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PRACTICAL INVERSE ANALYSIS IN METAL FORMING

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Abstract

This article is thought as a contribution to the practical use of inverse analysis in the field of metal forming. The first part of the paper gives a short overview of the field of inverse analysis and the different algorithms to solve the mathematical problems; in the second part the application to different problems in metal forming is discussed. These include the validation of Finite Element Models, i.e. cross rolling, the determination of damage parameters and the determination of heat transfer coefficients for heat treatment simulations. It is shown that inverse analysis is a comfortable tool when lots of analysis data need to be handled efficiently.

Key words: inverse analysis, levenberg-marquardt, simulated annealing, evolutionary algorithms

1. INTRODUCTION

Inverse Analysis is closely related to the minimization or maximization of a cost function, hence a branch of optimization and typically used for the determination of material parameters or heat transfer coefficients in metal forming and related fields. In the following section a brief introduction to the issues of optimization is given, which is followed by examples of use in the field of metal forming.

2. MATHEMATICAL PRELIMINARIES

In the following section, we deal with finding the extremum of a scalar valued function $\chi^2 \in R^+$ which is defined as the sum of the square of the difference between a measured value V^M and a simulated one V^S:

$$d\chi^2 = \left(V^M(a_i,\Omega) - V^S(a_i,b_j,\Omega)\right)^2 d\Omega \quad (1),$$

where the vectors $a_i \in R^m$ and $b_j \in R^n$ are defined and free parameters, respectively and Ω is the domain of validity.

Integrating equation (1) over the domain Ω yields:

$$\chi^2 = \int_\Omega \left(V^M(a_i,\Omega) - V^S(a_i,b_j,\Omega)\right)^2 d\Omega = \chi^2(b_j) \quad (2)$$

In the remaining part of this paper we assume that equation (2) is two times differentiable. Further on the function χ^2, its Gradient and Hessian exist with respect to all admissible b_j.

The gradient with respect to b_j and the Hessian are defined in R^n according to equations (3) and (4), respectively.

$$\nabla\chi^2 = \left(\frac{\partial\chi^2}{\partial b_1},\ldots,\frac{\partial\chi^2}{\partial b_n}\right)^T \quad (3)$$

$$H_{kl} = \frac{\partial^2\chi^2}{\partial b_k \partial b_l} \quad (4)$$

The optimization problem may be formulated as follows: Under all admissible b_j find $^*b_j$ which minimizes equation (2). In the next section an overview of nonlinear optimization procedures is given, the applied ones will be discussed.

3. NONLINEAR OPTIMIZATION

Figure 1 gives an overview of nonlinear optimization procedures, according to Schumacher (2004). They can be divided in search algorithms which are very often heuristic motivated and gradient-based methods which require the computation of the gradient or the Hessian.

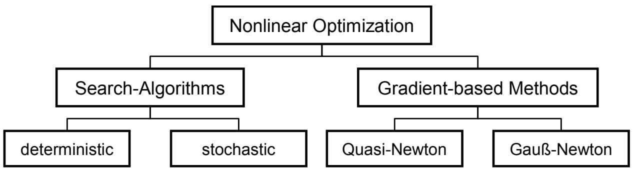

***Fig. 1.** Overview of nonlinear optimization methods.*

In the following subsections two stochastic search-algorithms (simulated annealing and evolutionary algorithms) and the Levenberg-Marquardt method, a gradient-based Gauß-Newton method, are described.

3.1. Levenberg-Marquardt

The Levenberg-Marquardt method is an elegant method for varying smoothly between the inverse-Hessian method (5) and the steepest descent method (6).

$$\Delta b_l = -\nabla \chi^2 H_{kl}^{-1} \tag{5}$$

$$\Delta b_l = -const. \cdot \nabla \chi^2 \tag{6}$$

This is done by introduction a regularization parameter λ. After some thought experiments it is clear that for large values of λ equation (7) reduces to equation (6) and for small values of λ to equation (5). The extraordinary robustness of the algorithm is due to the fact that large values of λ result in a small constant in equation (6):

$$\Delta b_l = -\frac{\nabla \chi^2}{(\lambda \cdot \delta_{kl} + H_{kl})} \tag{7}$$

