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THE SENSITIVITY ANALYSIS OF DEEP DRAWING PROCESS USING FLOW APPROACH

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Abstract

The objective of the paper is the sensitivity analysis of deep drawing simulations using flow approach with respect to any design parameter. The sensitivity analysis is being implemented into the MFP code developed by IPPT PAN under project Numpress.

First, the finite element formulation of flow approach in drawing simulations is presented. The mathematical formulation includes the consistent tangent viscosity matrix. As it is shown in the final sensitivity formulation, the tangent viscosity matrix is necessary in sensitivity calculations. Unfortunately the tangent matrix is asymmetric. The influence of the asymmetric contributions to viscosity matrix in sensitivity analysis is considered.

The semi-analytical Direct Differential Method is used. This means that design derivatives of some terms are estimated by finite difference method and next substituted into analytical sensitivity formula.

The mathematical formulae and the algorithm to build consistent tangent matrix and to perform sensitivity analysis are presented.

Key words: deep drawing, flow approach, sensitivity analysis

1. INTRODUCTION

The flow approach to numerical modelling of large plastic deformation processes (Oñate & Zienkiewicz, 1983; Agelet de Saracibar, 1990; Oñate & Agelet de Saracibar, 1990) is a numerically efficient alternative to traditional displacement-based finite element formulations, both implicit quasi static and explicit dynamic. In this approach, rigid-viscoplastic model is assumed and elastic strains are neglected. This leads to a constitutive equation in which stress is a function of strain rate and the form of the equation is analogous to nonlinear elasticity, with strain replaced by strain rate. The finite element formulation is thus analogous to that of nonlinear elastic problems, however, the nodal velocity, rather than displacement vector appears as an unknown here.

The formulation is suitable, in particular, to model deep drawing of metal sheets (Sosnowski, 1995) or other plastic metal forming processes, like rolling or extrusion (Antunez, 1998, 2001). The results are obviously only approximate and cannot compete against elasto-plastic formulations regarding accuracy (for instance, springback of a sheet is very difficult to be estimated this way). However, numerical efficiency of the method encourages to use it in e.g. parameter studies or optimization procedures, where multiple runs of the analysis for different parameter values are necessary while high accuracy is not critical.

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The most efficient optimization algorithms are known to be gradient methods. To effectively use them, information of design sensitivity has to be provided. Design sensitivity analysis has been widely discussed regarding traditional displacement FE formulations (Kleiber et al., 1997), including also large elasto-plastic deformations (Kowalczyk, 2006b,a). Although the issues of sensitivity analysis in flow approach formulations of sheet metal forming have also been addressed in the literature (Antunez et al., 1997; Kleiber & Sosnowski, 1995; Sosnowski, 2003), a complete formulation of the problem, including the geometric nonlinearities and full form of the consistent tangent matrix, has not yet been developed.

The present paper is an attempt to fill this gap in the state of the art. The results, when implemented in the finite element code dedicated to sheet metal forming simulations, may allow to fully integrate it with a gradient-based optimization system and thus accelerate the process of optimization of tool geometry and stamping control parameters.

2. FORMULATION OF RIGID-VISCOPLASTICITY PROBLEM

2.1. Constitutive formulation

The fundamentals of the flow approach in metal forming problems with the rigid-viscoplastic material model (Perzyna, 1966) are presented in (Agelet de Saracibar et al., 1992). The virtual work equation (equilibrium equation in the weak form) to be solved reads:

$$\int_{\Omega} \sigma_{ij}\delta\dot{\varepsilon}_{ij}\Omega = \int_{\Omega} f_i\delta v_i\Omega + \int_{\partial\Omega_t} t_i\delta v_i(\partial\Omega), \qquad i,j=1,\ldots,3 \tag{1}$$

where v_i denotes velocity field, f_i is the distributed volumetric load, t_i is the traction on the boundary an integrals are taken over the current, deformed body volume element $d\Omega$ or its surface element $\mathrm{d}(\partial\Omega)$, respectively.

Strain rates are presented as

$$\dot{\varepsilon}_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) \tag{2}$$

Stresses are calculated from the constitutive equation

$$\sigma_{ij} = s_{ij} + p\delta_{ij}, \qquad s_{ij} = 2\mu^*\dot{\varepsilon}_{ij} \tag{3}$$

where s_{ij} is the Cauchy stress deviator, p denotes the mean stress and δ_{ij} is the Kronecker delta. The viscosity coefficient μ^* being a function of strain rate, is defined in the flow problem as

$$\mu^*(\dot{\varepsilon}_{ij}) = \frac{\bar{\sigma}}{3\dot{\bar{\varepsilon}}} = \frac{1}{3\dot{\bar{\varepsilon}}}\left[\sigma_y + \left(\frac{\dot{\bar{\varepsilon}}}{\gamma}\right)^{\frac{1}{n}}\right] \tag{4}$$

Here, n $\bar{\sigma} = \sqrt{\frac{3}{2}s_{ij}s_{ij}}$ is the equivalent stress, $\dot{\bar{\varepsilon}} = \sqrt{\frac{2}{3}\dot{\varepsilon}_{ij}\dot{\varepsilon}_{ij}}$ is the effective inelastic strain rate, σ_y is the current static uniaxial tensile yield stress of the material and γ, n are physical parameters of the viscoplastic constitutive model (Perzyna, 1966). For plastic materials with strain hardening, the yield limit σ_y is a function of the effective inelastic strain $\bar{\varepsilon}$, and $\bar{\varepsilon}$ has to be computed as the time integral of $\dot{\bar{\varepsilon}}$.

