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LOCAL NUMERICAL HOMOGENIZATION IN MODELING OF HETEROGENEOUS VISCO-ELASTIC MATERIALS

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Abstract

The main objective of this paper is to present the prospects of application of local numerical homogenization to visco-elastic problems. Local numerical homogenization is one of the computational homogenization methods, proposed by Jhurani in 2009 for linear problems. Its main advantage is that it can be used in the case of modeling of heterogeneous materials with neither distinct scales separation nor periodic microstructure. The main idea of the approach is to replace of a group of many small finite elements by one macro element. The coarse element stiffness matrix is computed on the basis of the fine element matrices. In such a way one obtains a coarse mesh approximation of the time consuming fine mesh solution.

In this paper we use the Burgers model to describe inelastic deformations, however any other constitutive equations may be applied. In the 1D case the Burgers model is interpreted as a combination of a spring and a dashpot and it is mainly used for bituminous materials (e.g. binders or asphalt mix). Because of rheological effects a transient analysis is necessary.

Integration of local numerical homogenization with Burgers model should improve modeling of heterogeneous visco-elastic materials. The approach we propose can reduce the computational cost of the analysis without deterioration of the modeling reliability. We present numerical results of 1D and 2D analysis for selected problems that provide comparison between the 'brute force' FEM approach and local numerical homogenization in application to modeling of heterogeneous visco-elastic materials in order to validate the technique.

Key words: local numerical homogenization, visco-elasticity, Burgers model

1. INTRODUCTION

Most of new materials are composites of different kinds. Before their implementation they are thoroughly tested. Numerical tests can significantly reduce the cost of design process by eliminating some laboratory or 'in situ' experiments. However, numerical modeling of heterogeneous materials is a challenging task, especially in the case of inelastic deformations and non-periodic material microstructure. 'Brute force' FEM analysis, which accounts for all details of the material heterogeneity, is either highly time consuming or even impossible. Therefore, various approaches to evaluation of effective material properties and composite response are proposed (e.g. Geers et al., 2003; Mang et al., 2008). One of them is the computational homogenization. It is used to bridge 'neighboring' analyses scales by the concept of representative volume element (RVE). This approach was developed for example by Geers and his collaborators (Geers et al., 2003). However, we make use of the local numerical homogenization not based on RVE concept (Jhurani, 2009; Jhurani & Demkowicz, 2009; Klimczak & Cecot, 2011), which is presented briefly below.

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2. LOCAL NUMERICAL HOMOGENIZATION

Local numerical homogenization is one of computational homogenization methods. It was proposed by Chetan Jhurani (Jhurani, 2009) for linear problems. Mainly linear elasticity was discussed. Unlike other computational homogenization methods, local numerical homogenization is not based on the concept of RVE. The main advantage of this method is that no separation of scales condition has to be fulfilled. It means that the ratio of microscale characteristic dimension and macroscale characteristic dimension does not have to be much smaller than the unity. Moreover, periodicity of the material is not required. Therefore this method is suitable to model asphalt pavement structures, which is the subject of our interest.

Local numerical homogenization is thoroughly presented in Jhurani's dissertation (Jhurani, 2009). We would like to give an overview of this method and its main steps in context of linear problems.

Algorithm of the method consists of the following steps:

– assume 'trial' effective material properties of the analyzed heterogeneous domain,
– solve the auxiliary coarse mesh problem (it is advised to use adaptive FEM but it is not obligatory),
– refine the coarse mesh within its every element in order to match all the heterogeneities (fine and coarse meshes are naturally compatible then),
– find the coarse mesh element effective matrices knowing the fine mesh element matrices,
– assemble coarse mesh element effective matrices,
– solve the coarse mesh problem.

The core of the algorithm is evaluation of effective coarse mesh element matrices. Let us focus on a single coarse mesh element of the analyzed domain. Then we refine the mesh within this coarse mesh element to capture all details of the heterogeneity. For a stiffness matrix $K \in R^{NxN}$, and load vector $f \in R^N$, the fine mesh local FEM equation is:

$$Ku = f \tag{1}$$

Its solution is equal to :

$$u = K^{\dagger} f + u_0 \tag{2}$$

where $K^{\dagger}$ denotes the Moore – Penrose pseudo-inverse of K and u_0 is an arbitrary vector in the null space of K.

