Publishing House
A K A P I T

COMPUTER METHODS IN MATERIALS SCIENCE
Informatyka w Technologii Materiałów

Vol. 11, 2011, No. 1

OPTIM – LIBRARY OF BIOINSPIRED OPTIMIZATION ALGORITHMS IN ENGINEERING APPLICATIONS

WACŁAW KUŚ[1]*, ADAM DŁUGOSZ[1], TADEUSZ BURCZYŃSKI[1, 2]

[1] *Silesian University of Technology, Department for Strength of Materials and Computational Mechanics, Gliwice, Poland*
[2] *Cracow University of Technology, Institute of Computer Science, Cracow, Poland*
**Corresponding author: waclaw.kus@polsl.pl*

Abstract

The paper is devoted to engineering applications of Optim library. The library, created by paper authors, is implements bioinspired optimization algorithms. The Optim contains classes for optimization with use of evolutionary algorithm (single and multiobjective optimization), artificial immune system and particle swarm optimization. The Optim library is well suited for engineering problems with floating point representation of design variables. The paper contains description of library and methodology of transferring data between Optim and auxiliary programs used for objective function calculation. The numerical examples of optimization of mechanical structures by using Finite Element Method for objective function evaluation are shown in the paper.

Key words: optim library, bioinspired optimization algorithms, evolutionary algorithm, identification

1. INTRODUCTION

Bioinspired algorithms become a powerful optimization tool in many different engineering disciplines. The main advantage of the bioinspired algorithms is no need for using the gradient of fitness functions. The information about values of an objective (fitness) function is sufficient for this group of algorithms. Moreover the great probability, easy implementation and flexible operators rapidly increase popularity of such techniques. Among many different types of bioinspired algorithms the most popular are: EA – Evolutionary Algorithms (genetic real coded algorithms) (Michalewicz, 1996), AIS – Artificial Immune Systems (de Castro and Timmis, 2003) and PSO – Particle Swarm Optimization (Kenndy et al., 2001). Many papers are devoted to the application of mentioned algorithms in different problems, but authors usually focus only on the one of them. The present paper is the proposal of combining three main algorithms of the bioinspired techniques in one library and usage this library in optimization of practical engineering problems. As an optimization tool the *Optim* library is proposed. The library implements EA as single and multiobjective type of the algorithm, AIS and PSO. The description of bioinspired algorithms and the important components of them are presented in the paper. It allows the usage of the *Optim* library in many different engineering and bioengineering problems. This paper presents the *Optim* library and the description of the methods limited only to the single and multiobjective evolutionary algorithms. The numerical example of applications the *Optim* library in identifi-

 ISSN 1641-8581

cation of trabeculae material parameters and a piezoelectric actuator are presented in the paper.

2. THE BIOINSPIRED ALGORITHMS

2.1. Single objective optimization problem

An optimization problem with the single objective is a method of finding a vector of design variables $\mathbf{x}$ which minimize or maximize an objective function $f(\mathbf{x})$. The vector of design variables satisfies a set of constrains $g(\mathbf{x}), h(\mathbf{x})$:

Find: $\mathbf{x} = [x_1, x_2, \dots x_n]^T$ (1)

To minimize or maximize: $f(\mathbf{x})$ (2)

Subject to: $g_i(\mathbf{x}) \geq 0 \quad i = 1, 2, \dots, m$ (3)

$$h_i(\mathbf{x}) = 0 \quad i = 1, 2, \dots, p \tag{4}$$

where n is the number of design variables, k the number of objective functions, m the number of inequality constraints, and p the number of equality constraints.

The bioinspired methods are widely used in global optimization of multimodal functions. The palette of bioinspired methods contain evolutionary algorithms, artificial immune systems, particle swarm optimization and others.

The flowchart of the evolutionary algorithm for single objective optimization is presented in figure 1a. The evolutionary algorithm operates on a population of chromosomes. The design variables are coded into each chromosome, and each chromosome is potential solution of the optimization problem. The initial population of chromosomes is created in most cases in the random way in the first step of the evolutionary algorithm. Then the objective function (fitness function) values for all chromosomes are calculated. In the next step changes of chromosome genes values are performed by using evolutionary operators. The new generation is performed on the basis of the offspring population created during the selection process. The algorithm iterates until the end condition is fulfilled (expressed e.g. as a maximum number of iterations).