The algorithm can be represented as follows:

1) guess an appropriate starting vector ${}^s b_l$
2) set $b_l := {}^s b_l$
3) set $\lambda = 0.001$

do

4) evaluate H_{kl} and $\nabla \chi^2$ for b_l
5) solve the set of linear equations $(\lambda \cdot \delta_{kl} + H_{kl}) \Delta b_l = -\nabla \chi^2$ for Δb_l
6) if $\chi^2(b_l + \Delta b_l) < \chi^2(b_l)$ then $b_l := b_l + \Delta b_l$ and $\lambda := \lambda / 10$
   else: $\lambda := \lambda \cdot 10$

while $\lambda > 10^{-8}$

For details see Press et al. (2007).

3.2. Simulated annealing

The simulated annealing (SA) algorithm in its original form is based on the idea that all particles in an annealed solid arrange themselves in the low energy state of a corresponding lattice, provided the maximum temperature is sufficiently high and the cooling is carried out sufficiently slowly. Starting from a high temperature T, the solid is allowed to reach thermal equilibrium, characterised by a probability of being in a state with energy E given by the Boltzmann distribution. As the temperature decreases, the Boltzmann distribution concentrates on the states with lowest energy and finally, when the temperature approaches zero, only the minimum energy states have a non-zero probability of occurrence. The algorithm can be represented as follows:

1) define a cooling schedule $T = T(t)$
2) generate a vector b_j randomly

do

3) perform an admissible variation of b_j defined as δb_j
4) calculate a random number u [0,1]
5) if $u \le \exp\left(\frac{\chi^2(b_j + \delta b_j) / \chi^2(b_j) - 1}{T(t)}\right)$ then $b_j := b_j + \delta b_j$
6) $t := t + 1$

while $T(t) > T_0$

For details see Press et al. (2007) or Laarhoven and Aarts (1987).

3.3. Evolutionary qlgorithms

In contrast to SA, where the inspiring example is the cooling of matter, Evolutionary Algorithms (EA) use biology as analogy. Here a certain number N of individuals is generated which represent the parents for the next generation. Equation (2) is considered to be the fitness function. For each individual an admissible set of parameters b_j is generated at random, its fitness function is evaluated and the posterity is reproduced by two individuals copying their information by crossover or linear combination and altering the genes by mutation. The algorithm can be represented as follows:

1) generate a number of N individuals with random values $b_{ji}, i = 1...N$
2) counter = 0
3) define a maximal number of generations

do

4) evaluate $\chi_i^2(b_{ji})$
5) take the fittest as parents and produce N posterity individuals
6) perform mutations
7) counter: = counter + 1

while counter < maximal number of generations.

For details see Eiben and Smith (2003).

4. EXAMPLES

In this section three examples are presented where the algorithms, explained in section 3, are applied.

4.1. Contour recognition

In a recently completed project, software was developed for the validation of a finite element model of the cross rolling process. The task was to ascertain the contour of a slice of a tube (inner and outer contour) and to fit these contours by elliptic equations. Starting from the recognized contours a least square was formulated according to equation:

$$\chi^2(x_m, y_m, a, b, \beta) = \sum_{i=1}^{N}\left[d_i(x_i, y_i, x_m, y_m) - r(x_m, y_m, a, b, \beta)\right]^2 \quad (8),$$

where N is the number of data points, x_i, y_i, are the coordinates of the measured contour, x_m, y_m, a, b and β are the parameter defining the elliptic equation according to figure 2. This approach is slightly different since b_j can also be found in the measured value.

The minimization of χ^2 was done using the Levenberg-Marquardt method. The computing time for one picture is approximately 3 s, while the behaviour of the algorithm for the calculation of figure 3 is depicted in figure 4.