The formal analogy between the plastic flow equations (3) and a formulation of incompressible elasticity allows to solve the pure plastic flow problem with a numerical code developed for nonlinear elasticity. Rates of large plastic strains are treated in the same way as elastic strains. The incompressibility condition must only be satisfied.

In sheet metal forming, the shell theory is used as the simplification of 3D problems. Plane stress assumptions are used in the shell theory so that p in equation (3) can be determined and incompressibility is enforced by adjusting shell thickness during consecutive steps of the solution.

2.2. Finite element formulation

Introducing the standard FE interpolation of displacements

$$q_i(x_k) = N_{i\alpha}(x_k)q_\alpha = \mathbf{N}_i^T(\mathbf{x})\mathbf{q} \tag{5}$$

where $\mathbf{q}$ is a vector of nodal displacements (and, possibly, also rotations, depending on a particular kinematic formulation), one can transform equation (1) to the vector-matrix form,

$$\bar{\mathbf{K}}(\dot{\mathbf{q}})\dot{\mathbf{q}} = \mathbf{F} \tag{6}$$

in which $\bar{\mathbf{K}}$ and $\mathbf{F}$ denote the secant viscosity matrix and the external force vector, respectively, and dependence of $\bar{\mathbf{K}}$ on $\dot{\mathbf{q}}$ is due to the local dependence of μ^* on $\dot{\varepsilon}_{ij}$.

The external force vector $\mathbf{F}$ is expressed as

$$\mathbf{F} = \mathbf{f} + \mathbf{f}^c - \bar{\mathbf{K}}^{pr}\dot{\mathbf{q}}^{pr} \tag{7}$$

where $\mathbf{f}$ is the vector of prescribed external nodal loads (e.g. pressure acting on sheet), $\mathbf{f}^c$ is the vector of contact forces and the expression $\bar{\mathbf{K}}^{pr}\dot{\mathbf{q}}^{pr}$ corresponds to reaction forces due to prescribed nodal velocities $\dot{\mathbf{q}}^{pr}$ (kinematic boundary conditions) possibly imposed on some nodes. Generally, $\mathbf{F}$ may also depend on $\dot{\mathbf{q}}$.

The terms in equation (6) are computed by summing up their contributions related to particular finite elements or mesh nodes:

$$\bar{\mathbf{K}} = \mathbf{J}^T\left(\sum_{e=1}^{N_E}\mathbf{J}^{eT}\bar{\mathbf{K}}^e\mathbf{J}^e\right)\mathbf{J} \tag{8}$$

$$\bar{\mathbf{K}}^{pr} = \mathbf{J}^T\left(\sum_{e=1}^{N_E}\mathbf{J}^{eT}\bar{\mathbf{K}}^e\mathbf{J}^e\right)\mathbf{J}^{pr} \tag{9}$$

$$\mathbf{f} = \mathbf{J}^T\sum_{e=1}^{N_E}\mathbf{J}^{eT}\mathbf{f}^e \tag{10}$$

$$\mathbf{f}^c = \mathbf{J}^T\sum_{k\in S^c}\mathbf{J}^{kT}\mathbf{f}^{ck} \tag{11}$$

where N_E is the number of elements in the mesh, S^c is the current set of nodes with active contact constraints, and $\mathbf{J}$, $\mathbf{J}^{pr}$, $\mathbf{J}^e$, $\mathbf{J}^k$ are boolean assignment matrices used to switch between global and local, or free and prescribed nodal degrees of freedom. The shell element secant viscosity matrix appearing in equations (8)-(9) reads

$$\bar{\mathbf{K}}^e = \mathbf{T}^T\left(\int_{S^e}\mathbf{B}^T\bar{\mathbf{D}}^S\mathbf{B}\right)\mathbf{T} \tag{12}$$

where $\bar{\mathbf{D}}^S$ is the constitutive viscosity matrix appearing in equations (3)-(4) written in a vector-matrix formulation for a shell section, $\mathbf{B}$ is a geometric matrix of shape function gradients and $\mathbf{T}$ is a rotation operator between local and global coordinate sets.