2.2. Multiobjective optimization problem

The optimization process based on a collection of objective functions is called multi-objective

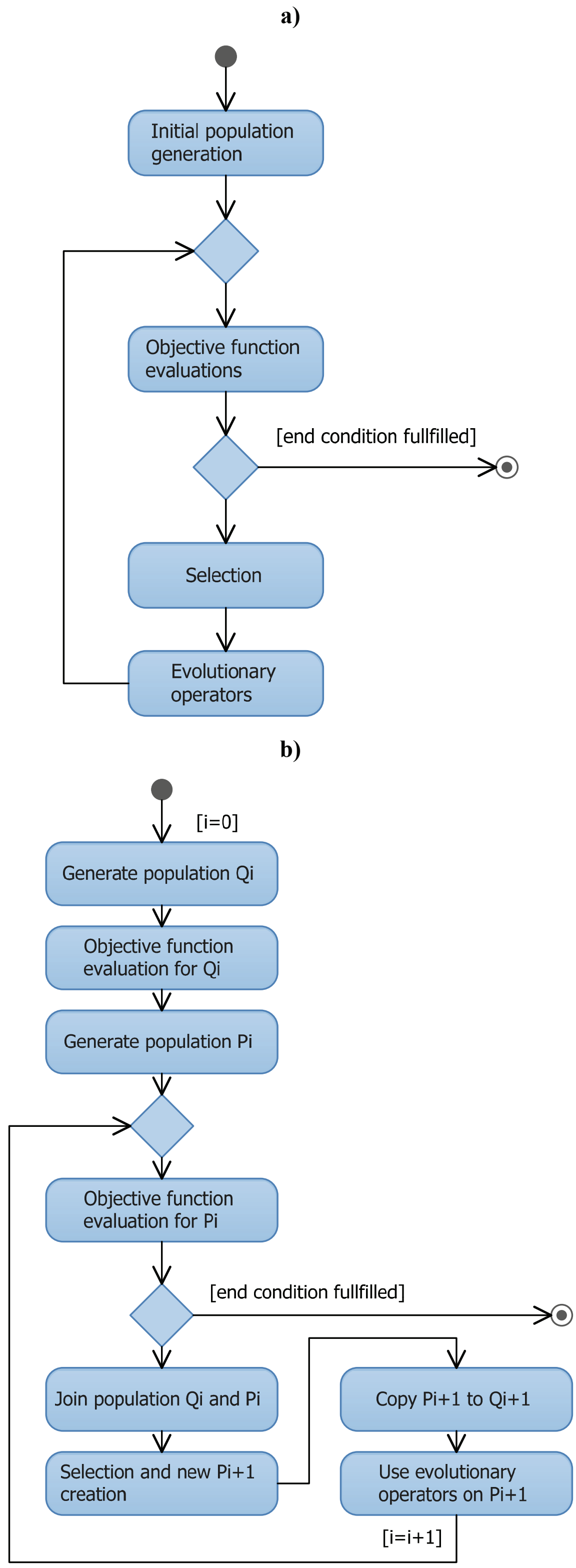


Fig. 1. The flowcharts of the evolutionary algorithms: a) for single objective and b) multiobjective optimization

optimization (MOO). A multiobjective optimization problem can be defined as a task of finding a set of design variables $\mathbf{x}$ which optimize a set of objective functions $f(\mathbf{x})$ and simultaneously satisfies a set of constrains $g(\mathbf{x}), h(\mathbf{x})$ (3)(4). One can be written as follows:

Find: $\mathbf{x} = [x_1, x_2, \dots x_n]^T$ (5)

To minimize or maximize:

$$f(\mathbf{x}) = \left[f_1(\mathbf{x}), f_2(\mathbf{x}), \dots, f_k(\mathbf{x}) \right]^T \tag{6}$$

where n is the number of design variables, k the number of objective functions, m the number of inequality constraints, and p the number of equality constraints.

Multi-objective optimization deals with multiple conflicting objectives and usually the optimal solution of one of the objectives is not necessarily the optimum for any of the other objectives. In such an approach, instead of one optimal solution a number of solutions are optimal. These solutions are called the Pareto-optimal solutions. The evolutionary algorithm is an ideal candidate for finding the Pareto optimal solution because it works on the population of individuals in each generation.

