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NUMERICAL SOLUTION OF THE THREE POINT BENDING OF GLASS IN APPLICATION TO THE INVERSE ANALYSIS OF THIS TEST

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Abstract

Numerical model of glass deformation is presented in this paper. The model is based on the solution of the Maxwell approach and, beyond the viscosity flow of the glass, it also describes the stress relaxation behavior at elevated temperatures. This model is applied to the three point bending test. 2D and 3D solutions are performed using Abaqus finite element software.

The general objective of the paper is application of the developed model to identification of the material parameters. Three point bending experiments were performed at various temperatures and inverse problem was formulated. The relaxation functions, shear modulus and bulk modulus were identified by Prony series. The properties of viscoelastic material determined at one particular temperature is transposed to another temperature using Williams-Landell-Ferry equation. Comparison of the 2D and 3D simulations is the particular objective of the work. Inverse calculations involve increase of the computing costs, which are particularly high in the case of the 3D solution of the direct problem. Therefore, the error connected with application of the 2D model is evaluated in the paper and recommendations regarding accuracy of the inverse analysis based on the 2D direct problem model are given.

Key words: viscosity, Williams-Landell-Ferry equation, glass, thermal bending, flexure

1. INTRODUCTION

The glass forming stage is a complicated process because of a large quantity of parameters which can influence its run and also because of the shape of the manufactured products. Viscosity is the most significant physicochemical property of glass, which has influence on forming process. This property determinates two technical parameters: mass flow and rate of its solidification. Forming stage take place together with the cooperation of molten glass flow and continuous solidification at the same time. The processes listed above are described by the following parameters:

– operational range of viscosity of molten glass and appropriate range of forming temperature,
– time of solidification – time of viscosity change in operational range of forming.

The glass forming process has to be optimized in such a way that the solidification time is exactly the same or a little bit longer than forming. Solidification time can be controlled by a change in chemical composition or by intensity of cooling of the glass in mould. The time of forming can be regulated in a limited range with the adequately chosen forming

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force value. Because of differences in wall thickness of the products and various heat transfer speed, the forming process on individual parts can proceed with different technical parameters. It can lead to the increase of the local stress, surface micro cracks and also to not achieving the proper shape of the product or deformation after taking out from the mould.

Having mentioned above facts in mind, the objective of this work was formulated as elaboration of numerical model of glass deformation in a range of temperatures which refers to viscosity glass flow. Numerical simulations could optimize the forming process and glass bending, making a contribution in reducing the process losses and in increasing the production capacity (Gregoirea et al., 2007; Giannopapa, 2006).

Numerical solutions for the glass deformation, which are known from the literature, require large experimental bases and do not include such an important parameter as temperature fluctuations. Creating the numerical model of glass deformation in fluctuating temperature conditions for different glass grades using finite element method is the main output of this work.

This work is a continuation of researches in which 2D solution was made. (Pernach et al., 2010)

2. RHEOLOGICAL MODEL OF GLASS

The rheological model allows to determine the relationship between stress, strain, displacement and strain rate. In this model equation of state is given by:

$$f\left(\sigma,\frac{d\sigma}{dt},\varepsilon,\frac{d\varepsilon}{dt}\right)=0 \qquad (1)$$

where: σ – stress, ε – strain, t – time.

According to the Maxwell rheological model the total strain of viscoelastic material, such as glass, is a sum of elastic and viscous parts (Christensen, 2003; Kolenda, 2007):

$$\varepsilon=\varepsilon_s+\varepsilon_l \qquad (2)$$

Equation (2), after differentiating with respect to time and loading Hooke's law and Newton's law, is described in the following form:

$$\frac{d\varepsilon}{dt}=\frac{1}{E}\frac{d\sigma}{dt}+\frac{1}{\eta}\sigma \qquad (3)$$

where: E – Young's modulus, η – coefficient of viscosity.