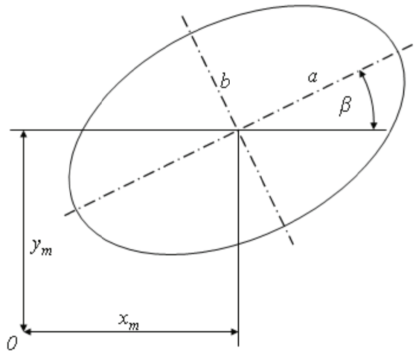

Fig. 2. Sketch of the elliptic equation.

A benchmark result for the software is depicted in figure 3 showing the capability of picture filtering and contour recognition.

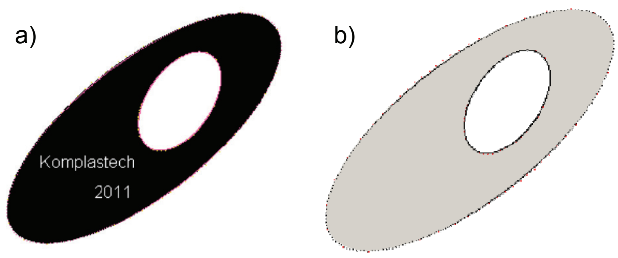

Fig. 3. Results for picture filtering and contour recognition, a) original picture, b) after filtering and contour recognition.

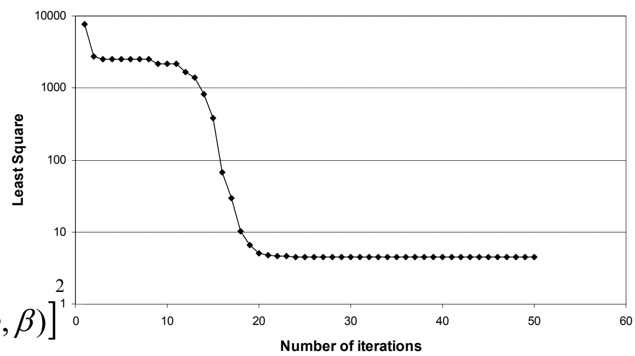

Fig. 4. Least Square Error as a function of the number of iterations.

4.2. Damage parameters identification

A further application in metal forming is the determination of material parameters for the estimation of flow curves or the identification of damage parameters. Damage parameters were determined using inverse analysis, where the quadratic norm of the error χ^2 (difference between measured and calculated force) was integrated over the displacement x using the trapezium rule. The error χ^2 is thus a function of all material parameters. Provided, that all material properties except the damage parameters

φ_d, φ_r and D_c are known, the error χ^2 is a function of these parameters .

Hot tension tests on a Gleeble 1500® thermo-mechanical testing device were conducted at 1150 and 1250 °C and at strain rates of 0.01, 0.05 and 0.1 s^{-1}. The hot tension tests were modelled, whereas the temperature distribution along the longitudinal direction was considered to be parabolic with a horizontal tangent in the centre of the specimen as it is depicted in figure 5. The elastic modulus of the material was considered to be temperature dependent which results in a more accurate strain distribution in the forming zone. The flow curve was modelled using a Haensel-Spittel equation. The material model was elasto-viscoplastic.

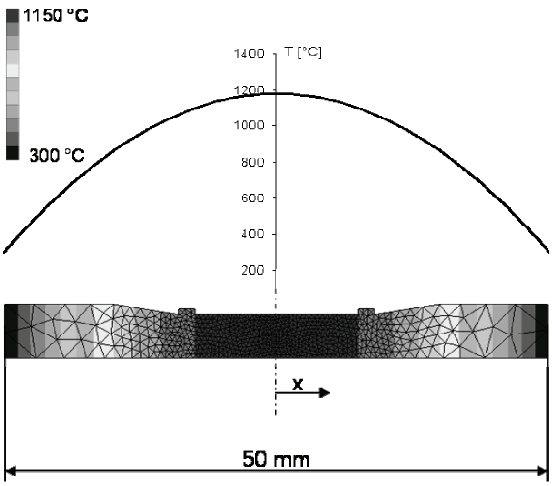

***Fig. 5.** Temperature profile for the specimen at 1150°C.*

Here, software was developed which is linked to the commercial Finite Element Code Forge2008 enabling to fit parameters using the thermo-mechanical response of the finite element calculation. The key to a successful calculation is the correct computation of the error square between measured and simulated load displacement curve. The structure of the software is depicted in figure 6.