Analyzing the same problem at macroscale level we use effective stiffness matrices of $\hat{K} \in R^{MxM}$ (M≤N) and coarse scale load vector defined in terms of f as $\hat{f} = A^T f$ ($A \in R^{NxM}$ - a chosen interpolation operator for a respective element). The coarse – scale solution is expressed in the following way:

$$\hat{u} = \hat{K}^{\dagger} \hat{f} + \hat{u}_0 \tag{3}$$

where $\hat{K}^{\dagger}$ denotes the Moore – Penrose pseudoinverse of $\hat{K}$, and $\hat{u}_0$ is an arbitrary vector in the null space of $\hat{K}$. The difference between (2) and (3) is equal to:

$$u - A\hat{u} = (K^{\dagger} - A\hat{K}^{\dagger} A^T) f + (u_0 - A\hat{u}_0) \tag{4}$$

Thus, we can express , up to a constant, the error $e \in R^N$ as:

$$e = (K^{\dagger} - A\hat{K}^{\dagger} A^T) f \tag{5}$$

Finally, minimization of the above expression (enhanced with the regularization term) with respect to $\hat{K}^{\dagger}$ (Jhurani, 2009) leads to the effective coarse mesh element stiffness matrix $\hat{K}$. This routine needs to be repeated for every coarse mesh element. Then the coarse mesh problem can be solved in the standard way.

1. BURGERS MODEL

Visco-elastic Burgers model is commonly used for modeling of bituminous materials. Its 1D scheme is presented in figure 1.

It is a material model, which efficiently simulates all of the most important response characteristics of bituminous materials, i.e. elastic, viscous, and visco-elastic. Additionally, it is may be easily implemented numerically. The total strain increment ($\Delta\varepsilon$) in Burgers model is the sum of the elastic

($\Delta\varepsilon_E$), visco-elastic ($\Delta\varepsilon_{VE}$) and viscous strain increments ($\Delta\varepsilon_V$) then:

$$\Delta\varepsilon = \Delta\varepsilon_E + \Delta\varepsilon_{VE} + \Delta\varepsilon_V \qquad (6)$$

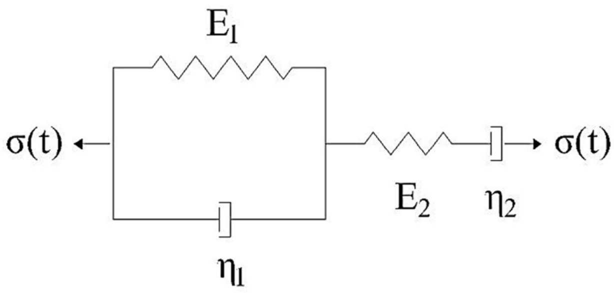

Fig. 1. *Burgers model.*

All of the above increments are presented in details by Collop et al. (2003) for both 1D and 3D case. Algorithm for time integration of the Burgers model is as follows:

– evaluate elastic solution as a trial one for a time step t_i ,
– calculate inelastic strain increments,
– update load vector considering the 'impact' of inelastic strain increments,
– solve the problem and calculate total strain increment,
– if the difference between updated total strain increment and a trial $\Delta\varepsilon$ is negligible and the difference between updated solution and a trial one is also negligible, go to the next time step; otherwise – go to the first iteration step.

2. INTEGRATION OF LOCAL NUMERICAL HOMOGENIZATION WITH BURGERS MODEL

Modeling of heterogeneous visco-elastic materials also requires time-consuming transient analysis. In this chapter we present the prospects of local numerical homogenization in application of visco-elastic Burgers model. Analysis becomes much more complex as we have to 'homogenize' at every time step.

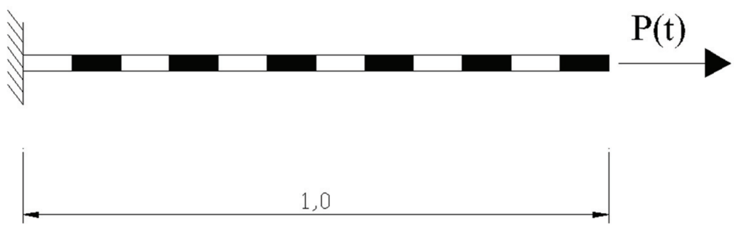

Fig. 2. *Analyzed heterogeneous visco-elastic 1D domain.*

Algorithm of the proposed approach for known load history and known constituents characteristics is as follows (for each time step):

– solve the elastic problem using local numerical homogenization according to the routine presented in chapter 2,
– consider each coarse mesh element to be an independent problem: refine the mesh within this element, assume boundary conditions on the basis of elastic solution and solve this local problem at time t_i,
– update the coarse mesh load vector considering inelastic contribution,
– assemble coarse mesh element matrices and updated load vectors,
– solve the coarse mesh problem.

Whole routine requires then solving several local visco-elastic problems instead of the global problem.

3. NUMERICAL RESULTS

In this chapter we present preliminary results of 1D and 2D numerical tests for visco-elastic materials. In figure 2 analyzed 1D domain is presented. All material data (for 'white' material) are the same as for the test performed by Woldekidan (2011). 'Black' material is characterized by two times weaker parameters. Cross-sectional area is equal to 50 cm^2. Analysis period is equal to 60 s. Load P is equal to 1.5 kN for t ≤ 15 s, then it is removed.