According to the Maxwell model a sudden application of load induces an immediate deformation by elastic spring, which is followed by creep of the dashpot. On the other hand, a sudden strain produces an immediate reaction by the spring, which if followed by stress relaxation. In consequence, the solution of equation (3) is function of stress relaxation:

$$\varepsilon(t)=const\Rightarrow\sigma(t)=\sigma_0\exp\left(-\frac{t}{t_R}\right) \qquad (4)$$

The function of creep is described as:

$$\sigma(t)=const\Rightarrow\varepsilon(t)=\varepsilon_0+\frac{\sigma}{\eta}t \qquad (5)$$

where: t_R - relaxation time (the rate of stress decrease) is given by:

$$t_R=\frac{\eta}{E} \qquad (6)$$

In static load case the effect of stress relaxation is determined by relaxation modulus $G(t)$:

$$\sigma(t)=G(t)\varepsilon \qquad (7)$$

The effect of creep is described by bulk modulus $J(t)$:

$$\varepsilon(t)=J(t)\sigma \qquad (8)$$

The viscoelastic properties are time and temperature dependent. Time and temperature are correlated according to superposition rule The relaxation modulus varies in the same way at different temperatures except for shift in time.

$$G(t,T)\rightarrow G[t_r(T)] \qquad (9)$$

where: t_r– relaxation time.

The time-temperature transformation is implemented by a shift factor a_T.

$$a_T=\frac{t}{t_r} \qquad (10)$$

Williams-Landell-Ferry (WLF) equation allows for the estimation of the temperature shift factor for temperatures other than those for which the material was tested (Kolenda, 2007; Williams et al., 1955):

$$log(a_T)=\frac{-C_1(T-T_g)}{C_2+T-T_g} \qquad (11)$$

where: T – temperature, T_g – reference temperature (glass transition temperature + 50°C), C_1,C_2

– material parameters dependent on temperature T_g, η_0 – viscosity at glass transition.

The constitutive equations for viscoelastic materials and different temperature conditions can be expressed in the tensor notation by (Christensen, 2003):

$$\sigma_{ij}(x,t) = \int_{-\infty}^{t} \bar{G}_{ij}(x,t-\tau)\frac{\partial \varepsilon_{kl}}{\partial \tau}(x,\tau)\,d\tau$$

$$\varepsilon_{ij}(x,t) = \int_{-\infty}^{t} \bar{J}_{ij}(x,t-\tau)\frac{\partial \sigma_{kl}}{\partial \tau}(x,\tau)\,d\tau \qquad (13)$$

where: σ_{ij} – stress tensor, ε_{ij} – strain tensor, $\bar{G}_{ij}$ – relaxation function tensor, $\bar{J}_{ij}$ – creep function tensor.

The relaxation tensor is divided into the elastic part G_0, which is time independent, and time dependent relaxation function G_{ij}. An assumption is made that for t<0,$\sigma_{ij} = 0$, $\varepsilon_{ij} = 0$, and $\varepsilon_{ij} \neq 0$ for $t = 0$, so equations (12) becomes:

$$\sigma_{ij}(x,t) = G_0(x)\varepsilon_{kl}(x,0) + \int_{0}^{t} G_{ij}(x,t-\tau)\frac{\partial \varepsilon_{kl}}{\partial \tau}(x,\tau)d\tau \qquad (14)$$

Equation (13) is written in the following form:

$$\varepsilon_{ij}(x,t) = J_0(x)\sigma_{kl}(x,0) + \int_{0}^{t} J_{ij}(x,t-\tau)\frac{\partial \sigma_{kl}}{\partial \tau}(x,\tau)d\tau \qquad (15)$$

Equations (14) and (15) in varying temperature conditions are given by:

$$\sigma_{ij}(x,t) = G_0(x)\varepsilon_{kl}(x,0) + \int_{0}^{t} G_{ij}(x,\xi-\zeta)\frac{\partial \varepsilon_{kl}}{\partial \tau}(x,\zeta)\,d\zeta \qquad (16)$$

$$\varepsilon_{ij}(x,t) = J_0(x)[\sigma_{kl}(x,0) - \alpha_0\theta(t)]\sigma_{kl}(x,0) + \int_{0}^{t} J_{ij}(x,\xi-\zeta) - \alpha_0\theta(\zeta)\frac{\partial \sigma_{kl}}{\partial \tau}(x,\zeta)d\zeta \qquad (17)$$

where:

$$\zeta = \int_{0}^{\tau} a_T[T(t)]dt, \quad \xi = \int_{0}^{\tau} a_T[T(\tau)]d\tau \qquad (18)$$

$\theta(t)$ – pseudotemperature, α_0 – coefficient of thermal expansion.