2.3. Iterative computation scheme

The solution of the nonlinear equation system (6) is obtained with the use of the Newton iteration scheme. The solution procedure consists in calculation of the secant viscosity matrix $\bar{\mathbf{K}}$ and the force vector $\mathbf{F}$ for consecutive approximations of $\dot{\mathbf{q}}$ and determination of its correctors $\partial\dot{\mathbf{q}}$, according to

$$\frac{\mathrm{d}}{\mathrm{d}\dot{\mathbf{q}}}\left(\bar{\mathbf{K}}\dot{\mathbf{q}} - \mathbf{F}\right)\delta\dot{\mathbf{q}} = \mathbf{F} - \bar{\mathbf{K}}\dot{\mathbf{q}}, \qquad \dot{\mathbf{q}} := \dot{\mathbf{q}} + \delta\dot{\mathbf{q}} \tag{13}$$

This can be written in the compact form as

$$\mathbf{K}\delta\dot{\mathbf{q}} = \mathbf{r} \tag{14}$$

where $\mathbf{K}$ and $\mathbf{r}$ are expressed as

$$\mathbf{K} = \bar{\mathbf{K}} + \frac{\mathrm{d}\bar{\mathbf{K}}}{\mathrm{d}\dot{\mathbf{q}}}\dot{\mathbf{q}} + \frac{\mathrm{d}\bar{\mathbf{K}}^{pr}}{\mathrm{d}\dot{\mathbf{q}}}\dot{\mathbf{q}}^{pr} - \frac{\mathrm{d}\mathbf{f}}{\mathrm{d}\dot{\mathbf{q}}} - \frac{\mathrm{d}\mathbf{f}^c}{\mathrm{d}\dot{\mathbf{q}}} \tag{15}$$

$$\mathbf{r} = \mathbf{f} + \mathbf{f}^c - \bar{\mathbf{K}}\dot{\mathbf{q}} - \bar{\mathbf{K}}^{pr}\dot{\mathbf{q}}^{pr} \tag{16}$$

and are called, respectively, the algorithmic tangent viscosity matrix and the vector of unbalanced (residual) nodal forces corresponding to current approximation of $\dot{\mathbf{q}}$.

The iterative Newton scheme is repeated until the convergence criterion is fulfilled. The nodal displacement vector is obtained by the following integration scheme

$$\mathbf{q}^{n+1} = \mathbf{q}^n + \left[(1-\vartheta)\dot{\mathbf{q}}^n + \vartheta\dot{\mathbf{q}}^{n+1}\right]\Delta t \tag{17}$$

where n denotes number of the last time increment considered (for which the solution is known) and ϑ is the implicit integration parameter and takes values $\vartheta \in (0,1]$. Note that the above integration formula implies that, for any variable dependent on both $\mathbf{q}$ and $\dot{\mathbf{q}}$,

$$\frac{\mathrm{d}(\cdot)}{\mathrm{d}\dot{\mathbf{q}}} = \frac{\partial(\cdot)}{\partial\dot{\mathbf{q}}} + \vartheta\Delta t\frac{\partial(\cdot)}{\partial\mathbf{q}}. \tag{18}$$

2.4. Algorithmic tangent matrix

The tangent viscositymatrix $\mathbf{K}$ is divided into three components: $\mathbf{K}^{sh}$ corresponding tomechanical properties of the sheet, $\mathbf{K}^f$ dependent on external forces and $\mathbf{K}^c$ comes from the contact conditions. Let us write them down as

$$K^{sh}_{\alpha\beta} = \bar{K}_{\alpha\beta} + \frac{\partial\bar{K}_{\alpha\gamma}}{\partial\dot{q}_\beta}\dot{q}_\gamma + \frac{\partial\bar{K}^{pr}_{\alpha\mu}}{\partial\dot{q}_\beta}\dot{q}^{pr}_\mu, \tag{19}$$

$$K^f_{\alpha\beta} = -\frac{\partial f_\alpha}{\partial\dot{q}_\beta}, \tag{20}$$

$$K^c_{\alpha\beta} = -\frac{\partial f^c_\alpha}{\partial\dot{q}_\beta}, \tag{21}$$

$\alpha,\beta,\gamma = 1,\ldots, N$, $\mu = 1,\ldots,N^{pr}$ (free and prescribed degrees of freedom, respectively).

The first of them can be derived as follows

$$\frac{\partial \bar{K}_{\alpha\gamma}}{\partial \dot{q}_\beta}\dot{q}_\gamma = \sum_{e=1}^{N_E} J_{\bar{\alpha}\alpha} J^e_{a\bar{\alpha}} \frac{\partial \bar{K}^e_{ac}}{\partial \dot{q}_\beta} J^e_{c\bar{\gamma}} J_{\bar{\gamma}\gamma}\dot{q}_\gamma ,$$

$$\frac{\partial \bar{K}^{pr}_{\alpha\mu}}{\partial \dot{q}_\beta}\dot{q}^{pr}_{\alpha\mu} = \sum_{e=1}^{N_E} J_{\bar{\alpha}\alpha} J^e_{a\bar{\alpha}} \frac{\partial \bar{K}^e_{ac}}{\partial \dot{q}_\beta} J^e_{c\bar{\gamma}} J^{pr}_{\bar{\gamma}\mu}\dot{q}^{pr}_{\mu} ,$$