Optim library is also prepared for solving multiobjective tasks (Długosz, 2011). It is implemented as a multiobjective evolutionary algorithm. The algorithm consists of two parts: an initialization and a main loop. figure 1b shows the flowchart of the multiobjective evolutionary algorithm. In the initialization step, besides determining all settings of the algorithm, populations Q_i and P_i are generated and the fitness functions are evaluated for population Q_i. In the main loop, after evaluation fitness functions for P_i and checking stop conditions, populations Q_i and P_i are combined. Selection is performed on the set R_i, which is two times bigger than P_i. The nondominated sorting procedure is used for classification of the individuals in population. Moreover to preserve diversity in the population a crowding coefficient is calculated for each solution (Deb et al., 2002). The individuals from the population R_i are put to P_{i+1} on the basis of the nondomination level and the crowding coefficient. Individuals from P_{i+1} are copied to Q_{i+1} and then evolutionary operators change the population P_{i+1}. Two types of mutation are used: uniform and Gaussian; and two types of crossover: simple and arithmetic. The algorithm works until stop condition in not fulfilled. Compared to the NSGAII the proposed implementation has more evolutionary operators. The other difference between these algorithms is related to the formation of population P_{i+1}. There is no binary tournament selection operator in *Optim* library. The algorithm was tested on several benchmark problems and some engineering problems. The results obtained by using proposed library in most cases are better comparing to the results obtained by using NSGAII (Długosz, 2011).

3. OPTIM LIBRARY

The library is written in C++. The classes for bioinspired algorithms are prescribed. The library is under development and will contain also other bioinspired and classical algorithms.

The typical evolutionary algorithm implemented by user should follow the scheme:
- initialize of an evolutionary object,
- set the box constraints by using method `setboxconst`,
- execute method for random chromosome creation `rand_starting_pop`,
- do in loop:
- evaluation of objective function,
- execute a method connected with the selection procedure e.g. `range_selection`,
- execute a method connected with evolutionary operators `eaoperators`.

The multiobjective evolutionary algorithm implemented by user should follow the scheme:
- initialize of an evolutionary object,
- set the box constraints by using the method `setboxconst`,
- determine parameters of the evolutionary algorithm, e.g. probability of the uniform mutation
- set the size of the population, number of generation, number of objective functions `init`,
- execute a method for the random chromo some creation of the population Q_i `rand_starting_pop`
- execute a method for evaluation of the objective functional Q_i `evaluateobjectivedesigns`
- execute a method for the random chromosome creation of the population P_i `copytoprev`, `rand_starting_pop`

do in loop:
- execute a method for evaluation of the objective functional P_i `evaluateobjectivedesigns`
- execute a method for combining populations Q_i and P_i `joindesignsprevdesigns`
- execute a method connected with the selection procedure `multio_selection`.
- execute a method connected for the copying population P_{i+1} to Q_{i+1}
- execute a method for applying evolutionary operators `eaoperators`

The objective evaluation can be performed in the parallel way (Kuś and Burczyński, 2008). The library contains some examples of the parallel evaluation of an objective function with use of threads. The examples can be adopted by the user.

4. FORMULATION OF ENGINEERING OPTIMIZATION PROBLEMS

4.1. Identification of material properties of trabeculae in femur bone

The femur bone is build from the cortical and trabecular bone (Ilic et al., 2010, Tsubota et al., 2003). The trabecular bone is a porous structure containing trabeculae. The goal of identification formulated as an optimization problem, is to obtain material properties for the single trabeculae. The identification is based on the homogenized orthotropic material properties of representative volume element characteristic for the trabecular bone. The computational homogenization is used in the example (Madej et al., 2008, Terada et al., 2001). The more detailed description of multiscale modelling of the bone with use of multiscale analysis can be found in (Burczyński et al., 2010). One should identify Young's modulus E and Poisson's ratio ν of the single trabeculae having information about measured material parameters c_{ij} in the considered problem. The problem is formulated as a minimization task:

$$\min_{E,\nu} F \tag{7}$$

where the objective function is formulated as follows:

$$F = \sum_{i=1}^{n} \left| a_i - \hat{a}_i \right| \tag{8}$$

a_i are computed homogenized RVE material properties,

$\hat{a}_i$ are RVE homogenized material properties from the macromodel.

Table 1. The homogenized material properties of the RVE.