Time dependent part of the relaxation function is determined in a simple shear test by deformation of the sample and stress measurement at a constant strain. Problem of Maxwell model identification is equivalent to approximation of measurement data by sum of exponentials. Prony method (12) leads to identification scheme, which is simple for implementation. In consequence, the relaxation functions: shear modulus and bulk modulus, are typically modeled with Prony series (Povolo, 1988):

$$G(t) = G_0\left(1 - \sum_{i=1}^{n} p_i\, exp\left(-\frac{t}{\tau_i}\right)\right) \qquad (19)$$

$$J(t) = J_0\left[1 - \sum_{i=1}^{n} p_i \exp\left(-\frac{t}{\tau_i}\right)\right] \qquad (20)$$

where: p_i – Prony constants, τ_i – relaxation time.

3. EXPERIMENT

In order to obtain samples, a peace of commercial tableware glass was cut off to get 6x8 mm plates of 0.7, 1.35 and 2.0 mm thickness, respectively. Both parallel surfaces of the plates were polished mechanically.

Thermo-mechanical analysis (TMA) was conducted on Perkin-Elmer TMA-7 with a sapphire 3-point bending kit. The applied method is a modification of the standard test method for strength of glass by flexure. For all series of samples three loads of 100, 200 and 400 mN were applied, respectively. The load was constant and the temperature was increased with a rate of 5°C/min during the experiment. From the obtained curves of displacement as a function of temperature, the point of softening and the value of viscous strain was calculated for the applied loads.

4. NUMERICAL SOLUTION

The numerical model of glass deformation in the test of temperature dependent 3-point bending was created in this work. Program Abaqus was employed for analysis of this problem in 2D and 3D domains. Solution domain and the finite element mesh created in the sample are presented in figure 1. Assuming the symmetry in 3D domains, only half of the sample is considered.

The program of numerical simulations was convergent with the range of experimental tests of glass deformation. Values of the glass properties used in this model are shown in table 1.

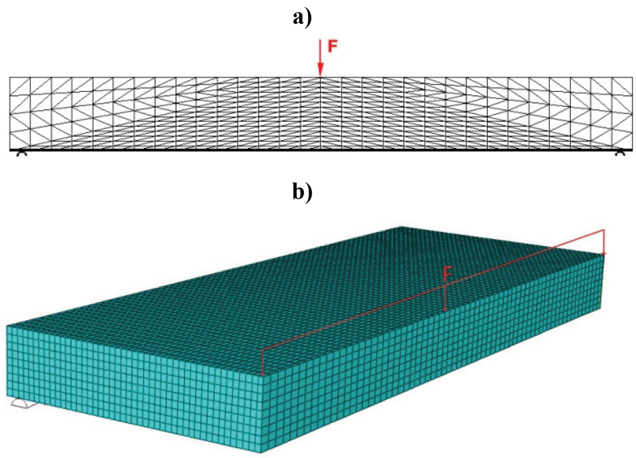

Fig. 1. Solution domain and the finite element mesh: a) 2D, b) 3D.

Table 1. Properties of glass used in the FE model

Density	2497.5 [kg/m^3]
Coeff. of thermal expansion	92×10^{-7} [1/K]
Reference temperature (transformation temperature)	490^{o}C

5. RESULTS

The values of the coefficients C_1 and C_2, which give the best agreement between measurements and predictions, were the objective of the simulations. The idea of inverse analysis of the experimental tests was applied with the vector of optimization variables $\mathbf{p} = \{C_1,C_2\}^T$. The inverse algorithm proposed in (Szeliga et al., 2006) is used. The objective function is formulated as the square root error between measured and calculated location of the top central point of the sample (the point, at which the load is applied):

$$\Phi = \sqrt{\frac{1}{Nt}\sum_{i=1}^{Nt}\frac{1}{Ns}\sum_{j=1}^{Ns}\left[\frac{y_{mij}\left(T\right)-y_{cij}\left(\mathbf{p},T\right)}{y_{mij}\left(T\right)}\right]^2} \qquad (21)$$

where: y_{mij}, y_{cij} – measured and calculated location of the considered point, respectively, Nt – number of performed tests, Ns – number of sampling points in one test.

Minimum of the objective function (21) was obtained for $C_1 = 26.3$ and $C_2 = 62.7$.

In order to validate the model, simulations of all tests using the optimum coefficients C_1 and C_2 were performed. Results of the numerical simulations of viscous flow performed for various thickness of samples (0.7, 1.35, 2.0 mm) and three loads (100, 200, 400 mN) were compared with the experimental data (Figures 2, 3 and 4).