Results for an arbitrary time step t_i are presented in figure 3. The whole domain was discretized by 10 fine mesh elements and 5 coarse mesh elements. Thus, two fine mesh elements were homogenized into one coarse mesh element. Distribution of inclusions is periodic for the sake of simplicity.

2D analysis was carried out for the domain presented below in figure 4. It is a 2m by 4m square analyzed in plane strain state. Its bottom edge is fixed, left and right hand side edges can displace only in the vertical direction. Uniformly distributed tensile load (1 kN/m^2) is applied to the upper edge. Material data were assumed in the same manner as for the 1D test. The Poisson ratio both for the inclusion and the matrix is equal to 0.3.

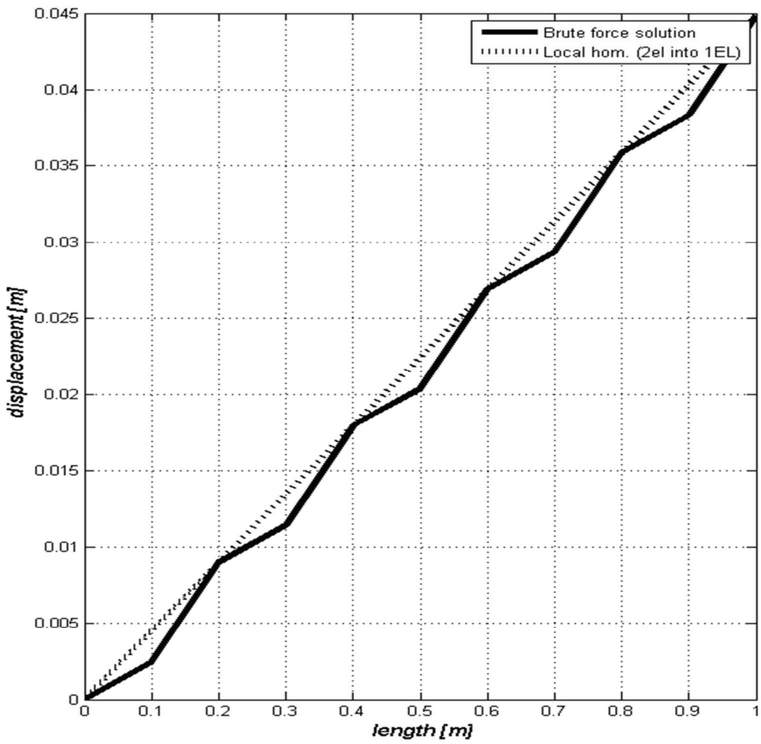

Fig. 3. 1D example. Displacements at arbitrary time t_i.

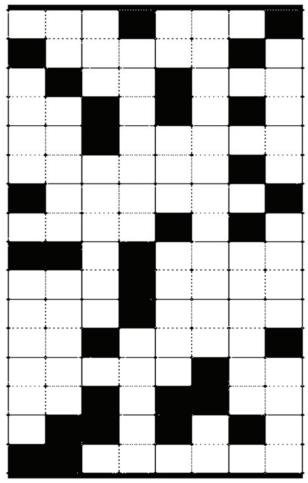

Fig. 4. Analyzed 2D domain with randomly distributed inclusions.

Vertical displacements of the upper edge are presented in figure 5.