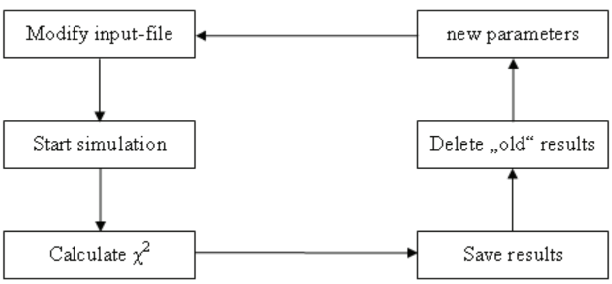

***Fig. 6.** Structure of the software for "Damage Parameter Identification".*

The optimisation for parameter identification was performed using a factorial design since the numerical calculation of a gradient or the Hessian is very expensive. Hence a domain of validity and the number of intermediate steps between the boundaries of the domain for the parameters was defined for each parameter. The damage evolution law was defined as follows

$$\dot{D} = \frac{D_c}{\varphi_r - \varphi_d} R_v(\xi)\dot{\varphi} \qquad (9)$$

where $\dot{D}$ denotes the time derivative of the scalar damage variable, D_c the critical damage value, φ_d the equivalent plastic strain at damage initiation, φ_r the equivalent plastic strain at rupture, R_v the triaxiality function, ξ the triaxiality and $\dot{\varphi}$ the equivalent plastic strain rate.

The parameters which were fitted are D_c in the domain [0.1, 0.6], φ_d in the domain [0, 0.2] and φ_r in the domain [0.4, 0.6], the admissible values of the parameters were [0.1, 0.2, 0.3, 0.4, 0.5, 0.6], [0, 0.05, 0.1, 0.15, 0.2] and [0.4, 0.45, 0.5, 0.55, 0.6], respectively. Due to the full factorial design 900 Finite Element Simulations needed to be performed, 150 times six different tension tests. The computation time is approximately 900 times 30 sec., resulting in a total computation time of about 7.5 hours. The minimum damage value was found for the parameter set $\{D_c, \varphi_d, \varphi_r\}$ at {0.4, 0.05, 0.6}. The evolution of the quadratic norm as a function of simulation cycles can be found in figure 7. It can be seen that the minimum error arises in the cycle where $D_c=0.4$ and $\varphi_r=0.6$. The minimum error exists then for $\varphi_d=0.05$.

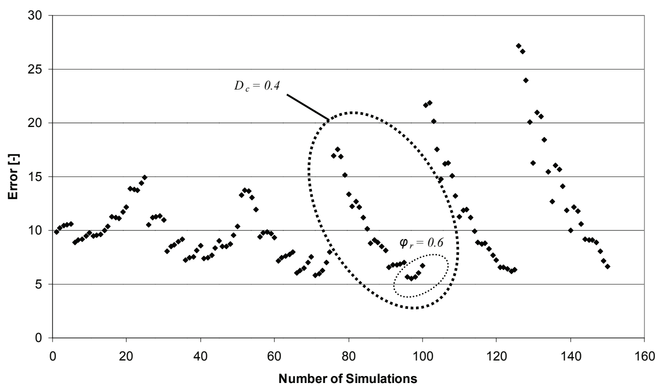

***Fig. 7.** Absolute error of the simulation cycles.*

4.3. Determination of heat transfer coefficients

The determination of heat transfer coefficients is a crucial element for the successful modelling of heat treatment operations. Since these parameters can vary over a wide range, simulated annealing and evolutionary algorithm were integrated into software which has a similar structure as depicted in figure 6.

In order to be able to conduct benchmark calculations an arbitrary axisymmetric part (∅800) was designed and the cooling was simulated with surface temperature dependent heat transfer coefficients - see Buchmayr and Kirkaldy (1992) – using the commercial finite element code Forge. Sensors were defined on the surface of the part which represent the surface measurement with thermocouples (cp. figure 8).