where $\bar{\alpha},\bar{\beta} = 1,\ldots, N + N^{pr}$, $a,b = 1,\ldots,N^e$ (number of nodal dof's in an element). Denoting by $\dot{\mathbf{q}}^e$ the element velocity vector, we can write

$$\frac{\partial \bar{K}_{\alpha\gamma}}{\partial \dot{q}_\beta}\dot{q}_\gamma + \frac{\partial \bar{K}^{pr}_{\alpha\mu}}{\partial \dot{q}_\beta}\dot{q}^{pr}_{\mu} = \sum_{e=1}^{N_E} J_{\bar{\alpha}\alpha} J^e_{a\bar{\alpha}} \frac{\partial \bar{K}^e_{ac}}{\partial \dot{q}_\beta}\dot{q}^e_c .$$

Moreover

$$\frac{\partial \bar{K}^e_{ac}}{\partial \dot{q}_\beta} = \frac{\partial \bar{K}^e_{ac}}{\partial \dot{q}^e_b} J^e_{b\bar{\beta}} J_{\bar{\beta}\beta}.$$

Finally, the sheet-dependent component of viscosity matrix can be formulated as

$$K^{sh}_{\alpha\beta} = \bar{K}_{\alpha\beta} + \frac{\partial \bar{K}_{\alpha\beta}}{\partial \dot{q}_\beta}\dot{q}_\gamma + \frac{\partial \bar{K}^{pr}_{\alpha\beta}}{\partial \dot{q}_\beta}\dot{q}^{pr}_\mu = \sum_{e=1}^{N_E} J_{\bar{\alpha}\alpha} J^e_{a\bar{\alpha}} K^e_{ab} J^e_{b\bar{\beta}} J_{\bar{\beta}\beta}. \tag{22}$$

$$K^e_{ab} = \bar{K}^e_{ab} + \frac{\partial \bar{K}^e_{ac}}{\partial \dot{q}^e_b}\dot{q}^e_c, \tag{23}$$

Where $\mathbf{K}^e$ is algorithmic tangent viscosity matrix of element e.

The next component depended on external loads can be written as

$$K^f_{\alpha\beta} = -\sum_{e=1}^{N_E} J_{\bar{\alpha}\alpha} J^e_{a\bar{\alpha}} \frac{\partial f^e_a}{\partial \dot{q}_\beta} = \sum_{e=1}^{N_E} J_{\bar{\alpha}\alpha} J^e_{a\bar{\alpha}} \bar{K}^{f\,e}_{ab} J^e_{b\bar{\beta}} J_{\bar{\beta}\beta} \tag{24}$$

$$\bar{K}^{f\,e}_{ab} = -\frac{\partial f^e_a}{\partial \dot{q}^e_b} \tag{25}$$

where $\mathbf{K}^{f\,e}$ is algorithmic tangent matrix of element e corresponding to external loads.

The last component of algorithmic viscosity matrix corresponds to contact conditions and is formulated as follows

$$K^c_{\alpha\beta} = -\frac{\partial f^c_\alpha}{\partial \dot{q}_\beta} = -J_{\bar{\alpha}\alpha}\frac{\partial f^{gc}_\alpha}{\partial \dot{q}^g_\beta} J_{\bar{\beta}\beta} = \sum_{k\in\{S^c\}} J_{\bar{\alpha}\alpha} J^k_{i\bar{\alpha}} D^{ck}_{ij} J^k_{j\bar{\beta}} J_{\bar{\beta}\beta} \tag{26}$$

where elements of $\mathbf{D}^{ck}$ matrix can be calculated for each contact node as

$$D^{ck}_{ij} = -\frac{\partial f^{ck}_i}{\partial \dot{u}^k_j} - \frac{\partial f^{ck}_i}{\partial X^k_l}\frac{\partial X^k_l}{\partial \dot{u}^k_j} \tag{27}$$

Since the location $\mathbf{X}^k$ of node k depends on its displacement $\mathbf{q}$ and since the contact reaction is the sum of its normal and tangent components, $\mathbf{p}$ and $\mathbf{t}$ we can write, cf. equation (18),

$$D^c_{ij} = -\frac{\mathrm{d}(p_i + t_i)}{\mathrm{d}\dot{u}_j} = -\frac{\partial t_i}{\partial \dot{u}_i} - \vartheta\Delta t\left(\frac{\partial p_i}{\partial u_i} + \frac{\partial t_i}{\partial u_i}\right). \tag{28}$$

It can be seen that some of terms in $\mathbf{K}$ are asymmetric. Starting from the end, this happens to the term $\frac{\partial t_i}{\partial u_i}$ in equation (28). However, it can be seen that this term is multiplied by the time step length Δt. If we assume that time step Δt is small enough, the asymmetric term is not significant and could be neglected.

