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AKAPIT

COMPUTER METHODS IN MATERIALS SCIENCE
Informatyka w Technologii Materiałów

Vol. 10, 2010, No. 4

INVESTIGATION ON THE EFFECTS OF PROCESS PARAMETERS ON THE THROUGH-THICKNESS SHEAR STRAIN IN SINGLE POINT INCREMENTAL FORMING USING DUAL LEVEL FE MODELLING AND STATISTICAL ANALYSIS

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Abstract

Single point incremental forming (SPIF) is a process with the capability to form complex geometries using a tool of very simple geometry, without the need for a matching die. At present, through-thickness modes of deformation and the effects of process parameters on through-thickness shear are not clear. The objectives of this report are firstly, to define the most critical working parameters that influence the through-thickness shear strains and secondly to obtain the optimal combination of these parameters that achieve maximum through-thickness shear deformation. Through-thickness shear strains are considered a direct indication of formability in the SPIF process. A design of experiment (DOE) approach is used to develop the study of various process parameters, in particular step-down size, sheet thickness, tool diameter, friction coefficient and strength coefficient. The example used is the manufacture of a truncated cone by SPIF. A dual-level FE modelling technique is used to simulate the process and obtain the corresponding shear strains for each combination of process parameters. The Analysis of Variance (ANOVA) method is used to analyze the results and obtain the most critical factors. The results show that the shear deformation, and hence the formability, could be increased by increasing the coefficient of friction and sheet thickness and decreasing the step-size down and tool diameter.

Key words: Single point incremental forming, Finite element analysis, through-thickness deformation modes, Design of experiments, statistical analysis, optimization

1. INTRODUCTION

Single point incremental forming (SPIF) refers to a die-less sheet forming process which can be used to form complex shapes using simple tools. The sheet metal is formed progressively by localized deformation imparted with a hemispherical forming tool. Thus, a variety of complex 3-D shapes can be formed as the tool moves through carefully controlled paths. A schematic illustration of the process is shown in figure 1.

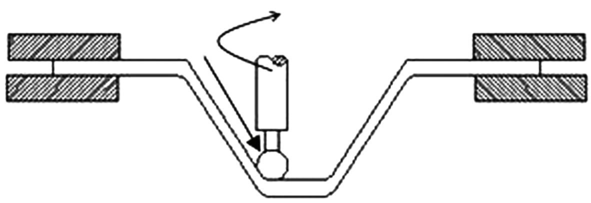

Fig. 1. *Single point incremental forming configuration.*

Knowledge of the process mechanics is crucial to understand how the final properties of the part develop and to identify the process forming limits.

ISSN 1641-8581

Many of the previous investigations have focused on an analysis of deformation mechanics through experimental investigations or on computational modelling using the finite element (FE) method. The work however, is contradictory with different views on the detailed deformation modes. In the analysis of stress and strain history reported by Bambach et al. (2003), Hirt et al. (2003) and later Ambrogio et al. (2004), it was suggested that stretching and thinning are the dominant modes of deformation and all shear strain components through the thickness are negligible. Conversely, the experimental measurements of Allwood et al. (2007) indicated that high values of transverse shear are present through the thickness. Jackson and Allwood (2009) also suggested that significant shear strain on planes perpendicular to, and parallel to, the tool direction could be a major contribution to the increase in the forming limit of the material in this process.

These shear strains have also been found using FE modelling, but only when the models contained a sufficient number of through-thickness elements (Essa & Hartley, 2010a). In order to avoid excessive computational times a dual-level FE model was constructed for a truncated cone made by SPIF. The first-level involved the use of two elements through the thickness for a model of the full sheet, from which a segment was extracted and a second-level model used with seven elements through the thickness. It was found that the shear deformation through the sheet thickness increases as the tool diameter decreases and the friction coefficient increases.

The use of statistical tools such as design of experiments (DOE) and analysis of variance (ANOVA), have been shown to be useful approaches to study the effect of working parameters on formability in SPIF. Fratini et al. (2004), Ham and Jeswiet (2006, 2007), Filice et al. (2006) and Ambrogio et al. (2007) used statistical analysis methods to investigate the effect of process variables such as feed rate, spindle speed, step-down size, tool diameter and sheet thickness on formability in SPIF. In general, they concluded that, the formability increases by decreasing the feed rate, step-down size and tool diameter and increasing the sheet thickness. Hussain et al. (2009a) used the same statistical method, i.e. DOE and ANOVA and found similar results except for the effects of tool diameter and step-down size; they reported that the formability of a titanium sheet made by SPIF could be improved by using a large step-down size and large tool diameter. This agrees with the results obtained by Le et al. (2008). Fratini et al. (2004) examined the influence of material properties such as strength coefficient, strain hardening exponent, anisotropy, ultimate tensile strength and percentage change in area. Among these, the strain hardening exponent and percentage change in area were the most significant mechanical properties. A similar investigation by Micari (2004) reported that the strain hardening exponent and the interaction between strength coefficient and strain hardening are the only influences on the formability. Later, Hussain et al. (2009b) found that percentage tensile reduction could be related to formability in SPIF.

