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NUMERICAL ACCURACY AND EFFICIENCY OF ALGORITHMS FOR SPRINGBACK CALCULATION

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Abstract

The objective of this work has been to study numerical accuracy and efficiency of algorithms of springback calculation. The finite element program Stampack has been used as a solver. It has capability of an explicit dynamic simulation of forming while the springback can be analysed using either an explicit dynamic or quasi-static implicit approach. Draw bending of a U profile has been used as a case study problem. Two methods of springback calculation have been compared, their advantages and disadvantages have been discussed. Numerical results have been verified by comparison with experimental data. The influence of selected numerical parameters on the accuracy and efficiency of the springback calculation has been investigated.

Key words: springback, explicit method, implicit method, metal forming

1. INTRODUCTION

Springback is one of the most serious formability problems influencing the quality of products stamped from metal sheet. It is caused by elastic recovery of internal stress after unloading. Nowadays numerical analysis of springback is indispensable in the design of stamping process.

The springback itself is a difficult design problem, and so is its analysis. There are many factors that affecting springback such as material properties, tool die gap, radius on die corners, blank holder force, lubrication conditions, etc. (Laurent et al., 2009 and Laurent et al. 2010). Geng and Wagoner (2001) investigated the role of plastic anisotropy and its evolution in springback. Chen and Koc (2007) showed that variation in the mechanical and numerical properties causes springback variation. Investigation of influence of punch speed is presented by Firat et al. (2010). The effect of friction coefficients on springback has been studied by Carden (2002).

The physical phenomena must be accurately considered in the numerical models in order to get correct results of springback calculation. There are also many possible errors resulting from the numerical algorithm. Generally, there are two approaches in the numerical analysis of springback problem, the explicit dynamic approach and quasi-static implicit one. Full methodology to analyze metal forming processes including the springback has been described by Rojek and Oñate (1999), Zimniak (2005) and Gronostajski et al. (2001).

In numerical modeling of springback we search an accurate solution but at reasonable computational cost, therefore we try to find the best compromise between the accuracy and numerical efficiency. In this work we verify correctness of numerical algorithms for springback calculation based on the explicit dynamic and implicit quasi-static approach. By

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investigation of the influence of selected numerical parameters on accuracy and efficiency of numerical solution an optimum algorithm and appropriate numerical parameters for springback calculation have been determined.

2. THEORETICAL FORMULATION

2.1. Modeling of sheet metal forming operations

Typically in a finite element model of a sheet metal forming operation we have a sheet and three basic tools: punch, die, blankholder. Tools can be treated as rigid bodies which excite blank sheet deformation by the contact forces under prescribed kinematic and loading conditions. Friction has big importance in metal forming operations so appropriate friction model must be included in the contact model. During the forming process a metal sheet undergoes large elastoplastic deformations. Constitutive material model has to be able to consider large elastoplastic deformations with plastic anisotropy taken into account.

2.2. Explicit dynamic formulation

The program Stampack is based on the explicit solution of the equations of motion. The theoretical formulation is presented by Oñate et al. (1995) and Rojek et al. (1996). The discretized equations of motion are written in the following form:

$$\mathbf{M}\ddot{\mathbf{a}} + \mathbf{C}\dot{\mathbf{a}} = \mathbf{p} - \mathbf{f}\ , \tag{1}$$

where $\mathbf{M}$ and $\mathbf{C}$ are the mass and damping matrices, $\ddot{\mathbf{a}}$ and $\dot{\mathbf{a}}$ are the vectors of the nodal accelerations and velocities, $\mathbf{p}$ and $\mathbf{f}$ are the vectors of external loads and internal forces, respectively. The element internal force vector is calculated from the following relation

$$\mathbf{f}^{(e)} = \int_{V^{(e)}} \mathbf{B}^T \boldsymbol{\sigma} dV\ , \tag{2}$$

where $\boldsymbol{\sigma}$ is the Cauchy stress tensor, $\mathbf{B}$ is the strain-displacement operator matrix and $V^{(e)}$ is the element volume.

