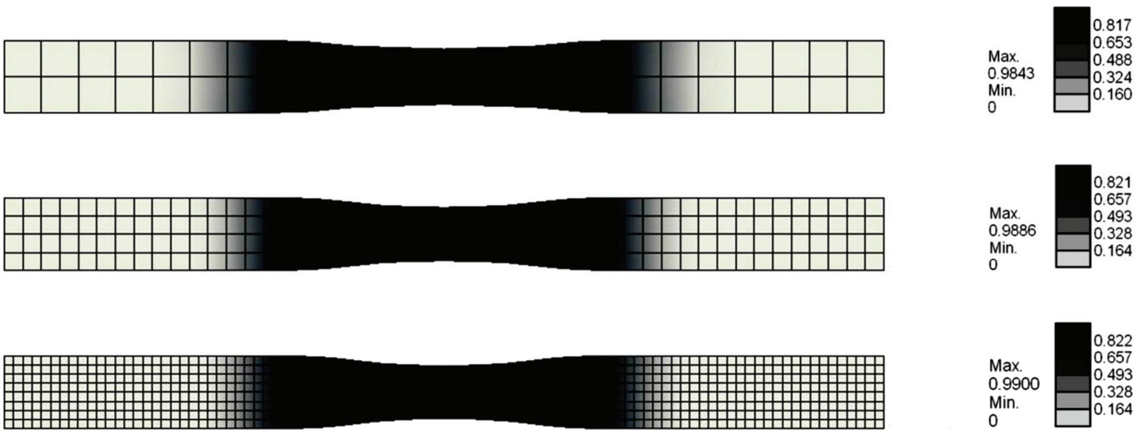

Fig. 4. Deformed mesh and distribution of damage variable ω for three discretizations (hyperelastic–gradient–damage model).

4.2. Hyperelasticity coupled with plasticity

Firstly, the test is performed for an ideal bar with a constant square cross-section along the length. The dimensions and the boundary conditions of the sample are the same as in the previous section and three meshes are taken into account: 20x2x2, 40x4x4 and 80x8x8 elements. The material parameters applied in the test are as follows: E = 200 GPa, ν = 0.3, σ_y = 300 MPa (perfect plasticity). The Huber-Mises-Hencky yield criterion is applied and the *F - bar* approach is incorporated to avoid locking (de Souza Neto et al., 2008).

The graph of the reactions sum vs end displacement is depicted in the first diagram of figure 5. It can be observed that in the plastic regime the diagram is descending although ideal plasticity is assumed. It is caused by taking into account the change of a cross-section during deformation. For all discretizations the loss of stability can be observed: for the coarse mesh the phenomenon is observed the earliest, whereas for the medium and the fine mesh the necking occurs at the same time.

Figures 6 and 7 present the deformed sample with the final accumulated plastic strain distribution and the evolution of the Green strain E_{xx} along the bar length respectively. It can be noticed that the loss of stability manifests itself in multiple necking. For each discretization the number and the arrangement of the strain localization zones is different.

The test for the same sample but with the assumed imperfection in the middle of the bar is also performed. The imperfection is prescribed as the reduction of the yield stress to the value σ_y = 290 MPa. The sum of reactions diagram is presented in the second graph of figure 5. We can observe that the results depend on the adopted finite element mesh and the finer the discretization is, the less stiff the model is. In figure 8 it can be noticed that the strains localize in the middle part of the sample where the imperfection is assumed.

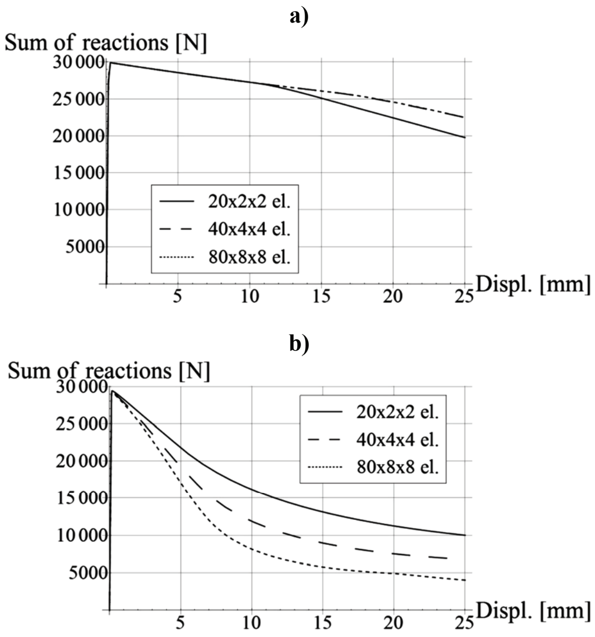

Fig. 5. *Sum of reactions vs. displacement for a) the ideal bar and for b) the bar with imperfection – hyperelastic–plastic model.*

