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LEMAITRE'S DAMAGE NONLOCAL MODEL PERFORMANCE UNDER CRACK INITIATION AND PROPAGATION

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Abstract

Modelling of ductile fracture is a rather challenging task due to the complexity of the physical phenomena involved. In ductile metals, material progressive degradation, which is strongly associated with large plastic straining, may be captured using the Lemaitre damage model. Nevertheless, to model the propagation of macro-cracks, the use of a discontinuous approach is in general imperative. Therefore, in this work, the Lemaitre Damage model is combined with the XFEM for a complete description of ductile failure. In addition, as in general continuous softening models suffer from pathological mesh dependence, a non-local formulation is implemented and its effects on crack initiation and propagation are evaluated.

Key words: ductile fracture, XFEM, nonlocal model

1. INTRODUCTION

The behaviour of ductile metallic materials may be described by continuum models, combining damage and plasticity. Most of these models fall within a thermodynamic framework defined in Lemaitre and Chaboche (1990), known as Continuum Damage Mechanics.

In terms of numerical simulation, continuum models may be implemented in a regular finite element formulation providing a successfully simulation of the hardening part of a material's traction curve. These models are also often regularized to deal with the proceeding softening part (Andrade et al. 2009). Nevertheless, they are incapable of modelling material behaviour up to rupture realistically and they are not very robust as they may present spurious damage growth patterns and require very refined meshes to follow a crack path.

An accurate description of fracture should include the initiation and propagation of discontinuity surfaces within a structure. In recent years, the simulation of crack growth became highly predictive in this framework due to the development of numerical strategies such as the Extended Finite Element method by Belytschko and Black (1999). XFEM is well adapted to propagation of existing cracks and has been extensively applied to Linear Elastic Fracture Mechanics, however, the discontinuous models are incapable of predicting the initiation of new cracks.

Therefore, in order to combine the best of both words, in this work, the Lemaitre damage model is enhanced with discontinuous XFEM elements to obtain a realistic failure model. The transition from the continuous model to the discontinuous model is performed by a numerical damage criterion. The damage model is implemented in a non-local inte-

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gral formulation and the effects of the regularization length are inspected in the pre and post fracture stages through some numerical examples.

2. DAMAGE MODEL

Ductile failure is characterized by the presence of moderate to large plastic straining, associated to material degradation, leading to macro-crack propagation. The Lemaitre model assumes that material degradation may be described by an internal variable, the damage variable D, in a thermodynamic framework (Lemaitre and Chaboche 1990).

For an isothermal process, the *Helmholtz free energy*, which characterizes the thermodynamic state of a given point at time t, may be considered to have the following form:

$$\psi = \psi(\boldsymbol{C}, \boldsymbol{C}^p, R, D) \tag{1}$$

where $\boldsymbol{C}$ is the Green-Lagrange strain tensor, $\boldsymbol{C}^p$ is the plastic part of the Green-Lagrange strain tensor and R is the isotropic hardening variable.

Assuming a multiplicative plasticity framework, the set of internal variables may be reduced to

$$\psi = \psi(\boldsymbol{b}^e, R, D) \tag{2}$$

where $\boldsymbol{b}^e$ is the left Cauchy-Green strain tensor, given by:

$$\boldsymbol{b}^e = \boldsymbol{F}(\boldsymbol{C}^p)^{-1}\boldsymbol{F}^T \tag{3}$$

and $\boldsymbol{F}$ is the deformation gradient.

From the free energy it is possible to derive the associated variables as follows:

$$\boldsymbol{\tau} = \rho 2 b^e \frac{\partial \psi}{\partial \boldsymbol{b}^e} \tag{4}$$

$$X = \rho \frac{\partial \psi}{\partial R} \tag{5}$$

$$Y = -\rho \frac{\partial \psi}{\partial D} \tag{6}$$

where $\boldsymbol{\tau}$ is the second Piolla-Kirchhoff stress tensor, X are the thermodynamic forces conjugated with hardening, Y is the damage energy release rate and ρ is the mass density. To complete the description of a ductile damage process and derive the evolution equations, a complementary energy dissipation potential, φ, has to be defined. Here the effects of plastic dissipation and damage are considered in a decoupled way, defining it as:

$$\varphi = \varphi^p + \varphi^d \tag{7}$$

For the plastic part, φ^p, assuming the strain equivalence principle (Lemaitre and Chaboche 1990), the von Mises yield function is adopted:

$$\varphi^p = \frac{1}{1-D}\sqrt{\frac{3}{2}\boldsymbol{\tau} : \boldsymbol{\tau}} - \tau_y(R) \tag{8}$$

where τ_y is the yield strength. For the damage dissipation potential, φ^d, the following function based in the work of Lemaitre is employed:

$$\varphi^d = \frac{r}{(s+1)(1-D)}\left(\frac{-Y}{r}\right)^{s+1} \tag{9}$$

where r and s are scalar material parameters associated with damage evolution. Finally, making use of the normality rule it is possible to determine the evolution equations:

$$\mathcal{L}_v \boldsymbol{b}^e = -2\dot{\gamma}\frac{\partial \varphi}{\partial \boldsymbol{\tau}}\boldsymbol{b}^e \tag{10}$$

$$\dot{D} = \dot{\gamma}\frac{1}{1-D}\left(\frac{-Y}{r}\right)^s \tag{11}$$

$$\dot{R} = \dot{\gamma} \tag{12}$$

where $\mathcal{L}_v$ represents the spatial velocity Lie derivative and $\dot{\gamma}$ represents the plastic multiplier. The model is concluded with the Kuhn-Tucker conditions

$$\dot{\gamma} \geq 0;\ \boldsymbol{F}^p \leq 0;\ \dot{\gamma}\boldsymbol{F}^p = 0 \tag{13}$$

which must hold globally in the domain, V.

2.1. Non-local formulation

The Lemaitre damage model, as continuum softening models in general, suffers from pathological mesh dependence, when implemented in a FEM formulation. This problem may be attenuated by resorting to non-local formulations where one or various internal variables at a point are evaluated taking into account the influence of surrounding material points.

Following an integral approach, the local damage variable, D, is replaced by its non-local counterpart, $\overline{D}$, as follows:

$$\overline{D}(\boldsymbol{x}) = \int_V \beta(\boldsymbol{x}, \boldsymbol{\xi}) D(\boldsymbol{\xi}) dV(\boldsymbol{\xi}) \tag{14}$$

The damage variable is therefore averaged in a finite volume V, with radius l_r ,which is often termed as regularisation or characteristic length. In equation (14) $\beta(\boldsymbol{x}, \boldsymbol{\xi})$ is an averaging operator defined as:

$$\beta(\boldsymbol{x}, \boldsymbol{\xi}) = \frac{\alpha(x,\xi)}{\Omega(\xi)} \tag{15}$$

in which $\alpha(\boldsymbol{x},\boldsymbol{\xi})$ is the following weighting function

$$\alpha(\boldsymbol{x},\boldsymbol{\xi}) = \langle 1 - \frac{\|x-\xi\|^2}{l_r^2}\rangle^2 \quad (16)$$

and Ω is usually designated representative volume and is defined as:

$$\Omega(\boldsymbol{x}) = \int_V \alpha(\boldsymbol{x},\boldsymbol{\xi}) dV(\boldsymbol{x}) \quad (17)$$

In these equations, $\bar{D}(\boldsymbol{x})$ represents the damage measurement at a material generic point denoted by $\boldsymbol{x}$, which has been averaged over the defined volume V, containing the set of points $\boldsymbol{\xi}$.

The averaging operator β is assumed to be independent of the deformation history and therefore the damage evolution is readily determined as

$$\dot{\bar{D}}(\boldsymbol{x}) = \int_V \beta(\boldsymbol{x},\boldsymbol{\xi})\dot{D}(\boldsymbol{\xi})dV(\boldsymbol{\xi}) \quad (18)$$

As damage is chosen to be the only variable enhanced by non-locality, the local constitutive model can be directly extended to the non-local case.

3. MODELLING DISCONTINUITIES WITH THE XFEM

Discontinuities in the problem fields may be introduced independently from the finite element mesh using the XFEM in which the nodes of the elements crossed by a discontinuity are enriched.