Other asymmetric terms appear in $\mathbf{K}^{sh}$. Note that $\mathbf{K}^e$ in equation (12) depends on both nodal velocities and nodal locations. Thus, according to equation (18), its derivative appearing in formula (23) can be divided into two terms:

$$K^e_{ab} = \int_{S^e} \bar{B}_{Ka}\, D\sec_{KL}\, \bar{B}_{Lb}\,\mathrm{d}S + K^{*e}_{ab}\,\vartheta\Delta t, \qquad \bar{B}_{Ka} = B_{Kp}T_{pa} \tag{29}$$

where

$$D^S_{KL} = \bar{D}^S_{KL} + \frac{\partial \bar{D}^S_{KM}}{\partial \dot{E}_L}\bar{B}_{Ld}\dot{q}^e_d \tag{30}$$

$$\begin{aligned} K^{*e}_{ab} = \frac{\partial \bar{K}^e_{ac}}{\partial x^e_B}\frac{\partial x^e_B}{\partial q^e_b}\dot{q}^e_c = \Bigg[& \int_{S^e} \frac{\partial \bar{B}_{Ka}}{\partial x^e_B}\bar{D}^S_{KM}\,\bar{B}_{Mc}\,\mathrm{d}S \\ & + \left(\int_{S^e} \bar{B}_{Ka}\frac{\partial \bar{D}^S_{KM}}{\partial \dot{E}_L}\frac{\partial \bar{B}_{Md}}{\partial x^e_B}\bar{B}_{Mc}\mathrm{d}S\right)\dot{q}^e_d \\ & + \int_{S^e} \bar{B}_{Ka}\,\bar{D}\sec_{KM}\frac{\partial \bar{B}_{Mc}}{\partial x^e_B}\,\mathrm{d}S\Bigg] J^{xe}_{Bb}\,\dot{q}^e_c \end{aligned} \tag{31}$$

and $\mathbf{J}^{xe}_{Bb}$ is a boolean matrix. As it can be seen from formula (31), the term K^{*e}_{ab} is asymmetric. Fortunately, it is again multiplied by time step length Δt and thus it can be neglected if the time step is small enough.

We do not discuss possible asymmetry of the matrix K^f_{ab} – this depends on a particular type of loads applied.

In consequence we do not obtain tangent viscosity matrix, but only its symmetric part which dominates the neglected terms. The known solvers for symmetric matrices are significantly faster than solvers for asymmetric matrices. The computational effort consumed for solving additional terms and then asymmetric systems of equations will be significantly larger that effort used for one or two more iterations in Newton scheme using fast solver for symmetric matrices.

3. SENSITIVITY ANALYSIS

In this paper, the sensitivity analysis formulation for metal forming process using flow approach is developed in general form. We do not make any assumptions what type the design parameter is of. It could be e.g. one of the constitutive parameters (i.e. viscosity or flow function parameters), boundary conditions (i.e. prescribed nodal velocity, external load) or geometric parameters (i.e. sheet thickness, shape parameters of tools). The widely described Direct Differentiation and semi-analytical methods (Kleiber et al., 1997; Kowalczyk, 2006a) are used. The sensitivities of nodal velocities with respect to design parameter is obtained as a solution of a conjugate system of equations.

After the last Newton iteration in each time step actual nodal velocities vector $\dot{\mathbf{q}}$ fulfills the equilibrium equation (6), which can be rewritten here

$$\bar{\mathbf{K}}\dot{\mathbf{q}} = \mathbf{f} + \mathbf{f}^{c} - \bar{\mathbf{K}}^{pr}\dot{\mathbf{q}}^{pr} \tag{32}$$

After differentiation of system of equations (32) with respect to a design parameter h one can obtain

$$\frac{\mathrm{d}\bar{\mathbf{K}}}{\mathrm{d}h}\dot{\mathbf{q}} + \bar{\mathbf{K}}\frac{\mathrm{d}\dot{\mathbf{q}}}{\mathrm{d}h} = \frac{\mathrm{d}\mathbf{f}}{\mathrm{d}h} + \frac{\mathrm{d}\mathbf{f}^{c}}{\mathrm{d}h} - \frac{\mathrm{d}\bar{\mathbf{K}}^{pr}}{\mathrm{d}h}\dot{\mathbf{q}} - \bar{\mathbf{K}}^{pr}\frac{\mathrm{d}\dot{\mathbf{q}}^{pr}}{\mathrm{d}h}. \tag{33}$$

Note that all terms in equation (32) depend on h either explicitly or through other variables that are design-dependent. Among those variables, some are known at the beginning of the time step (so their design derivatives are also known) and some – and here we are talking about $\dot{\mathbf{q}}$ – are not, and the derivative $\frac{\mathrm{d}\dot{\mathbf{q}}}{\mathrm{d}h}$ is unknown. For simplicity of notation, let us write down, for any variable $a(\dot{\mathbf{q}},\ldots,h)$:

$$\frac{\mathrm{d}a}{\mathrm{d}h} = \frac{\mathrm{d}a}{\mathrm{d}\dot{\mathbf{q}}}\frac{\mathrm{d}\dot{\mathbf{q}}}{\mathrm{d}h} + \frac{\tilde{\mathrm{d}}a}{\mathrm{d}h} \tag{34}$$

where the last term will be called 'explicit design derivative' of a and contains all terms known at the beginning of the time step computations,

$$\frac{\tilde{\mathrm{d}}a}{\mathrm{d}h} = \left.\frac{\mathrm{d}a}{\mathrm{d}h}\right|_{\dot{\mathbf{q}}=const}.$$