Material parameter	c_{11}	c_{12}	c_{13}	c_{22}	c_{23}	c_{33}	c_{44}	c_{55}	c_{66}
Value [MPa]	1002.0	321.0	239.0	902.0	239.0	604.0	633.0	457.0	457.0

For the homogenized orthotropic material (# of material parameters is 9):

$$a_i = \{c_{11}, c_{22}, c_{33}, c_{12}, c_{13}, c_{23}, c_{44}, c_{55}, c_{66}\} \tag{9}$$

The searched materials parameters – Young's modulus E and Poisson's ratio ν of the single trabeculae create a chromosome

$$ch = [g_1, g_2] \tag{10}$$

where g_i (i = 1,2) are genes: g_1 – Young's modulus E, g_2 – Poisson's ratio ν.

Table 2. Constraints for the design variables and the evolutionary algorithm parameters.

Design variable	Range	Parameter	Value
E	<1000 ÷ 10000>	# of individuals	20
		# of iterations	100
ν	<0.1 ÷ 0.4>	probability of simple crossover	0.9
		probability of Gaussian mutation	0.9
		probability of uniform mutation	0.1

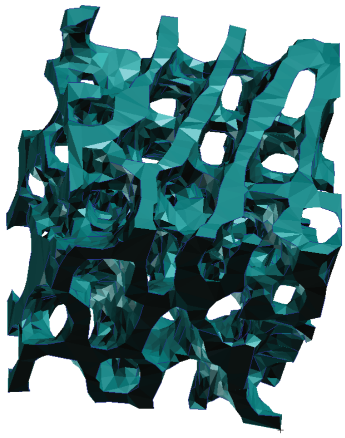


Fig. 2. The RVE model for trabecular bone.

Table 3. Actual and found material parameters of the trabeculae.

Material parameters	Actual	Found	Error %
E [MPa]	3300.0	3305.5	0.16
ν	0.330	0.329	0.30

The RVE geometry is presented in figure 2. The homogenization analysis is performed with use of FEM (Zienkiewicz et al., 2005). The orthotropic material properties for RVE are shown in table 1, whereas table 2 contains parameters of the algorithm. The results obtained after identification process are shown in table 3.

4.2. Multiobjective shape optimization of piezoelectric structures

The piezoelectric materials couple mechanical and electric fields together (Tiersten 1969). An electric field is produced when a structure is deformed and conversely it deforms when it is subjected to an electric field. Piezoelectric materials are widely used as sensors and actuators in smart structures and micro-electro-mechanical systems (MEMS). The piezoelectric phenomenon is also applied in ultrasonic transducers, electromechanical filters and microphones. Static analysis of piezoelectric materials requires the solution of coupled electric and mechanical partial differential equilibrium equations (Gaul et al 2000). These equations, arbitrary geometries and boundary conditions, are usually solved by numerical methods. To solve such a problem the finite element method (FEM) is used. Coupled field equations of static piezoelectricity can be expressed as follows (Kögl et al 2002):

$$\begin{bmatrix} \mathbf{K}_{uu} & \mathbf{K}_{u\varphi} \\ \mathbf{K}_{\varphi u} & \mathbf{K}_{\varphi\varphi} \end{bmatrix} \begin{bmatrix} \mathbf{u} \\ \mathbf{\Phi} \end{bmatrix} = \begin{bmatrix} \mathbf{F}_u \\ \boldsymbol{\rho}_\varphi \end{bmatrix} \tag{11}$$

where $\mathbf{K}_{uu}$ is the mechanical stiffness matrix, $\mathbf{K}_{u\varphi}$, $\mathbf{K}_{\varphi u}$ is the piezoelectric stiffness matrix, $\mathbf{K}_{\varphi\varphi}$ is the dielectric stiffness matrix, $\mathbf{F}_u$ is the force vector and $\boldsymbol{\rho}_\varphi$ is the charge flux vector.

Solution of the problem must be completed with the proper boundary conditions, both mechanical and electric. The first one is specified by displacements and loads, whereas the second one is specified by the electric potential and the charge flux density.