The current paper focuses on the through-thickness shear strain as the main indicator of formability in the SPIF process. An evaluation is conducted of five process parameters in SPIF of a truncated cone; these are step-down size, sheet thickness, tool diameter, friction coefficient and strength coefficient. The dual-level FE approach is used in conjunction with a statistical analysis of the results. The results are analyzed using the ANOVA method to identify the most critical working parameters. Additionally, using a min-max optimization method, the optimum working parameter setting that allows the maximum shear strain deformation is determined.

2. THE DUAL-LEVEL FE MODEL

A finite-element model is required that provides an accurate description of the through-thickness shear strains. It has been shown that a minimum of seven elements through the sheet thickness is necessary (Essa & Hartley, 2010a). Fewer than this will result in an under-estimation of the shear strains. However, the cpu time involved means that it is unrealistic to complete a full model of the process with this level of refinement.

The problem may be overcome through the use of a dual-level approach. In this technique, the full model is simulated for a number of successive loops of tool movement until a sufficient amount of plastic strain is generated. A segment of the sheet surrounding the forming tool, which includes both deformed and un-deformed regions, is then extracted. For this second-level FE model, the geometry of the sheet, the tool position and boundary conditions are defined by the full model. The deformation in the second-level model then continues as in the full model. The dual-level FE procedure is summarised below, and described in more detail by Essa and Hartley (2010a).

2.1. First-level FE model

In the example of a truncated cone (with a wall angle of 45° and a major diameter of 90 mm), a rectangular sheet with an edge length of 170 mm by 170mm and thickness of 1mm is used. The hemispherical forming tool has a diameter of 15mm. The initial configuration is shown in figure 2. In the numerical simulation, the forming tool is modelled as a rigid body while the blank sheet is has elastic-plastic material properties of aluminium alloy (Al-5251-H22). The Young's modulus is 70 GPa and Poisson ratio 0.34. The plastic behaviour of the material is assumed to be isotropic with a stress-strain curve of, $\sigma = 400\varepsilon^{0.19}$ MPa, the initial yield stress of 165 MPa. Where, σ is the flow stress and ε is the plastic strain. The density of the sheet material is 2700kg/m^3. For simplicity, anisotropic, thermal and rate effects are not included in the present model.

The initial sheet blank is completely fixed at the sheet flange where it is constrained by displacement boundary conditions such that it cannot move in any of the XYZ directions. The flange is clamped to give a smaller region of 150 mm x 150 mm free to deform. The tool paths for the forming tool are generated using Matlab software and applied in Abaqus/CAE. The forming tool is assigned to move at 30 mm/s. The tool path is a complete circular path through 360º followed by a downward translation of 1 mm. The radius of the circular path reduces each time the tool moves down by the step-down size, i.e. 1 mm. Surface to surface contact between the forming tool and sheet surface is assumed and coulomb friction is set with a friction coefficient of 0.05. In the first-level (full) FE model, the initial sheet was meshed with 17298 elements (8-node solid element with reduced integration and hourglass control, C3D8R) with two elements through the thickness. The simulations were performed using Abaqus/Implicit on an Intel® CoreTM Dual computer with a 3 GHz CPU.

Figure 3 shows the von Mises stress distribution in the deformed sheet at the end of the process. A largely uniform stress distribution develops along the cone wall with higher values around the last tool path.