With this notation, equation (33) can be rewritten after transformations as

$$\mathbf{K}\frac{\mathrm{d}\dot{\mathbf{q}}}{\mathrm{d}h} = \mathbf{R} \tag{35}$$

Where $\mathbf{K}$ is the already introduced algorithmic tangent matrix (formula (15)) while the right hand side is

$$\mathbf{R} = \frac{\tilde{\mathrm{d}}\mathbf{f}}{\mathrm{d}h} + \frac{\tilde{\mathrm{d}}\mathbf{f}^{c}}{\mathrm{d}h} - \frac{\tilde{\mathrm{d}}\bar{\mathbf{K}}}{\mathrm{d}h}\dot{\mathbf{q}} - \frac{\tilde{\mathrm{d}}\bar{\mathbf{K}}^{pr}}{\mathrm{d}h}\dot{\mathbf{q}} - \bar{\mathbf{K}}^{pr}\frac{\mathrm{d}\dot{\mathbf{q}}^{pr}}{\mathrm{d}h}, \tag{36}$$

or, sh!ortly

$$\mathbf{R} = \frac{\tilde{\mathrm{d}}\mathbf{r}}{\mathrm{d}h}$$

where $\mathbf{r}$ is defined by equation (16).

Tangent viscosity matrix $\mathbf{K}$ is already implemented in MFP code. The so formulated sensitivity problem reduces to calculation of the right hand side $\mathbf{R}$ given in equation (36).

3.1. The right-hand side vector

Let us derive subsequent terms in equation (36). According to definitions (8)–(9) we can write

$$\frac{\tilde{\mathrm{d}}\bar{\mathbf{K}}}{\mathrm{d}h}\dot{\mathbf{q}} + \frac{\tilde{\mathrm{d}}\bar{\mathbf{K}}^{pr}}{\mathrm{d}h}\dot{\mathbf{q}} = \mathbf{J}^{T}\sum_{e=1}^{N_E}\mathbf{J}^{eT}\frac{\tilde{\mathrm{d}}\bar{\mathbf{K}}^{e}}{\mathrm{d}h}\dot{\mathbf{q}}^{e} \tag{37}$$

The element secant viscosity matrix depends on the constitutive viscosity matrix $\mathbf{D}^{S}$, matrix of shape function differentials $\mathbf{B}$, coordinates transformation matrix $\mathbf{T}$ and design parameter h.

$$\bar{\mathbf{K}}^{e} = \bar{\mathbf{K}}^{e}\left(\mathbf{D}^{S},\mathbf{T},\mathbf{B},h\right) \tag{38}$$

Explicit differentiation with respect to design parameter h and constant velocities $\dot{\mathbf{q}}$ leads to

$$\frac{\tilde{\mathrm{d}}\bar{\mathbf{K}}^{e}}{\mathrm{d}h} = \frac{\partial\bar{\mathbf{K}}^{e}}{\partial h} + \frac{\partial\bar{\mathbf{K}}^{e}}{\partial\mathbf{D}^{S}}\frac{\tilde{\mathrm{d}}\mathbf{D}^{S}}{\mathrm{d}h} + \frac{\partial\bar{\mathbf{K}}^{e}}{\partial\mathbf{T}}\frac{\tilde{\mathrm{d}}\mathbf{T}}{\mathrm{d}h} + \frac{\partial\bar{\mathbf{K}}^{e}}{\partial\mathbf{B}}\frac{\tilde{\mathrm{d}}\mathbf{B}}{\mathrm{d}h} \tag{39}$$

The constitutive matrix $\bar{\mathbf{D}}$ depends on thickness b and parameters of the constitutive equation (yield curve parameters, viscosity parameters etc.). Generally we can express the explicit sensitivity as

$$\frac{\tilde{\mathrm{d}}\mathbf{D}^{S}}{\mathrm{d}h} = \frac{\partial\bar{\mathbf{D}}^{e}}{\partial b}\frac{\tilde{\mathrm{d}}b}{\mathrm{d}h} + \frac{\partial\mathbf{D}^{S}}{\partial\mu^{*}}\frac{\tilde{\mathrm{d}}\mu^{*}}{\mathrm{d}h}. \tag{40}$$