For the multicriteria optimization problem three different functionals are proposed: the minimum volume of the structure (12), the minimization of the maximal value of the equivalent stress (13) and the maximization of vertical deflection of the structure (14)

$$\min_x \quad f_1 \overset{def}{=} \int_\Omega d\Omega \tag{12}$$

$$\min_x \quad f_2 \overset{def}{=} \max\left(\sigma_{eq}\right) \tag{13}$$

$$\max_x \quad f_3 \overset{def}{=} \max\left(u_i\right) \tag{14}$$

Functionals f_1, f_2 and f_3 are calculated on the basis of results from numerical simulation. FEM software Ansys Multiphysics is used for obtaining solution of the piezoelectric problem. For the preparation of the geometry, finite element mesh, boundary conditions and for calculation of objective functionals some additional codes are written in APDL (Ansys Parametric Design Language).

The piezoelectric actuator presented in figure 3 is considered. The problem is considered as a two dimensional plane stress analysis task. The plate is 100 mm long, 1 mm thick and is made of aluminum. The left side is fixed in the space in all degrees of freedom. Three electrodes made of the piezoelectric material PZT4 are glued to the plate on the top of the surface. A potential difference 1000V is applied to the electrodes, which makes the plate bend. The multicriteria optimization problem is to determine the width, the thickness and locations of the piezoelectric parts. Nine design variables are assumed (figure 3). The multiobjective problem is solved taking into account three functionals (12) (13) (14) simultaneously. Table 3 contains limitations of the design variables and values of the multicriteria algorithm parameters. Figure 4 presents a set of the Pareto optimal solutions. Table 4 contains value of design variables for the three selected extreme points. Table 4 contains also final shapes, locations of the PZT4 electrodes and stress distribution in this area.

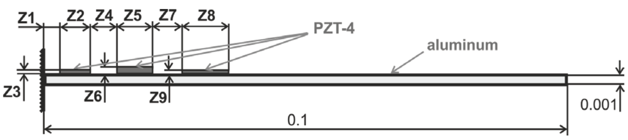


Fig. 3. *Model of piezoelectric actuator.*

Table 3. *The range of the design variables and values of the Optim parameter.*

Design variable	Range	Parameter	Value
Z1, Z4, Z7	<0.0001 ÷ 0.01>	# of individuals	200
		# of iterations	200
Z2, Z5, Z8	<0.001 ÷ 0.1>	probability of simple crossover	0.1
		probability of arithmetic crossover	0.1
Z3, Z6, Z9	<0.0005 ÷ 0.002>	probability of uniform mutation	0.1
		probability of Gaussian mutation	0.7

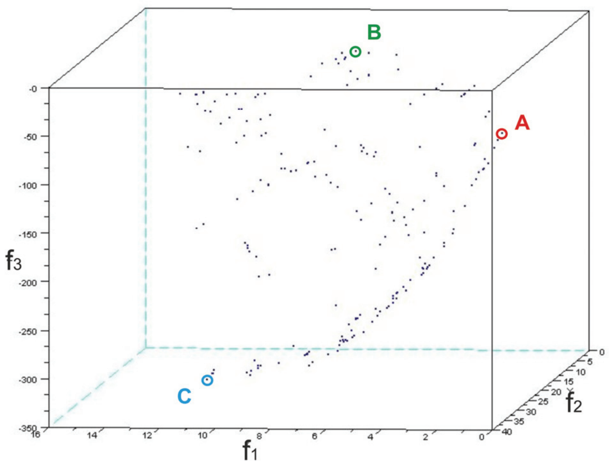
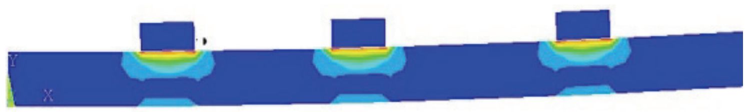
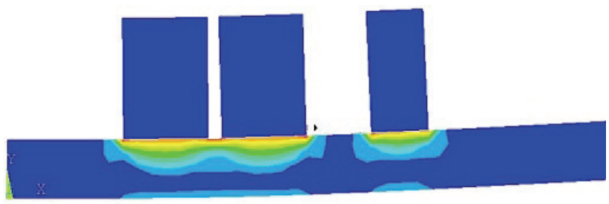


Fig. 4. Set of Pareto optimal solutions

Table 4. Results of the optimization for extreme points A, B and C.