The Heaviside function, H, is particularly suitable to represent the decohesion between the two surfaces of a crack. Defining a coordinate system associated to the crack, as represented in figure 1, this function is defined in terms of the coordinate normal to the crack, $\hat{\eta}$:

$$H(\hat{\eta}) = \begin{cases} 1: \hat{\eta} \geq 0 \\ -1: \hat{\eta} < 0 \end{cases} \quad (19)$$

Enriching finite elements only with the Heaviside function always causes the extension of the crack to the edge of the element. Therefore, to represent cracks truly mesh independent it is desirable to introduce a crack tip function. In this work, the following function denoted by T was employed, as follows:

$$T(\hat{\xi}) = \begin{cases} 1: \hat{\xi} \leq 0 \\ 0 \text{ otherwise} \end{cases} \quad (20)$$

The full displacement field approximation then reads:

$$u(\boldsymbol{x}) = \sum_{i=1}^{n} N_i \boldsymbol{u}_i + \sum_{j=1}^{n_{split}} N_j[H(\boldsymbol{x}) - H(\boldsymbol{x}_j)]\boldsymbol{a}_j + \sum_{k=1}^{n_{tip}} N_k[T(\boldsymbol{x}) - T(\boldsymbol{x}_k)][H(\boldsymbol{x}) - H(\boldsymbol{x}_k)]\boldsymbol{b}_k \quad (21)$$

where N_i represents the standard element shape functions, $\boldsymbol{u}_i$ represents the nodal displacements, $\boldsymbol{a}_j$and $\boldsymbol{b}_k$ represent, respectively, the extra degrees of freedom associated with the elements totally crossed by a crack (*split elements*) and the extra degrees of freedom associated with the elements containing a crack tip (*tip elements*). The enrichment functions are shifted relatively to the nodes to prevent them from spanning to neighbour elements.

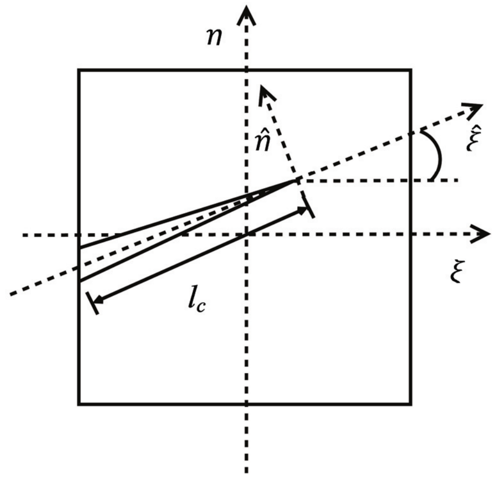

Fig. 1*. Crack tip coordinates.*

4. TRANSITION FROM DAMAGE TO FRACTURE

In this work, fracture is triggered by the evolution of damage: when a critical damage value, D_c, is reached, a crack is inserted through the XFEM.

In a problem discretised through the FEM, the values of the damage variable are stored at each Gauss point, from which the damage distribution pattern follows directly. This information is used to determine the crack characteristics, namely its initiation point, direction and length.

4.1. Crack initiation point

The crack initiation point lies in the region where damage firstly reached the critical value. The elements in which damage is higher than the critical value are selected and grouped into clouds. The point of the cloud with the highest damage is the crack initiation point.

To calculate the highest damage point, a cubic *B-spline* function (Cottrell et al. 2009) is constructed for interpolation points. Unlike Lagrange polynomials, this type of function does not introduce new maxima in the distribution and allows the incorporation of information of various finite elements in a single function. A more detailed description of this technique can be found in the reference (Seabra et al.).

4.2. Crack direction and crack length

After determining the crack initiation point, the maximum damage growth direction is calculated. In terms of numerical implementation, the crack initiation algorithm is a particular case of the crack propagation algorithm in which the starting point of the crack is unknown.

To derive the new segment direction a set of points is selected along a circumference, centred at the last crack tip, which contains all the elements where the critical damage value has been reached in at least one Gauss point. As it is unlike that the crack will snap back, the points behind the previous crack segment are excluded and, subsequently, the point with the highest damage value should be determined. The crack direction is then obtained by joining the last crack tip to the point with the highest damage. Moreover, for a good accuracy two more circumferences, with a slightly higher and slightly smaller radius, are tested and then the three directions are averaged. Finally, the furthest point with D_c is searched along the crack growth direction, using a mid-point algorithm.

5. NUMERICAL EXAMPLES

The presented numerical model for ductile fracture is now tested for the double notched specimen which is loaded in such a way that a shear-like failure mode would occur. The specimen is illustrated in figure 2 and the material and geometrical properties used are listed in table 1.

Besides the material parameters defined in table 1 the proposed model requires a value for the regularization length, l_r, and for the critical damage, D_c, that triggers the inclusion of a crack in the model. Therefore, different values of l_r (1.6 mm and 2.5 mm) are employed to evaluate its influence in crack initiation and propagation, as well as different critical damage values (0.2 and 0.5).