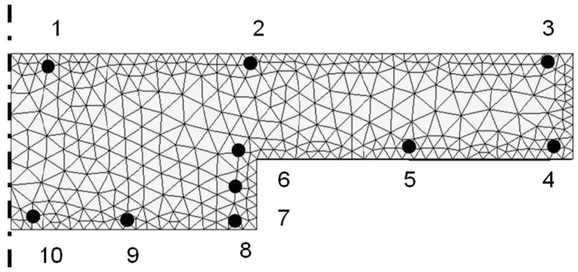

Fig. 8. Positions of sensors in an axis-symmetric part.

The task was to find the heat transfer coefficient as a function of time using evolutionary algorithms and simulated annealing. The cost function χ^2 was calculated as follows:

$$\chi^2(\alpha_i) = \sum_{s=1}^{N} \int_t \left[T_s^{ref}(t) - T_s^{calc}(t,\alpha_i)\right]^2 dt \quad (9)$$

where T^{ref} denotes the temperature progress of the reference calculation for thermocouple s, T^{calc} the temperature progress of the calculated curve for thermocouple s, α_i denotes the heat transfer coefficients and N the number of sensors.

The evolutionary algorithm was based on a (μ,λ)-evolution strategy, which means that the parents of generation (n+1) are the posterities of generation (n) and the elderly of generation (n-1) are completely forgotten. In the following example each generation consists of 100 individuals and the fittest 20 percent are selected for the combination of the next generation.

The average error square, minimum and maximum error for each generation is depicted in figure 9. It can be seen that the minimum error square is reached in the sixth generation, for which a number of approximately 600 finite element calculations needed to be performed.

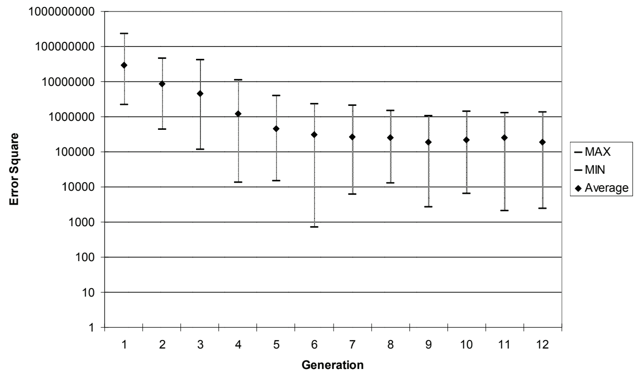

Fig. 9. Maximal, minimal and average error square plotted against the number of generations.

The parameter identification using SA reveals promising results. Figure 10 shows the result for the calculation. From the depiction it can be seen that as it gets "colder" the probability for the acceptance of a higher error square becomes smaller and the parameter set seems to reach the configuration to attain the minimum. It can be seen that about 4000 finite element calculations need to be performed.

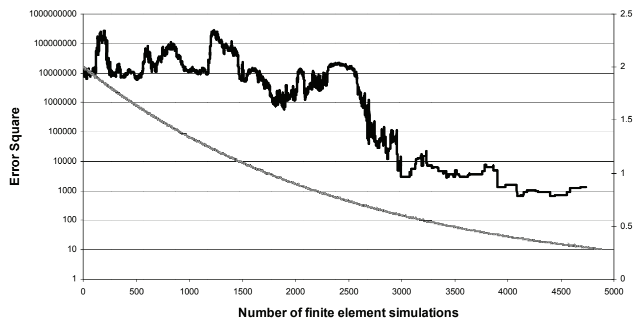

Fig. 10. Error square and temperature as a function of the number of simulations.

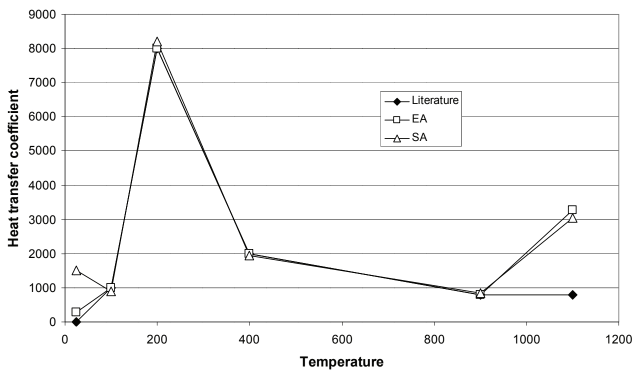

Fig. 11. Results for the identification of the heat transfer coefficient as a function of surface temperature for simulated annealing (SA) and evolutionary algorithms(EA) compared to Buchmayr and Kirkaldy (1992).