Equations (1) are integrated in time using an explicit scheme, in which the displacements a_{n+1} at time t_{n+1} are obtained from the equations for the known configuration at time t_n:

$$\ddot{\mathbf{a}}_n = \mathbf{M}_D^{-1}\left(\mathbf{p}_n - \mathbf{f}_n - \mathbf{C}\dot{\mathbf{a}}_n\right) \tag{3}$$

where $\mathbf{M}_D = diag\mathbf{M}$

$$\dot{\mathbf{a}}_{n+1/2} = \dot{\mathbf{a}}_{n-1/2} + \ddot{\mathbf{a}}_n \Delta t_{n+1/2} \tag{4}$$

where $\Delta t_{n+1/2} = \frac{1}{2}\left(\Delta t_n + \Delta t_{n+1}\right)$

$$\mathbf{a}_{n+1} = \mathbf{a}_n + \dot{\mathbf{a}}_{n-1/2}\Delta t_{n+1} \tag{5}$$

The effectiveness of the explicit dynamic formulation is based on the use of a diagonal mass matrix $\mathbf{M}_D$, so there is no need to solve a system of equations. Because of its efficiency in the analysis of large scale systems the dynamic explicit approach is a preferable option in industrial sheet stamping simulation. The dynamic analysis can be extended to the springback phase once the final deformation is obtained. The analysis can be continued with removed contact conditions and with adequate damping to obtain the deformed shape after springback. Experience, however, shows that this method of springback calculation is highly inefficient because the time required for the springback analysis with the explicit dynamic code is very long and it usually exceeds the time of the stamping analysis. To overcome this disadvantage the implicit springback calculation can be used.

2.3. Springback calculation by the implicit approach

In this approach the springback is treated as a quasistatic process. The implicit solution schemes for the quasistatic case are obtained by taking the equations of equilibrium at the unknown configuration at time $\mathbf{t}_{n+1}$ to get the solution for this time $\mathbf{a}_{n+1}$. The solution to the implicit problem can be obtained by an iterative procedure. In our analysis we used the modified Newton-Raphson method. The following system of equations is solved at the k-th iteration:

$$\mathbf{K}_{n+1}^{(0)} \delta\mathbf{a}_{n+1}^{(k)} = \mathbf{p}_{n+1} - \mathbf{f}_{n+1}\left(\mathbf{a}_{n+1}^{(k)}, \boldsymbol{\sigma}_{n+1}^{(k)}\right), \tag{6}$$

where $\mathbf{K}_{n+1}^{(0)}$ is the tangent stiffness matrix at the beginning of the (n+1)-th step. From Eq. (6) the correction $\delta\mathbf{a}_{n+1}^{(k)}$ is found that is used to obtain the k-th approximation $\mathbf{a}_{n+1}^{(k)}$ of the solution searched

$$\mathbf{a}_{n+1}^{(k)} = \mathbf{a}_{n+1}^{(k-1)} + \delta\mathbf{a}_{n+1}^{(k)}. \tag{7}$$

The iterations continue until the required convergence is obtained.

In the springback analysis there is no external loading, $\mathbf{p} = 0$. The residual forces appear from the residual stresses in the sheet at the end of forming. These stresses termed σ_{res} are the initial values for the solution of eq. (6) i.e. after removing the contact surfaces:

$$\boldsymbol{\sigma}_1^{(0)} = \boldsymbol{\sigma}_{res} \,. \tag{8}$$

Substituting eq. (8) into (6) would allow to perform the springback analysis in one step. In case of large springback effects more steps have to be used and at each one of them we apply only a fraction of the residual stresses $\boldsymbol{\sigma}_{res}$.

$$\boldsymbol{\sigma}_1^{(0)} = \frac{\boldsymbol{\sigma}_{res}}{N}, \quad \ldots, \quad \boldsymbol{\sigma}_1^{(0)} = \boldsymbol{\sigma}_n + \frac{\boldsymbol{\sigma}_{res}}{N}, \quad \ldots, . \tag{9}$$

N is the number of load steps. The methodology of springback calculation using a quasistatic approach is presented by Rojek and Oñate (1999).