The matrices **T** and **B** are dependent on actual sheet configuration. Current positions **X** of element nodes can be expressed as, cf. equation (17),

$$\mathbf{X}=\mathbf{X}^{n}+\mathbf{J}^{xe}\left[(1-\vartheta)\dot{\mathbf{q}}^{n}+\vartheta\dot{\mathbf{q}}\right]\Delta t \tag{41}$$

where $\mathbf{X}^n$ is the model (sheet) configuration from previous (n-th) time step, $\dot{\mathbf{q}}^n$ and $\dot{\mathbf{q}}$ are nodal velocities at previous and actual time step. We write it down as

$$\frac{\tilde{\mathrm{d}}\mathbf{T}}{\mathrm{d}h}=\frac{\partial\mathbf{T}}{\partial h}+\frac{\partial\mathbf{T}}{\partial\mathbf{X}}\frac{\tilde{\mathrm{d}}\mathbf{X}}{\mathrm{d}h} \qquad \frac{\tilde{\mathrm{d}}\mathbf{B}}{\mathrm{d}h}=\frac{\partial\mathbf{B}}{\partial h}+\frac{\partial\mathbf{B}}{\partial\mathbf{X}}\frac{\tilde{\mathrm{d}}\mathbf{X}}{\mathrm{d}h} \tag{42}$$

where

$$\frac{\tilde{\mathrm{d}}\mathbf{X}}{\mathrm{d}h}=\frac{\mathrm{d}\mathbf{X}^{n}}{\mathrm{d}h}+\mathbf{J}^{xe}\left[(1-\vartheta)\frac{\mathrm{d}\dot{\mathbf{q}}^{n}}{\mathrm{d}h}+\left(\dot{\mathbf{q}}-\dot{\mathbf{q}}^{n}\right)\frac{\mathrm{d}\vartheta}{\mathrm{d}h}\right]\Delta t \tag{43}$$

The sensitivity of prescribed velocity vector $\frac{\mathrm{d}\dot{\mathbf{q}}^{pr}}{\mathrm{d}h}$ is explicitly known.

It is assumed that external load vector **f** is generally dependent on nodal velocity. Its sensitivity with respect to design parameter h depends on the load type. Here it is approximated by

$$\frac{\tilde{\mathrm{d}}\mathbf{f}}{\mathrm{d}h}\approx\frac{\mathbf{f}^{*}-\mathbf{f}}{\Delta h} \tag{44}$$

where Δh is a small perturbation of design parameter, while $\mathbf{f}^*$ is the nodal load vector computed for perturbed $h + \Delta h$ but at frozen $\dot{\mathbf{q}}$.

Explicit design derivative of contact forces can be derived from equation (11).

$$\frac{\tilde{\mathrm{d}}\mathbf{f}^{c}}{\mathrm{d}h}=\mathbf{J}^{T}\sum_{k\in S^{c}}\mathbf{J}^{kT}\frac{\tilde{\mathrm{d}}\mathbf{f}^{c^{k}}}{\mathrm{d}h}=\mathbf{J}^{T}\sum_{k\in S^{c}}\mathbf{J}^{kT}\left(\frac{\tilde{\mathrm{d}}\mathbf{p}^{k}}{\mathrm{d}h}+\frac{\tilde{\mathrm{d}}\mathbf{t}^{k}}{\mathrm{d}h}\right) \tag{45}$$

where $\mathbf{p}^k$ and $\mathbf{t}^k$ denote normal and tangent contact reactions at node k, respectively. These depend on nodal position; the tangent force additionally on nodal velocity and friction coefficient μ,

$$\frac{\tilde{\mathrm{d}}\mathbf{p}}{\mathrm{d}h}=\frac{\partial\mathbf{p}}{\partial\mathbf{X}}\frac{\tilde{\mathrm{d}}\mathbf{X}}{\mathrm{d}h} \qquad \frac{\tilde{\mathrm{d}}\mathbf{t}}{\mathrm{d}h}=\frac{\partial\mathbf{t}}{\partial h}\frac{\tilde{\mathrm{d}}\mathbf{X}}{\mathrm{d}h}+\frac{\partial\mathbf{t}}{\partial\mu}\frac{\tilde{\mathrm{d}}\mu}{\mathrm{d}h}. \tag{46}$$

Explicit derivative of **X** is given by equation (43), with obviously different meaning of the boolean assignment matrix $\mathbf{J}^x$.

Resuming, the right hand side of system of equations (35) given by formula (36) is calculated as the sum of expressions given by equations (39)–(46).

3.2. Algorithm of sensitivity analysis

Computation of the right-hand side vector given in equation (36) for an arbitrary design parameter may appear a very difficult task. In order to ensure the highest possible level of generality of the program we assume that the sensitivity will be determined by semi-analytical differentiation method. The main system of equations is directly differentiated. Differentiation of some components of the right-hand side may appear tedious. For example, analytical formulation of the differentiation with respect to specific geometric parameters (i.e. radius of a hole or another detail) is possible, but extremely complex. In that case the sensitivity of an arbitrary variable a with respect to the given parameter is estimated using a finite difference quotient

$$\frac{\tilde{\mathrm{d}}a(h_0)}{\mathrm{d}h}\approx\frac{a(h_0+\Delta h)-a(h_0)}{\Delta h} \tag{47}$$

where Δh is perturbation of design parameter h.