Figure 11 shows the results for the heat transfer coefficient. It is obvious that the least square function χ^2 is most sensitive in the medium temperature regime, whereas the heat transfer coefficient at very high and very low temperature hardly influences the results. With highest probability these parameters can be determined correctly if the algorithms are to be refined.

5. CONCLUSION

This article exposes the capability of inverse analysis for the determination of parameters. It could be shown that there is a great need for methods like these that simplify practical work by testing and finding different solutions. From a pure mathematical point of view, uniqueness of the solution is the uttermost interest, from the engineer's point of view the applicability and physical soundness.

Generally the choice of the optimization procedure is very much dependent on the problem. In the first example computation time is very inexpensive and the starting value for the optimization problem can calculated very easily near the optimum. Hence a gradient based optimization procedure is first choice. In the second example, which deals with damage parameter identification, a starting value or the domain of interest may be known but the computation of the gradient or the Hessian is too time consuming. Here the full factorial design is very practical. Computing the heat transfer coefficient – using finite element calculations - methods based on stochastic algorithms are able to search a wide parameter domain. Here the behaviour of the quadratic norm in the domain of validity is hardly known and gradient based methods are supposed to search local minima, so simulated annealing and evolutionary algorithms promise to find a global solution. Both algorithms are able to find a solution near the global minimum while evolutionary algorithm find the solution in a shorter time. This is due to the fact that simulated annealing is dependent on the starting value and searches the valid domain to improve the error norm, while evolutionary algorithms generate individuals which are spread over the whole domain.

REFERENCES

Buchmayr, B., Kirkaldy, J.S., 1992, A Fundamental Based Microstructural Model for the Optimization of Heat Treatment Processes, *Proc. 1st Int. Conf. on Quenching and Control of Distortion*, Chicago, 221-227.

Eiben, A.E., Smith, J.E., 2003, Introduction to Evolutionary Computing, Springer, Berlin.

Laarhoven, P.J.M. van, Aarts, E.H.L., 1987, *Simulated Annealing: Theory and Applications*, Springer, Eindhoven.

Press, W.H., Teukolsky, S.A., Vetterling, W.T., Flannery, B.P., 2007, *Numerical Recipes 3rd Edition: The Art of Scientific Computing*, Cambridge University Press, Cambridge.

Pschera, R., Klarner, J., Sommitsch, C., 2010, Modelling the Forming Limit during Cross-Rolling of Seamless Pipes Using a Modified Continuum Damage Mechanics Approach, *Steel research international*, 81, 686-690.

Schumacher, A., 2004, *Optimierung mechanischer Strukturen; Grundlagen und Anwendungen*, Springer, Berlin.

PRAKTYCZNE ZASTOSOWANIE ANALIZY ODWROTNEJ W PRZERÓBCE PLASTYCZNEJ METALI

Streszczenie

Ideą artykułu jest omówienie praktycznych aspektów zastosowania rozwiązania odwrotnego w plastycznej przeróbce metali. W pierwszej części pracy przedstawiono krótkie omówienie problematyki rozwiązań odwrotnych i przedstawiono różne algorytmy dla matematycznego rozwiązania tych problemów. W drugiej części pracy przedstawiono aplikacje do różnych zadań w plastycznej przeróbce metali. Zaprezentowane przykłady to walidacja modelu MES dla walcowania poprzeczno-klinowego, przewidywania parametrów zniszczenia materiału oraz współczynnika wymiany ciepła w symulacjach procesów obróbki cieplnej. Wykazano, że analiza odwrotna jest użytecznym narzędziem kiedy duża liczba danych musi być przetwarzana w sposób efektywny.

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