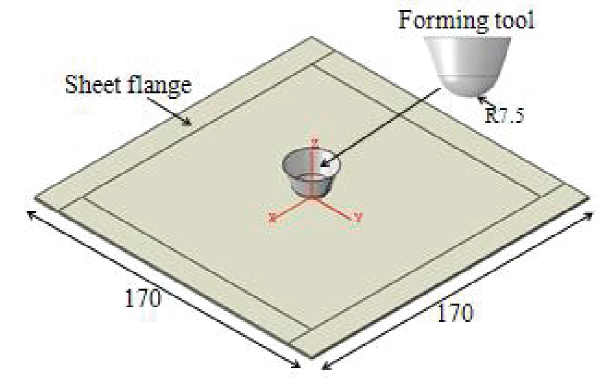

Fig. 2. The configuration of the full, first-level, 3-D FE model of SPIF to produce a truncated cone (dimensions in mm)

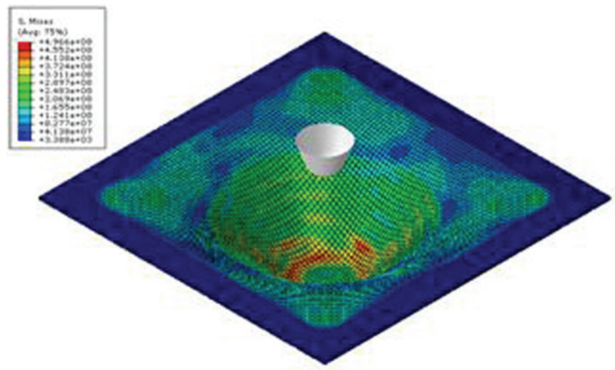

Fig. 3. Von Mises stress (Pa) distribution in the fully deformed truncated cone (Essa & Hartley, 2010a).

The thickness distribution along the central plane of the deformed sheet is shown in figure 4. It can be seen that the sheet thinning increases as the cone depth increases. At a certain point near to the cone base, less thinning is apparent and the thickness is very close to its original value.

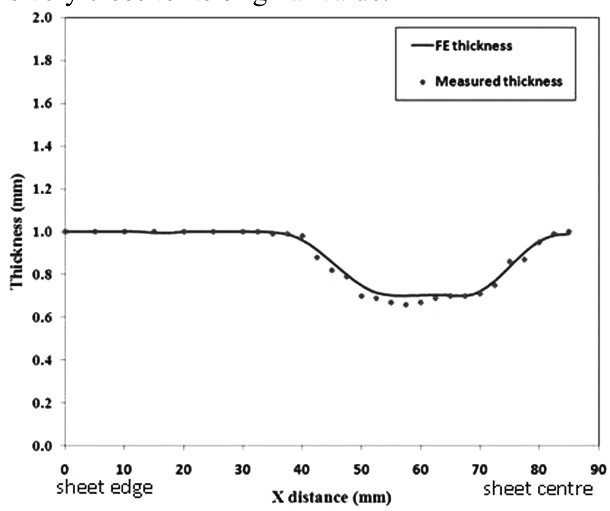

Fig. 4. Thickness distribution along central plane of the 45° truncated cone (FE data is from Essa and Hartley (2010a), experimental data was taken from a formed sample kindly provided by Julian Allwood and Omer Music at the Cambridge University Institute for Manufacturing).

2.2. The second-level FE model

A minimum of three successive paths of deformation in the full model were required to ensure that sufficient plastic strain was generated to provide a permanent deformed shape (figure 5a). A 55 mm x 55 mm segment of the sheet was then extracted (figure 5b) and the nodal coordinate points of this segment were used to define the surfaces of the new model, as shown in figure 5c, within which the second-level FE mesh was created. This model was meshed with 7 elements (type C3D8R) through the thickness, with a total of 25137 elements, as shown in figure 5(d). The forming tool is set to move a distance of 25mm along an arc at the same speed as in the full model. The sheet edges are constrained to have zero displacement, ensuring boundary conditions similar to the model of a full cone.

The Von Mises stress distribution in the second-level model, is shown in figure 6(a), revealing the distinct annulus of high localised stresses typical of the process. A cross-section along the T-Z plane, located near to the initial tool position, shown in figure 6(b), illustrates the shear deformation through the thickness. The enlarged view in figure 6(c) highlights the significant shear deformation compared to the elements in the un-deformed region. Figure 6(c) shows the shear deformation in the 1-3 plane, γ_{13}, and the 2-3 plane, γ_{23}, for this set of elements. A control element in the middle of this set, as shown in figure 6(c), is selected from which to extract the strain history.