In figure 3 the crack paths obtained for two different meshes (21 and 31 elements per side), considering a critical damage value of 0.5, a regularization length of 1.6 mm and applying a displacement of1.5 mm are represented. It can be observed that the paths are nearly the same for both cases. The non-local integral model avoids spurious localization of

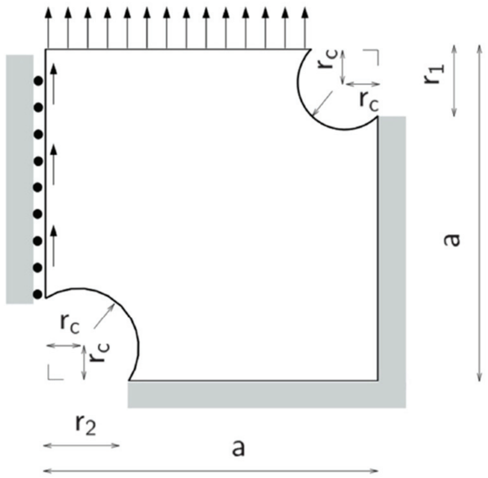

__Fig. 2.__ Double notched specimen geometry and boundary conditions.

__Table 1.__ Material and geometrical properties.

Property	Value
Elastic Modulus	E = 206.9 GPa
Poisson's ratio	ν = 0.29
Damage exponent	s = 1.0
Damage denominator	r = 1.25 MPa
Hardening function	$\tau_y(R) = 450 + 129.24R + 265(1e^{-16.93R})$ MPa
a	a = 10 mm
$\boldsymbol{r_c}$	r_c = 1.0 mm
$\boldsymbol{r_1}$	r_1 = 2.0 mm
$\boldsymbol{r_2}$	r_2 = 2.5 mm

damage, resulting in similar damaged areas for different mesh refinements. The subsequent insertion of the crack through the XFEM respects these damage contours independently from the mesh as well, indicating that this methodology is adequate to predict failure in specimens of arbitrary shape.

To inspect the effect of the regularization length, the *reaction force-applied displacement* curves for $D_c = 0.2$ for two different values of the regularization length and for same three different meshes (21, 31 and 41 elements per side) are shown in figure 5. It can be observed, that increasing the regularization length, the mesh dependence is strongly attenuated.

Nevertheless in the *reaction force-applied displacement* curves illustrated in figure 4, for three different meshes (21, 31 and 41 elements per side), it can be observed that some mesh dependence is still present.

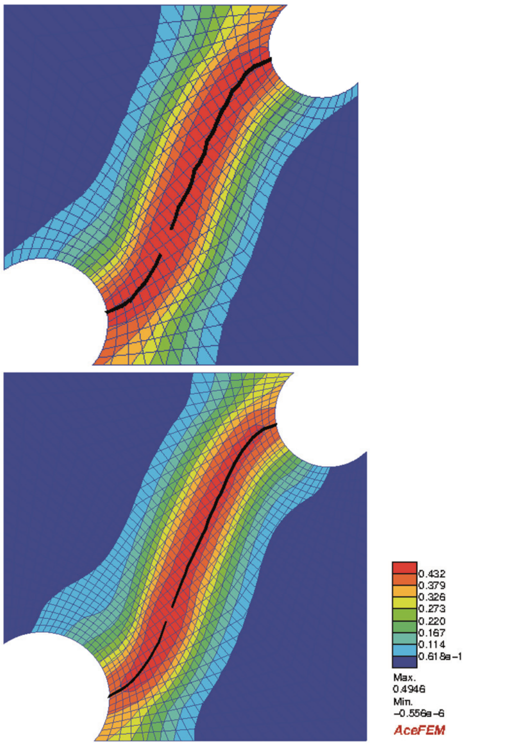

Fig. 3. Damage contours and final crack for two different meshes (21 and 31 elements per side).