3. NUMERICAL STUDIES

3.1. Description of the numerical problem

Draw bending of a U profile was used as a case study of springback calculation. The test definition was specified by Renault, which also provided experimental results. The tool geometry is shown in figure 1. A rectangular steel blank used in the forming process has the dimensions 420x200 mm. The material has the following properties: Young's modulus E = 206.86 GPa, Poisson's coefficient υ = 0.29, mass density ρ = 7820 kg/m^3, limit of elasticity Re(0^o) = 160 MPa, Re(45^o) = 172 MPa, Re(90^o) = 168 MPa, Lankford's coefficients R_0 = 1.8, R_{45} = 1.642, R_{90} = 2.2. Hollomon's coefficients K = 563 MPa, n = 0.256.

Two values of the blankholding force, 300 and 600 kN, were used in the experiments and analyses. The symmetry was taken into account, so a quarter of the blank was considered in the model only. The tools were modeled as rigid bodies. Sheet was discretized with triangular BST shell elements (Oñate et al., 1996, 1997, 1999). The finite element model is shown in figure 2. The blank was discretized uniformly with 16200 BST elements and 90 rigid triangular facets were used in the discretization of the tools. The friction between the sheet and tools was treated assuming the Coulomb model with friction coefficient μ = 0.05.

Four Gauss points were used for the integration of stresses across the blank thickness. Isotropic elastic behavior and transversal anisotropy in plasticity have been assumed. The Hill 48 yield criterion was used for the steel with the average Lankford coefficient $R = (R_0 + 2R_{45} + R_{90})/4$ =1.82. The stress-strain relationship was approximated by the following function $\sigma = 563\left(0.00888 + \bar{\varepsilon}^p\right)^{0.256}$ MPa, where σ is the Cauchy stress and $\bar{\varepsilon}^p$ – the effective plastic strain.

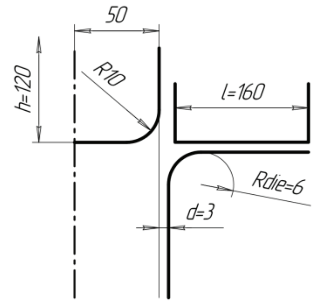

Fig. 1. *Geometry of U bending test.*

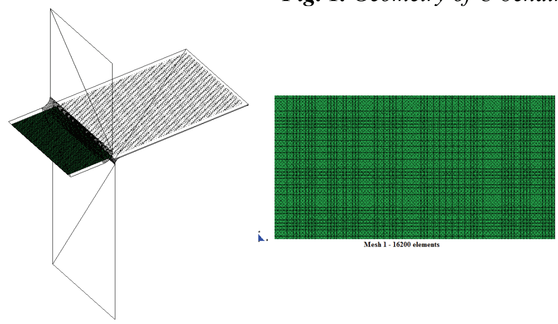

Fig. 2. *FE model of U bending test.*

3.2. Methodology used for the springback calculation

The analysis of the forming has been performed using the explicit dynamic model. The forming was simulated assuming the punch velocity of 10 m/s. The springback calculation for the case with 300 kN blankholding force was carried out using both the explicit dynamic and implicit quasistatic approaches. The implicit module developed for the Stampack was used in the implicit analysis of the springback. The Newton-Raphson (N-R) scheme was employed in the solution of nonlinear equations. The explicit dynamic solution was compared with the implicit one, and both solutions were compared with the experimental data. Then the quasistatic implicit approach was applied to the springback solution for the blankholding force of 600 kN. The influence of the numerical parameters (number of Gauss points and number of load increments in the quasistatic springback calculation) on the accuracy and efficiency of the solution was investigated using the quasistatic implicit approach.

3.3. Numerical results

The residual stresses at the end of forming are the input data for the springback analysis. The accuracy of the springback calculation depends on the accuracy of simulation of forming process. In the explicit dynamic solution the dynamic effects must be carefully controlled. The blankholding force must be applied with adequate damping. The value of the damping coefficient must be assumed very carefully so that the blankholder action is stable during the whole forming process. Figure 3a shows the plots of the contact force at the interface between the sheet and blankholder (the value of the force obtained for the quarter of the geometry is multiplied by 4). It can be observed that neglecting small oscillations the contact force has practically constant value corresponding to the external load applied to the blankholder. This confirms that the blankholder action is modeled correctly. Predicted punch forces during forming are plotted in figure 3b. The punch force obtained for the blankholding force of 600 kN is compared with the experimental result showing a very good agreement.