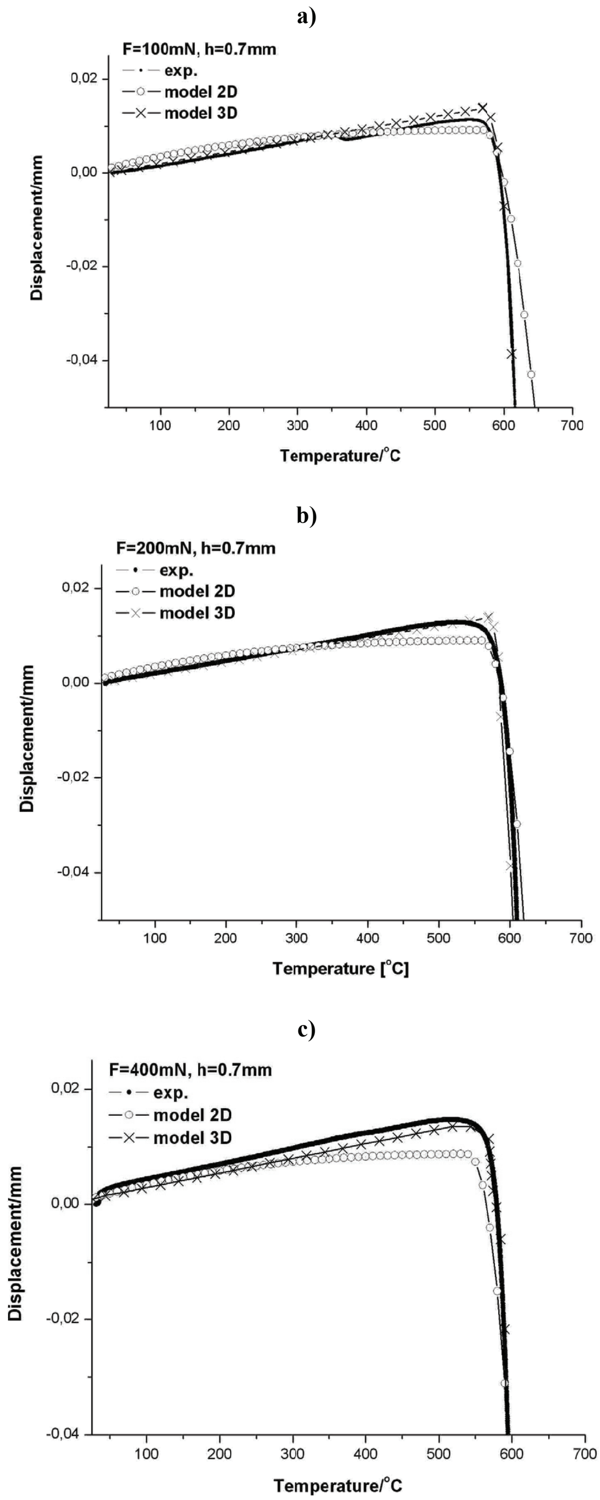

Fig. 2. Results of numerical simulations of deformation of glass (open points) compared with the measurements (filled points), for the different loads: a) 100 mN, b) 200 mN, c) 400 mN. Thickness of the sample 0.7 mm.