It should be also mentioned, that the same results are obtained for hyperelasticity-plasticity with hardening where the hardening modulus has sufficiently small value, for example 0.5%*E*. It seems that, even in the absence of damage, regularization is necessary for large deformations and ideal plasticity or small hardening.

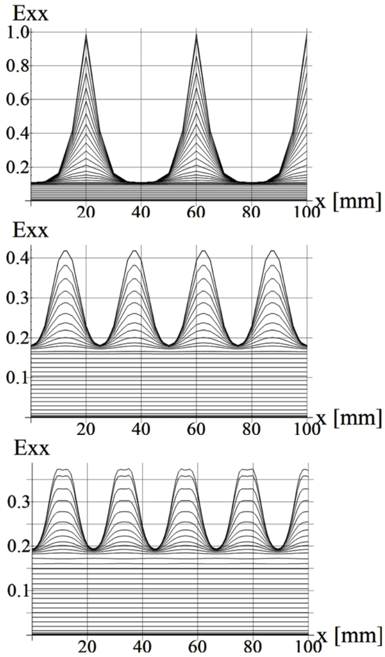

Fig. 7. *Evolution of Green strain Exx along the bar for three meshes and ideal bar (hyperelastic–plastic model).*

4.3. Hyperelasticity-plasticity coupled with gradient damage

The last test is performed for the material model which includes both plasticity and gradient damage. The tested sample is the bar with imperfection with the following material parameters: E = 200 GPa, ν = 0.3, κ_0 = 0.0002, α = 0.99, β = 1, σ_y = 300 MPa, h = 1%E. The imperfection is as in the previous subsection.

Figures 9 and 10 present selected results of the simulation. The reaction diagrams are close for each discretization and present the plasticity regime with hardening and reduction of the reaction forces due to damage. In the analysed test, the material softening is reproduced properly due to gradient regularization and geometrical softening does not occur because of a sufficiently large value of the hardening modulus.

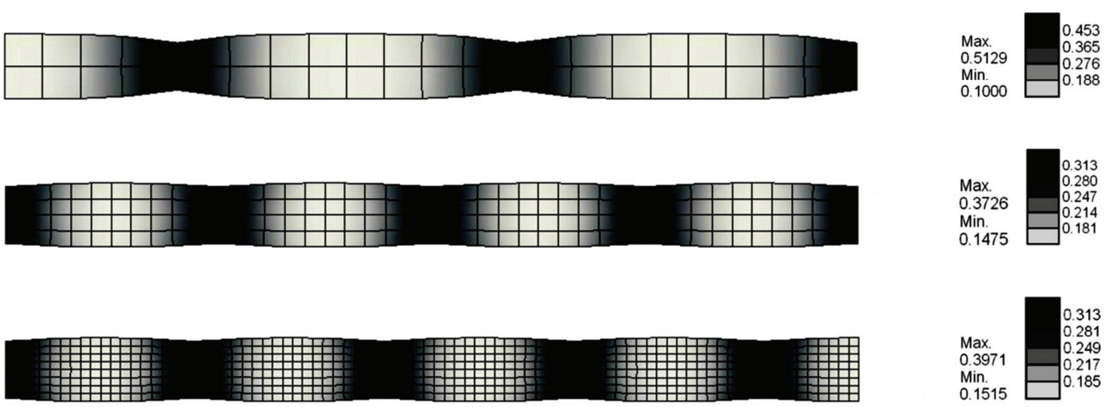

Fig. 6. *Final accumulated plastic strain for three meshes and ideal bar (hyperelastic–plastic model).*