The shape of the blank after forming is shown in figure 4a. This is the initial shape for the subsequent springback analysis. The case with the blankholding force of 300 kN was analysed using the explicit dynamic approach first.

The springback analysis consists in the solution of the problem of damped free vibrations. Introducing an adequate structural damping leads to an equilibrium state with the deformed shape after springback. A configuration can be regarded as steady when the vibrations in the lowest structural mode are damped out. A steady solution can be reached in an efficient way if the damping applied is close to the critical value for the lowest natural frequency. The lowest vibration frequency can be obtained by analysing the free vibrations of the system without damping prior to the springback analysis. The free vibrations of the part after forming are illustrated with the curve of the vertical displacement of the endpoint of the sheet shown in figure 5a. From this curve we can estimate the period, frequency and corresponding critical damping.

Then, the free vibrations with the damping equal to 45%, 78%, 90%, 100% and 200% of the critical damping were analysed. The sheet behaviour during the analysis of damped vibrations can be followed by plotting the curve of the vertical displacement of the endpoint given in figure 5b. It can be seen that the solutions for the damping equal to 45%, 78%, 90% and 100% of the critical damping converge to a steady state solution. The shape of the blank after springback is shown in figure 4b. The comparison with the experimental shape is presented in figure 6. A very good agreement can be observed. The springback for the same case was analysed using the quasi-static implicit approach described in Sec. 2.3. The residual stresses were applied in 10 steps. The solution obtained in this way is given in figure 6 showing a very good agreement with the two other curves. This confirms equivalence of the two algorithms of springback analysis.

At the same time the quasi-static implicit solution is much more efficient numerically. The solution took 31 min CPU (processor Intel Core 2 Quad Q9650, 3GHz) only, while the dynamic solution took more than 12 hours CPU. The forming simulation took in both cases about 40 min CPU. This makes the combination of the explicit forming simulation with the implicit springback calculation a preferable option in springback calculation. This approach was employed in further studies presented in this work.

The springback after forming with the blankholding force of 600 kN was calculated using the quasi-static implicit model. The predicted shape after springback is compared with experimental data in figure 7. It can be seen that as it was expected the springback effects decreased due to the increase of the blankholding force. The agreement between the numerical and experimental results is quite good although not so good as in case of 300 kN blankholding force.

Numerical model of sheet forming and springback is usually a compromise between accuracy and computational cost requirements. The influence of the number of Gauss points on the solution has been studied aiming to check if either the accuracy can be increased without great loss of numerical efficiency by taking more Gauss points than 4 or the numerical efficiency can be increased without considerable loss of accuracy by taking less Gauss points than 4. The results of this study for the case of 300 kN blankholding force are shown in the form of a histogram in figure 8a. The results demonstrate the convergence of the solution with the increasing of the number of Gauss points, however, in order to get the full convergence the number of Gauss points would have to be increased more.

Number of Gauss points has influence on the efficiency of the solution. Table 1 presents the information on the CPU times for different number of integration points through the shell thickness. Increasing the number of Gauss points by one increased the CPU time of the forming simulation approximately by 2 min, the CPU time for 4 Gauss points being 40 mins. Influence of the number of Gauss points on the efficiency of the implicit springback analysis is not so straightforward due to different convergence rate for different number of Gauss points. Number of Newton-Raphson iterations at one load step decreases with the increase of the number of Gauss points which is given in table 1. For 2 Gauss points the solution converged in 8 N-R iterations, while using 5 or 6 Gauss points the convergence was obtained in 5 iterations. Total CPU time is minimum for 4 Gauss points. The number of Gauss points assumed for the initial tests, 4, seems a reasonable choice providing the solution with quite a good accuracy at not excessive computational cost.