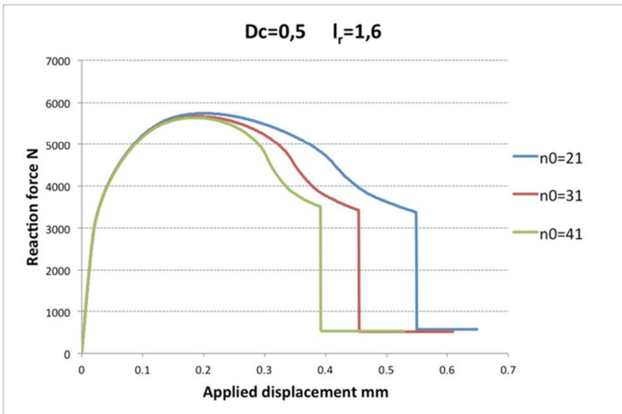

Fig. 4. Reaction force as a function of the applied displacement of the top nodes of the double notched specimen, considering $D_c = 0.5$ *and* $l_r = 1.6$ *mm.*

6. CONCLUSIONS

A numerical model for ductile fracture was presented. The model successfully captures all the stages of material behavior: elastic, plastic-hardening and plastic softening. As the XFEM is a highly flexible method, it allows the representation of arbitrary crack paths within a finite element mesh. In the particular case of this model, crack initiation and crack propagation are handled in a unified way, and closely related to damage evolution. Moreover, the location of the cracks does not need to be known in advance allowing the simulation of ductile failure in components of arbitrary shape.

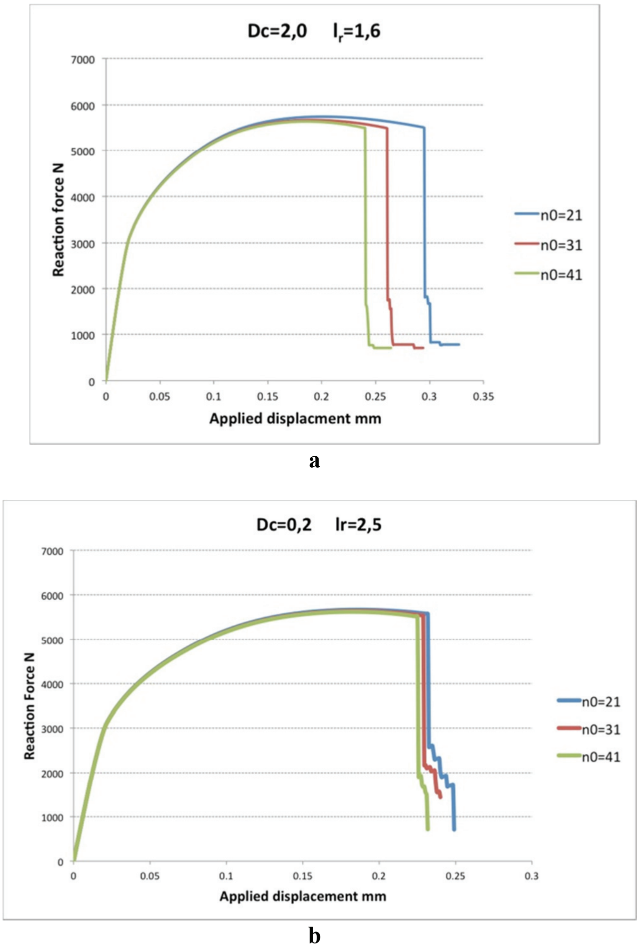

Fig. 5. Reaction force as a function of the applied displacement of the top nodes of the double notched specimen, considering a) $D_c = 0.2$ *and* $l_r = 1.6$ *mm b)* $D_c = 0.2$ *and* $l_r = 2.5$ *mm.*

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ZASTOSOWANIE NIELOKALNEGO MODELU LEMAITRE'A W MODELOWANIU POWSTAWANIA I ROZPRZESTRZENIANIA SIĘ PĘKNIĘĆ

Streszczenie

Modelowanie pęknięć plastycznych jest zagadnieniem trudnym ze względu na złożoność występujących zjawisk fizycznych. W metalach plastycznych postępujące zniszczenie materiału, związane ściśle z dużymi odkształceniami plastycznymi, może być opisane modelem opracowanym przez Lemaitre'a. Niemniej jednak model propagacji makro uszkodzeń wymaga uwzględnienia nieciągłości tych zjawisk. Dlatego w niniejszej pracy model uszkodzeń Lemaitre'a został połączony z modelem XMES, co pozwoliło na kompletny opis zjawiska pęknięcia plastycznego. Ponadto, ponieważ modele ciągłe stosowane do opisu mięknięcia materiału są uzależnione od siatki elementów skończonych, zastosowano sformułowanie nielokalne, co umożliwiło modelowanie zarówno inicjacji jak i rozprzestrzeniania się pęknięć.

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