Full implementation of derived equations (section 3) is difficult and can be reason of number of numerical problems. Some parameters are easier to implement (i.e. material constitutive data) others more difficult (e.g. shape parameters). The above presented formulation of sensitivity analysis allows to use approximate formula (47) at any level of computations.

For example, when the sheet thickness b is taken as a design parameter it is easy to approximate $\frac{\mathrm{d}b}{\mathrm{d}h}$ using formula (47), substitute into equation (40) and next into equations (39), (37) and (36). Finally, the right hand side of linear system (35) is obtained in simple, analytical way.

The case is different when the design parameter corresponds to the model shape. The calculation of $\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}h}$ and $\frac{\mathrm{d}\mathbf{T}}{\mathrm{d}h}$ (see equations (42)) could be difficult. In such a case the approximate differentials of **B** and **T** could be calculated using difference quotient and next substituted into equation (39).

In the most general case it is possible to approximate whole right hand side vector given in equation (36) by difference quotients by

$$R_i=\frac{F_i^{*}-r_i}{\Delta h} \tag{48}$$

where r_i is the right hand side vector (16) (if the solution is accurate, this should be zero), while its modification by design parameter perturbation Δh at frozen $\dot{\mathbf{q}}$ is denoted as F_i^* (this obviously is a non-zero value).

Let us note here that in our sensitivity analysis formulation the mesh topology in the FE model remains the same for both perturbed and primary models, even if geometry is design-dependent. The algorithm of sheet metal forming with sensitivity analysis with respect to the selected design parameter is conducted as follows:

1. Allocation of variables, reading of input data.
2. Identification of design parameter (or parameters). For each design parameter the perturbation is assumed.
3. Assuming of the initial value of the pseudo-time, usually $t = 0$.
4. The calculation and determining of temporary variables necessary for simulation.
5. Calculation of viscosity matrix and the right hand side vector.
6. Solution of the main linear system of equations (as the linearization of the main nonlinear system).
7. If the residual forces and the velocity norm are greater than assumed accuracy go to 5.
8. Read of the sensitivity input data, calculation of input data sensitivity with respect to design parameter.
9. Calculation of the right hand side of equation (35).
10. Solution of the linear system (35).
11. Print of the calculated sensitivities.
12. If another design parameters exists go to 8.
13. Next time step, go to 4.

The implementation of the presented sensitivity algorithm is not finished at the moment. The numerical examples and the discussion about results will be presented in the next papers.

4. FINAL REMARKS

– The paper deals with analytical and semi-analytical formulation of design sensitivity in metal forming simulations using flow approach.
– The consistent tangent viscosity matrix is necessary in sensitive computations. Unfortunately the pure tangent viscosity matrix is asymmetric. In numerical implementations only its symmetrical part is implemented. The influence of the asymmetric parts on design sensitivity is not yet studied, but when time step Δt is small enough the asymmetric part is also small (see equation(12)).
– The fully analytical implementation of design sensitivity is very difficult. Presented formulation allows to use difference quotient (equation (47)) at any level of sensitivity formulation: from calculation of input data differentials to approximate of whole right hand side given in equation (36).
– The numerical implementation of above presented formulation is not yet done. Authors are working on it. The first numerical results and discussion about it results will be presented in the next papers.

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ANALIZA WRAŻLIWOŚCI SYMULACJI TŁOCZENIA BLACH Z WYKORZYSTANIEM SZTYWNO-LEPKO-PLASTYCZNEGO MODELU MATERIAŁU

Streszczenie

Celem artykułu jest teoretyczne sformułowanie problemu analizy wrażliwości symulacji głębokiego tłoczenia blach na dowolnie wybrany parametr projektowy. Analiza wrażliwości następnie będzie zaimplementowana do programu MFP rozwijanego w IPPT PAN w ramach projektu *Numpress*.

Przedstawiony zostanie sztywno-lepko-plastyczny model materiału oraz jego zastosowanie w symulacjach MES plastycznego formowania metali. Uwzględniono tutaj styczną macierz lepkości modelu. Jak zostało dowiedzione w końcowym sformułowaniu analizy wrażliwości styczna macierz lepkości jest niezbędna w obliczeniach wrażliwości. Niestety pełna macierz lepkości jest niesymetryczna. W niniejszym artykule udowodniono, że człony niesymetryczne są małe (w stosunku do pozostałych wyrazów macierzy lepkości) i zostały pominięte.

W sformułowaniu analizy wrażliwości wykorzystano półanalityczną metodę bezpośredniego różniczkowania (semi-analytical Direct Differentiation Method). To znaczy, że pochodne niektórych wyrazów są obliczane za pomocą ilorazu różnicowego a następnie zostają podstawione do analitycznego sformułowania wrażliwości.

Przedstawiono matematyczne sformułowanie oraz algorytm budowy stycznej macierzy lepkości oraz analizy wrażliwości symulacji głębokiego tłoczenia blach z wykorzystaniem sztywnolepko-plastycznego modelu materiału.

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