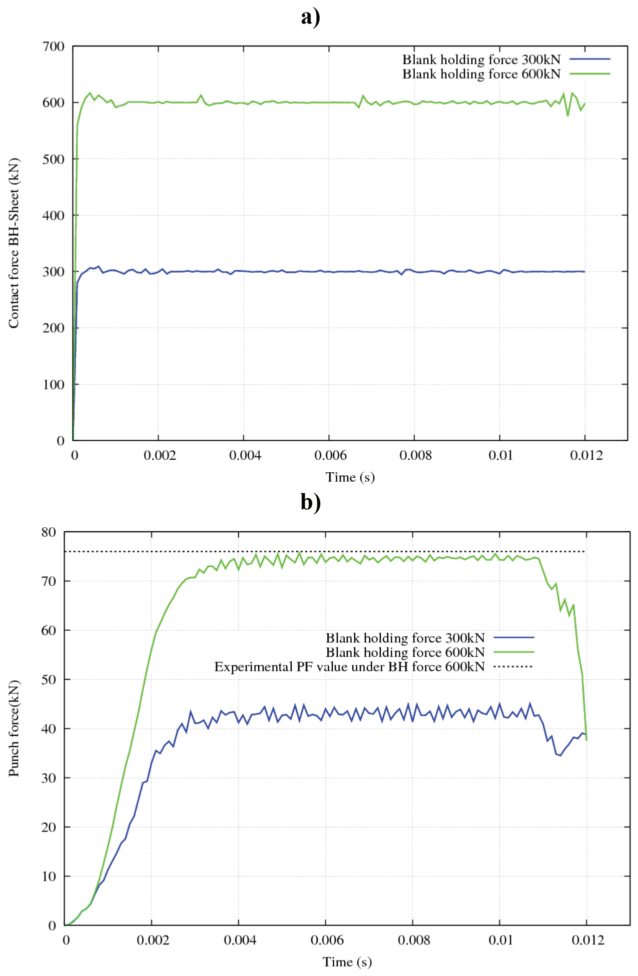

Fig. 3. History of contact forces: a) between the blank and the blankholder, b) between the blank and the punch.

Accuracy and efficiency of the nonlinear analysis in the quasistatic implicit approach depends on the number of load steps. The results presented until now were obtained taking 10 steps. The solution was performed for various numbers of load steps ranging from 3 to 25. The results of this study are plotted in figure 8b and CPU times are given in figure 9. The curve in figure 8b is nearly linear. The results obtained with 3 increments are close to those obtained with 25 steps. This shows that in this example we can use a small number of load steps. The residual stresses can sometimes be applied in one step.

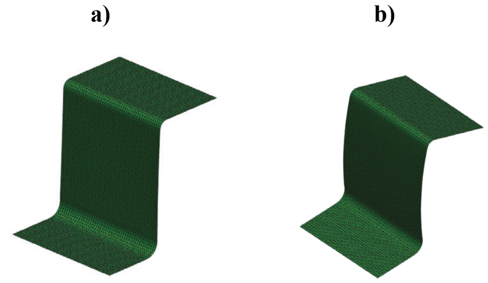

Fig. 4. Shape of the blank: a) after forming, b) after springback (blankholding force of 300 kN).