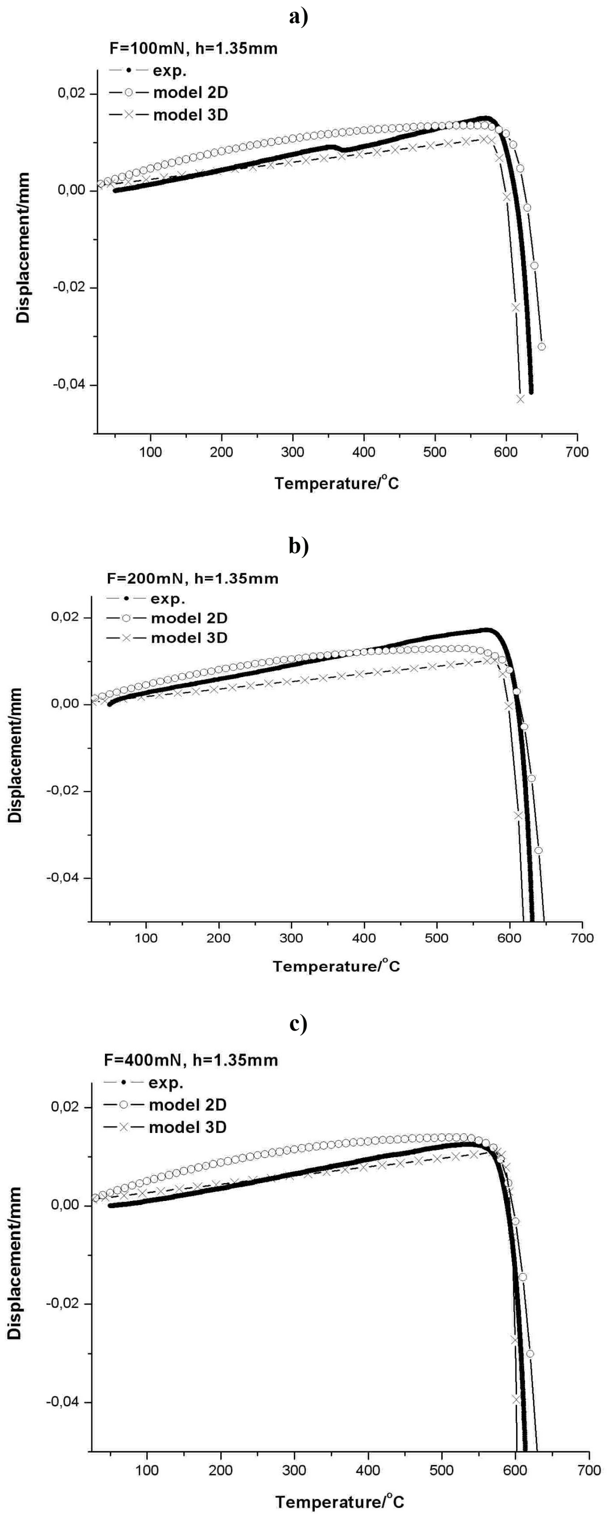

Fig. 3. *Results of numerical simulations of deformation of glass (open points) compared with the measurements (filled points), for the different loads: a) 100 mN, b) 200 mN, c) 400 mN. Thickness of the sample 1.35 mm.*

Figure 5 shows the stress component in the vertical direction (σ_{yy}) at the stage of initiation of viscous flow of glass for 200 mN load and 2.00 mm thick sample. The stresses are compressive above the supports and under the point where the load is applied. Tensile stresses occur in the remaining part of the sample.