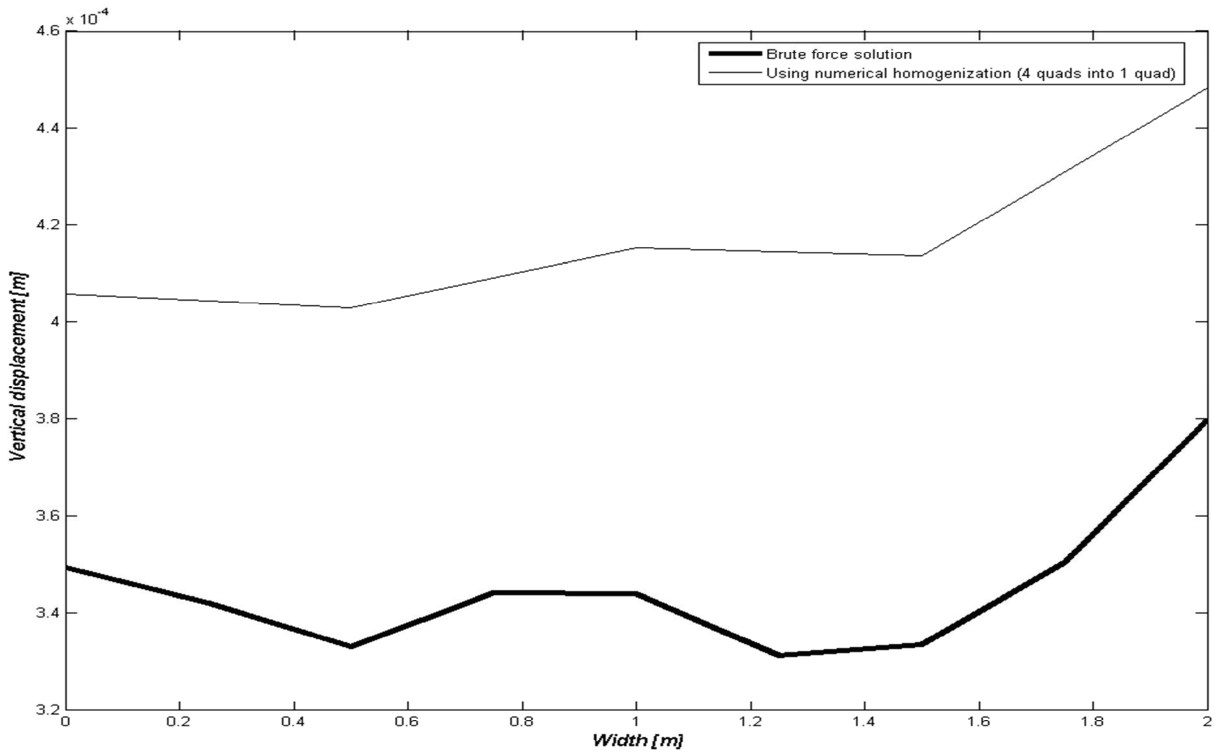

Fig. 5. 2D example. Vertical displacements along the upper edge.

6. CONCLUSIONS

Summing up we can conclude that for the tests presented in the paper integration of local numerical homogenization with visco-elastic material models:

- significantly reduced the computational cost of numerical analysis,
- did not introduce significant additional error to the solution.

These results encourage us to perform further tests and obtain an effective algorithm for analyses of heterogeneous visco-elastic materials.

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LOKALNA HOMOGENIZACJA NUMERYCZNA W MODELOWANIU NIEJEDNORODNYCH MATERIAŁÓW LEPKOSPRĘŻYSTYCH

Streszczenie

Głównym celem niniejszego artykułu jest prezentacja możliwości wykorzystania lokalnej homogenizacji numerycznej do zadań lepkosprężystych. Lokalna homogenizacja numeryczna jest jedną z metod homogenizacji komputerowej. Została zaproponowana przez Ch. Jhurani w roku 2009 do zagadnień liniowych. Jej główną zaletą jest to, iż może być wykorzystana do modelowania materiałów niejednorodnych, które nie wykazują wyraźnej rozdzielności skal ani nie charakteryzują się periodycznością mikrostruktury. Główną cechą tego podejścia jest wykonanie homogenizacji po dyskretyzacji analizowanego obszaru. Kluczowym krokiem algorytmu jest zastąpienie grupy elementów siatki gęstej jednym elementem siatki rzadkiej. Ostatecznie wystarczy rozwiązać zadanie w obszarze zdyskretyzowanym siatką rzadką, zamiast siatką gęstą.

W niniejszym artykule wykorzystujemy model Burgersa do opisania deformacji lepkosprężystych. Możliwe jest jednak zastosowanie innego równania konstytutywnego opisującego zachowanie materiału w czasie. W przypadku jednowymiarowym model Burgersa jest interpretowany jako kombinacja sprężyn i tłumików. Wykorzystywany jest głównie do modelowania zachowania materiałów bitumicznych (np. lepiszcza asfaltowe lub mieszanki mineralno-asfaltowe). Ze względu na reologię zagadnienia niezbędna jest wykonanie analizy niestacjonarnej. Powoduje to znaczne wydłużenie czasu obliczeń ze względu na konieczność rozwiązania zadania w każdej chwili czasu oraz iteracyjny charakter algorytmu.

Integracja lokalnej homogenizacji numerycznej z modelem Burgersa może poprawić sposób modelowania niejednorodnych materiałów lepkosprężystych. Proponowane przez nas podejście może ograniczyć czas obliczeń bez pogorszenia wiarygodności modelowania. Prezentujemy wyniki zadań 1D oraz 2D dla wybranych zagadnień. Porównane zostały one z wynikami podejścia "brute force", tj. wynikami obliczeń wykonanych za pomocą MES przy pełnym uwzględnieniu mikrostruktury materiału. Rezultaty porównań powyższych metod pokazują, że proponowane przez nas podejście może być z powodzeniem wykorzystane do modelowania niejednorodnych materiałów lepkosprężystych, gdyż nie wprowadza znacznego dodatkowego błędu do rozwiązania obniżając jednocześnie koszt wykonywanych obliczeń.

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