Point **A** (Z1; Z2; ...; Z9) 0.00238; 0.001; 0.0005; 0.00245; 0.001; 0.0005; 0.00305; 0.001; 0.0005	
Point **B** (Z1; Z2; ...; Z9) 0.00198; 0.00148; 0.00199; 0.00021; 0.00147; 0.00199; 0.00109; 0.001; 0.002	
Point **C** (Z1; Z2; ...; Z9) 0.0001; 0.00581; 0.0005; 0.0001; 0.00515; 0.0005; 0.0001; 0.00999; 0.0005	

5. CONCLUSIONS

The applications of the Optim library in optimization problems were presented in the paper. The single objective optimization algorithm was used in identification of material properties of the trabecular bone. The piezoelectric actuator was optimized with the use of the multiobjective evolutionary optimization algorithm. The Optim library is under development, but stable versions which include some of presented algorithms can be downloaded from web pages www.multiscale.polsl.pl.

ACKNOWLEDGEMENT

The research is partially financed from the Polish science budget resources as the projects N N519 383 836 and R07 0006 10.

REFERENCES

Burczyński, T., Kuś, W., Brodacka, A., 2010, Multiscale modeling of osseous tissues, *J. Theor. Appl. Mech.*, 48 (4), 855-870.

Deb, K., Pratap, A, Agarwal, S., and Meyarivan, T, 2002, A fast and elitist multi-objective genetic algorithm: NSGA-II, *IEEE Transaction on Evolutionary Computation*, 6 (2), 181-197.

de Castro, L. N, Timmis, J., 2003, Artificial immune systems as a novel soft computing paradigm, *Soft Computing*, 7 (8), 526-544.

Długosz, A., 2011, Multiobjective evolutionary optimization of MEMS structures, *Computer Assisted Mechanics and Engineering Sciences*, (in print)

Gaul, L., Kögl, M., 2000, A boundary element method for transient piezoelectric analysis. Engineering, *Analysis with Boundary Elements*, 24, 591-598.

Ilic S., Hackl, K., Gilbert, R., 2010, Application of the multiscale FEM to the modeling of cancellous bone; *Biomech. Model. Mechanobiol.*, 9, 87–102.

Kennedy, J., Eberhart, R. C., Shi Y., 2001, *Swarm Intelligence*, Morgan Kaufmann Publishers.

Kuś, W., Burczyński, T., 2008, Parallel bioinspired algorithms in optimization of structures, *Lecture Notes in Computational Sciences*, vol. 4967, Springer, 1285-1292.

Madej, Ł., Mrozek, A., Kuś, W., Burczyński, T., Pietrzyk, M., 2008, Concurrent and upscaling methods in multi scale modelling - case studies, Computer Methods in Material Science, 8 (1), 1-10.

Michalewicz, Z., 1996, *Genetic algorithms + data structures = evolutionary algorithms*, Springer-Verlag, Berlin.

Kögl, M., Gaul, L., 2002, *Smart structures: applications and related technologies, chapter: Piezoelectric analysis with FEM and BEM*, Springer Verlag.

Terada, K., Kikuchi, N., 2001, A class of general algorithms for multi-scale analyses for heterogeneous media, *Comp. Meth. Appl. Mech. Eng.*, 190, 5427-5464.

Tiersten, H.F., 1969, *Linear piezoelectric plate vibrations*, Plenum Press.

Tsubota, K., Adachi, T., Nishiumi, S., Tomita, Y., 2003, Elastic properties of single trabeculae measured by micro-three-point bending test, *Proc. Int. Conf on Advanced Technology in Experimental Mechanics*, Nagoya, CD ROM.
Zienkiewicz, O. C, Taylor, R. L., Zhu, J. Z., 2005, *The Finite Element Method: Its Basis and Fundamentals*, 6th Edition, Butterworth-Heinemann.

OPTIM – BIBLIOTEKA INSPIROWANYCH PRZEZ NATURĘ ALGORYTMÓW OPTYMALIZACYJNYCH W ZASTOSOWANIACH INŻYNIERSKICH

Streszczenie

W artykule przedstawiono bibliotekę Optim, dedykowaną obliczeniom inspirowanym biologicznie. Biblioteka ta jest dedykowana do stosowania w inżynierskich zadaniach optymalizacji. W artykule przedstawiono optymalizację jedno i wielokryterialną z użyciem algorytmów ewolucyjnych. Przedstawiono również zastosowania opisanych metod w zagadnieniu identyfikacji parametrów materiałowych kości beleczkowej oraz optymalizacji kształtu siłownika wykonanego w technologii MEMS.

Received: November 14, 2010
Received in a revised form: November 17, 2010
Accepted: November 22, 2010