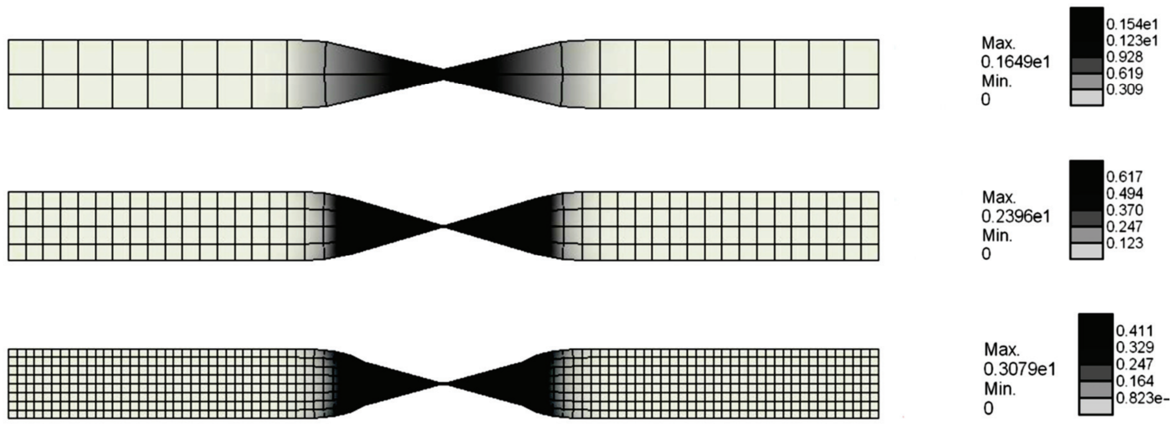

Fig. 8. *Final accumulated plastic strain for imperfect bar and three discretizations (hyperelastic–plastic model).*

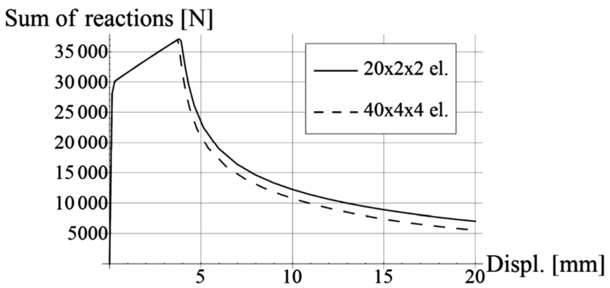

Fig. 9. *Sum of reactions vs. displacement for hyperelasticity–plasticity–gradient–damage model.*

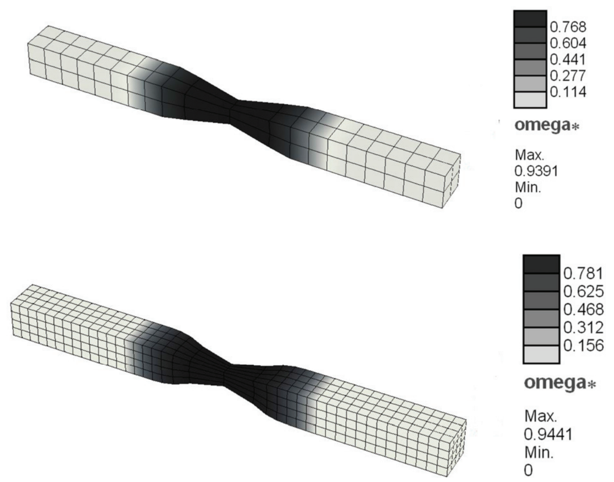

Fig. 10. *Deformed mesh and damage variable* ω *distribution for discretizations 20x2x2 and 40x4x4 (hyperelasticity–plasticity–gradient–damage model).*

5. CONCLUSIONS

In the paper the problem of strain localization for a material model including geometrical and material nonlinearities with the applied gradient regularization has been outlined. The considered model is briefly described and selected numerical results are presented. The simulations are performed for different variants of the model and exhibit geometrical or material softening. The hyperelastic-damage model exhibits material softening which can cause mesh-sensitivity observed in the presented simulation results. The gradient averaging procedure, incorporating an internal length parameter, allows one to properly reproduce the material behaviour.

The numerical tests reveal that for a model incorporating ideal plasticity in large strain regime the strain localization might occur. For a sample with imperfection one zone of large strains can be predicted in contrast to the ideal sample where multiple necks are formed. To prevent the numerical results from a pathological mesh-dependence, the application of the regularization of the plastic part of the model should be considered in the future work. Moreover, the work is planned to be extended towards thermo-mechanical coupling.