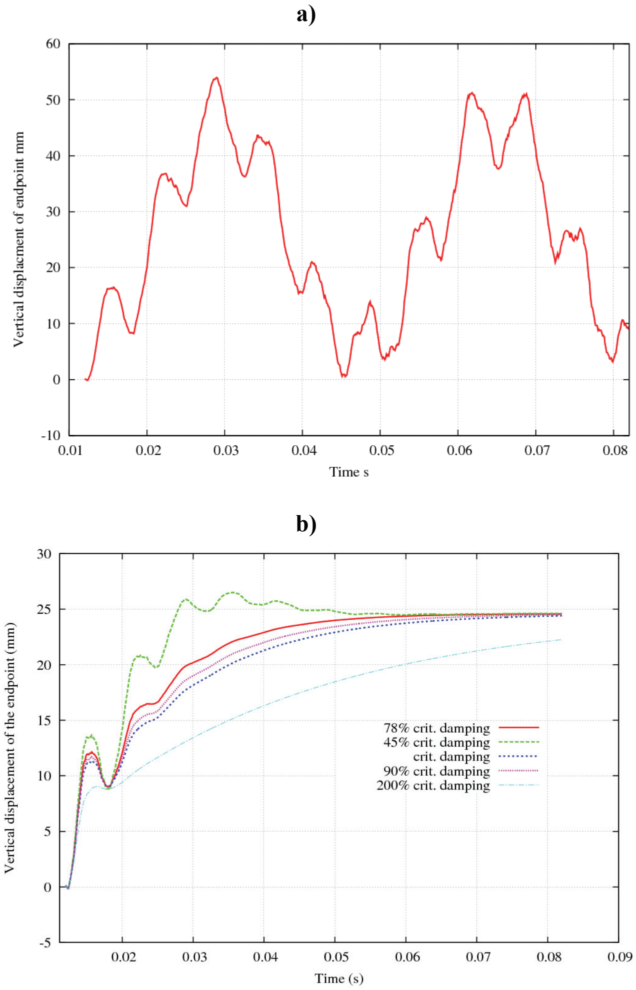

Fig. 5. Vertical displacement of the endpoint of the sheet for: a) undamped vibrations, b) different values of damping.

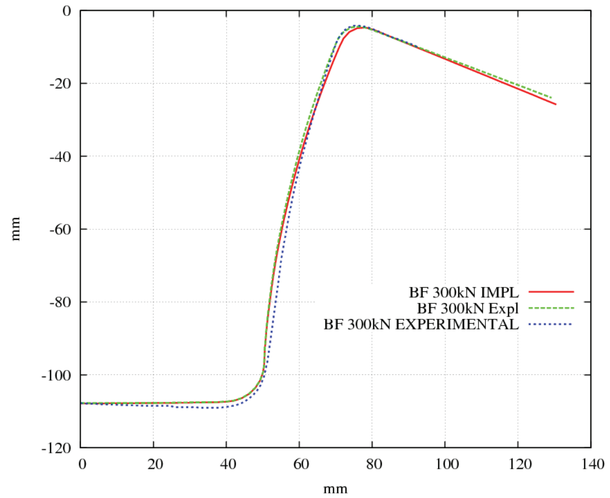

Fig. 6. Experimental and numerical results, blankholder force of 300 kN.

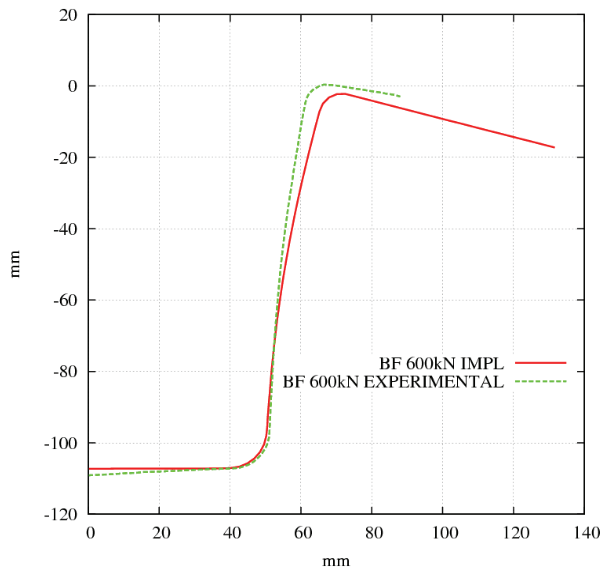

Fig. 7. Experimental and numerical results, blankholder force of 600 kN.

Table 1. *CPU time and number of N-R iterations during explicit stamping and implicit springback analysis using different number of integration points through thickness.*

Number of integration points through thickness	Time of stamping analysis [s]	Implicit springback analysis		Total time of expl.stamping and impl. Springback analysis [s]
		Number of N-R iterations/load step	Time of springback analysis [s]	
2	2164	8	2280	4444
3	2265	7	2220	4485
4	2456	6	1500	3956
5	2531	5	1440	3971
6	2773	5	1380	4153
7	2844	4	1320	4164