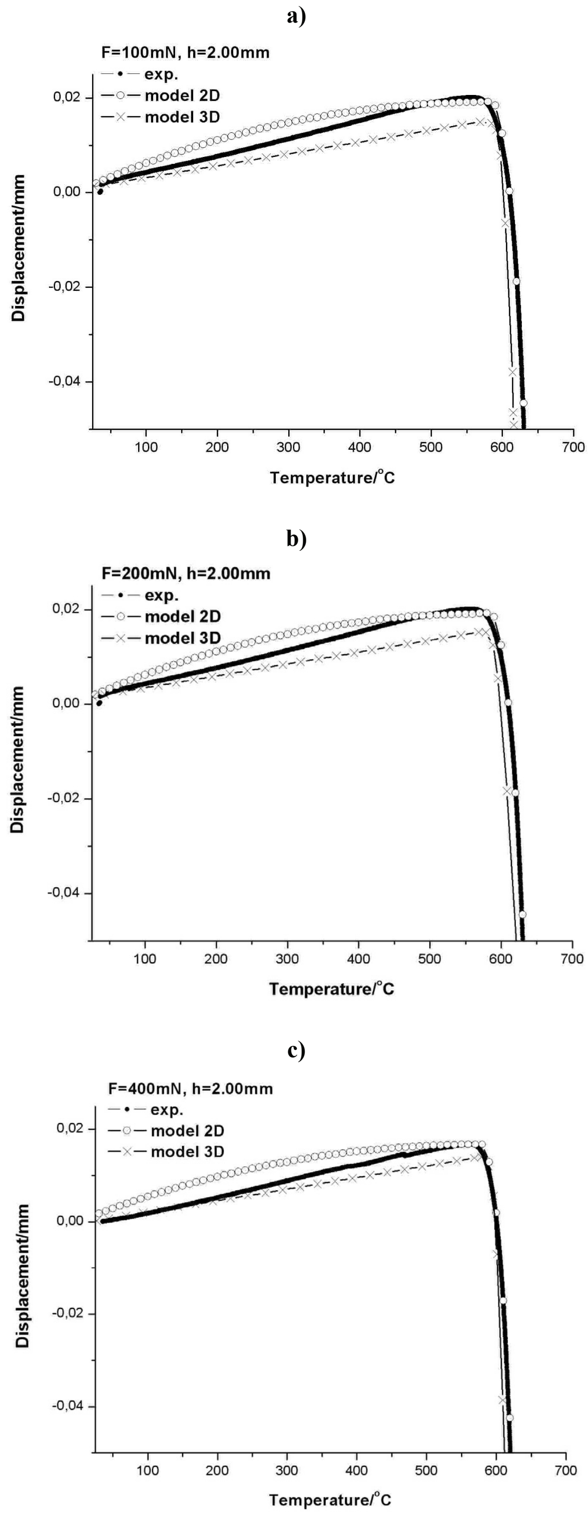

Fig. 4. *Results of numerical simulations of deformation of glass (open points) compared with the measurements (filled points), for the different loads: a) 100 mN, b) 200 mN, c) 400 mN. Thickness of the sample 2.0 mm.*

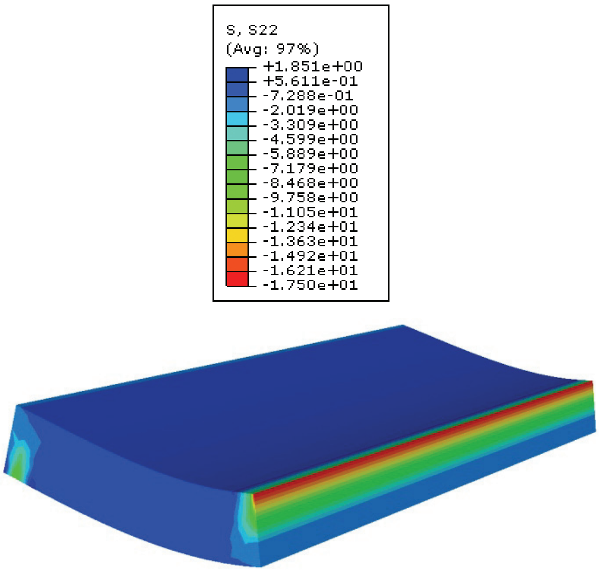

Fig. 5. *Calculated distribution of strain component for load 200 mN at the cross section of the sample in thickness 2 mm.*

6. CONCLUSIONS

Identification of properties of glass was performed on the basis of temperature dependent three point bending test and FE simulations of this test. Two parameters in the Williams-Landell-Ferry (WLF) equation were the optimization variables. Performed experiments and simulations yielded the values of these coefficients: $C_1 = 26.3$ and $C_2 = 62.7$.

Validation of the model with the optimized coefficients confirmed good agreement with the experimental data. Some discrepancies observed for the thinner samples in 2D model and big forces in 3D.

2D model can't be used in inverse analysis because deformation of material causing by thermal expansion is not correct.

3D solution in elastic range gives linear characteristic of curve and better agreement with the experiment.

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MODEL REOLOGICZNY SZKŁA I MOŻLIWOŚCI JEGO IDENTYFIKACJI

Streszczenie

W pracy przedstawiono numeryczny model odkształcania szkła. Model oparto na rozwiązaniu Maxwella, który pozwala opisać zjawisko relaksacji naprężeń w podwyższonych temperaturach. Model trójpunktowego zginania szkła w układzie 2D i 3D zaimplementowano w programie Abaqus.

Podstawowym celem pracy było stworzenie modelu, który pozwalałaby na identyfikację parametrów materiałowych szkła opisujących lepkościowe płynięcie szkła w trakcie jego odkształcania. Badania doświadczalne zginania szkła przeprowadzono dla zmiennych warunków temperaturowych. Funkcje relaksacji, moduł ścinania i ściśliwości określono za pomocą szeregów Prony'ego. Właściwości lepkosprężyste materiału wyznaczone w jednej temperaturze transponowano za pomocą równania Williama-Landella-Ferryego do innych temperatur. W pracy porównano wyniki symulacji w układzie 2D i 3D.

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