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REFERENCES

Areias, P., Cesar de Sa, J., and Conceicao, C., 2003, A gradient model for finite strain elastoplasticity coupled with damage, *Finite Elements in Analysis and Design*, 39, 1191-1235.

Auricchio, F., Taylor, R. L., 1999, A return-map algorithm for general associative isotropic elasto-plastic materials in large deformation regimes, *Int. J. Plasticity*, 15, 1359-1378.

Bonet, J., Wood, R. D., 2008, *Nonlinear continuum mechanics for finite element analysis*, Cambridge University Press, Cambridge.

de Souza Neto, E., Peric, D., Owen, D., 2008, *Computational methods for plasticity. Theory and applications*, John Wiley & Sons, Ltd, Chichester, UK.

Korelc, J., 2009, Automation of primal and sensitivity analysis of transient coupled problems, *Computational Mechanics*, 44, 631-649.

Mazars, J., Pijaudier-Cabot, G., 1989, Continuum damage theory - application to concrete, *ASCE J. Eng. Mech.*, 115, 345-365.

Peerlings, R., de Borst, R., Brekelmans, W., de Vree, J., 1996, Gradient-enhanced damage for quasi-brittle materials, *Int. J. Numer. Meth. Engng*, 39, 3391-3403.

Simo, J. C., Hughes, T. J. R., 1998, *Computational Inelasticity. Interdisciplinary Applied Mathematics Vol. 7*, Springer-Verlag, New York.

Steinmann, P., 1999, Formulation and computation of geometrically non-linear gradient damage, *Int. J. Numer. Meth. Engng*, 46, 757-779.

Wcisło, B., Pamin, J., Kowalczyk-Gajewska, K., 2012, Gradient-enhanced model for large deformations of damaging elastic-plastic materials, *Arch. Mech*. (to be published).

Żebro, T., Kowalczyk-Gajewska, K., Pamin, J., 2008, A geometrically nonlinear model of scalar damage coupled to plasticity, *Technical Transactions*, 20/3-Ś, 251-262.

NUMERYCZNE SYMULACJE LOKALIZACJI ODKSZTAŁCEŃ DLA MODELU USZKODZENIA SPRZĘŻONEGO Z PLASTYCZNOŚCIĄ W DUŻYCH ODKSZTAŁCENIACH

Streszczenie

Artykuł dotyczy zjawiska lokalizacji odkształceń w nieliniowych i nielokalnych modelach materiałowych. W szczególności, przedstawiony opis materiału zawiera nie tylko nieliniowe związki konstytutywne (uszkodzenie, plastyczność), ale również nieliniowości geometryczne (duże odkształcenia). Lokalizacja odkształceń w analizowanym modelu ma dwojakie źródło: efekty geometryczne (szyjkowanie) oraz osłabienie spowodowane uszkodzeniem materiału.

Zastosowanie standardowych modeli continuum nie prowadzi do poprawnej symulacji zachowania materiałów z osłabieniem. Spowodowane jest to utratą eliptyczności równań równowagi, gdy zależność między naprężeniami a odkształceniami wchodzi na ścieżkę opadającą. W takim przypadku odkształcenia lokalizują się w najmniejszej możliwej objętości materiału, która w symulacji numerycznej określona jest przez rozmiar elementu skończonego. Aby uniknąć patologicznej zależności wyników testów numerycznych od dyskretyzacji należy zastosować odpowiednią regularyzację. W niniejszej pracy zastosowano uśrednianie gradientowe, w którym istotną rolę odgrywa wewnętrzna skala długości. Jest to dodatkowy parametr materiału związany z jego mikrostrukturą, który może określać szerokość strefy lokalizacji odkształceń.

W artykule przedstawiono zwięźle opis analizowanego modelu sprężysto-plastycznego sprzężonego z uszkodzeniem przy dużych odkształceniach oraz zastosowanej regularyzacji gradientowej. Model ten został oprogramowany w pakiecie AceGen w środowisku Mathematica oraz przetestowany przy użyciu pakietu AceFEM. W pracy zaprezentowane są wybrane wyniki symulacji rozciągania pręta dla różnych wariantów przyjętego opisu materiału, w których można zaobserwować lokalizację odkształceń zarówno związaną z osłabieniem materiału jak i osłabieniem geometrycznym.

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