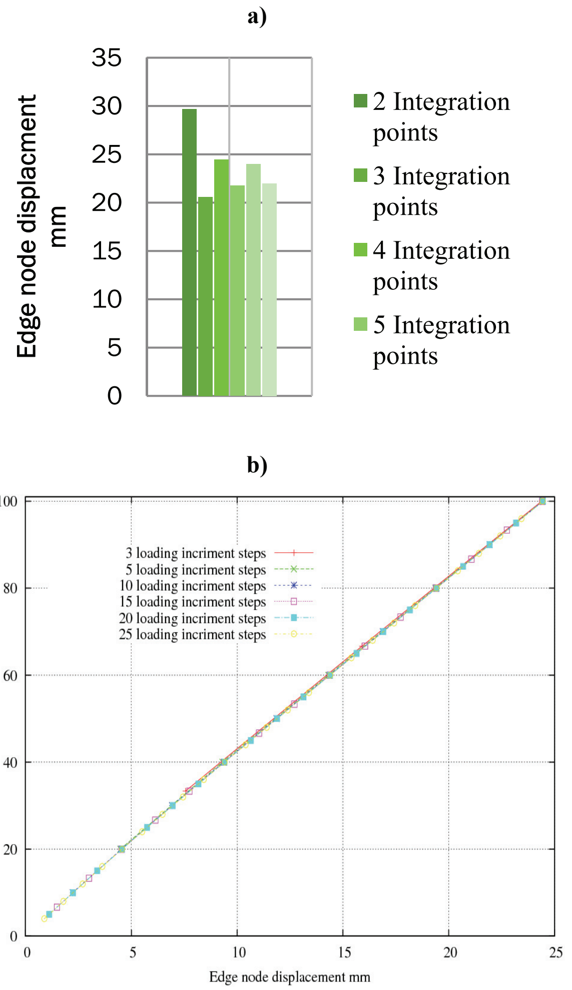

Fig. 8. Influence of numerical parameters on the solution: a) number of integration points through thickness, b) number of load steps.

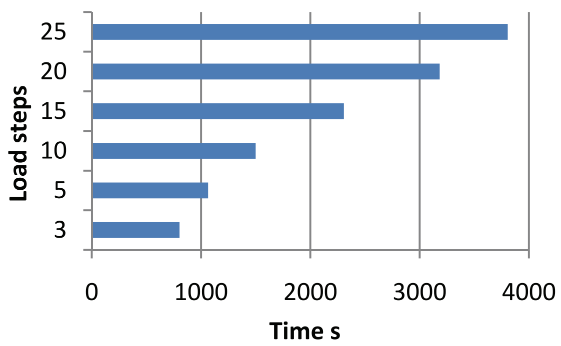

Fig. 9. Total time of CPU at implicit analysis using different number of load steps.

4. CONCLUSIONS

Comparison of the numerical and experimental results shows that the numerical model used in this work is capable to predict correctly the shape after springback. This allows us to use this model in the design to study the influence of physical parameters on springback.

Comparison of the explicit dynamic and implicit quasistatic models of springback calculations has shown that these methods are equivalent as far as the accuracy is concerned. Nevertheless the implicit quasistatic approach was found as more efficient in prediction of springback. The explicit approach is simple but requires more time and investigations to obtain stable numerical solution.

Investigations of the influence of numerical parameters have shown that typical forming and springback simulations are acceptably accurate with 4 through-thickness integration points. The implicit quasistatic analysis can be performed in quite few steps, in the case analysed as few as three.

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DOKŁADNOŚĆ I EFEKTYWNOŚĆ NUMERYCZNA ALGORYTMÓW OBLICZANIA SPRĘŻYNOWANIA POWROTNEGO

Streszczenie

Celem niniejszej pracy jest zbadanie dokładności i efektywności numerycznej algorytmów obliczania sprężynowania powrotnego po procesie kształtowania blach. W obliczeniach wykorzystano program metody elementów skończonych Stampack. Program ten jest oparty o algorytm jawnego całkowania równań ruchu względem czasu, co daje możliwość efektywnej symulacji procesu kształtowania. W analizie sprężynowania powrotnego porównano sformułowanie jawne dynamiczne z metodą wykorzystującą sformułowanie quasistatyczne niejawne. Przeprowadzono obliczenia dla testowego przykładu kształtowania profilu o kształcie U. Wyniki numeryczne porównano z wynikami doświadczalnymi otrzymując dużą zgodność. Zbadano wpływ wybranych parametrów numerycznych na dokładność i numeryczną efektywność obliczeń. Pozwoliło to określić optymalny model numeryczny dający zadowalająco dokładne wyniki przy niezbyt długim czasie obliczeń.

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