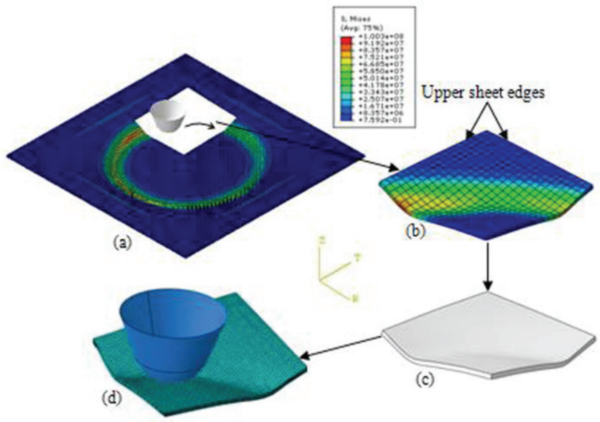

Fig. 5. The four stages in the process of constructing the second level FE model for a truncated cone (Essa and Hartley, 2010a).

The strain history of the selected element is shown in figure 7, which demonstrates an increasing strain until the tool passes the element location, after which no further plastic deformation is imposed. The distributions of the shear strains γ_{13} and γ_{23} through the thickness are shown in figure 8. The maxima are found at the top surface in contact with the tool, reducing to small values at the opposing surface.

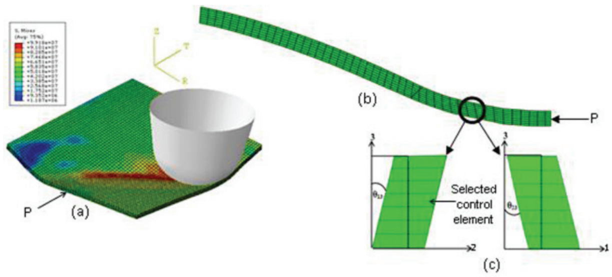

Fig. 6. (a) Von Mises stress (Pa) distribution, (b) edge view in the T-Z (1-3) plane near the initial tool position and (c) illustration of shear deformation (Essa and Hartley, 2010a).

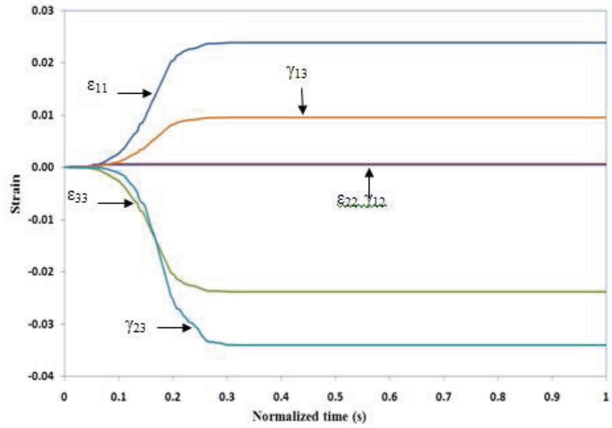

Fig. 7. Strain history of the second-FE model of the truncated cone.

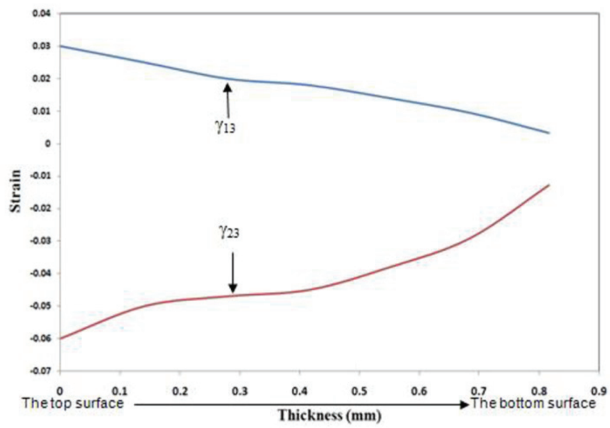

Fig. 8. Shear strain distribution along the sheet thickness of the second-level FE model of the truncated cone.

3. PARAMETRIC INVESTIGATION

In single point incremental forming there are a number of parameters that affect the process mechanics. These could be classified into process parameters (tool feed rate, tool diameter, incremental step-down and lubrication), material parameters (strength coefficient, strain hardening, anisotropy and Young's modulus) and design parameters (sheet thickness and final product geometry). Based on a preliminary study (Essa & Hartley, 2010a), the feed rate was excluded from the investigation as it did not show any significant effect on the shear deformation for the example shown here. Except for the feed rate, all process parameters are included in the present plan while the strength coefficient is chosen to represent the material parameters. Since the analysis will be conducted for a fixed product geometry, i.e. a truncated cone, the sheet thickness is chosen to represent the design parameters. A design of experiment (DOE) approach requires a range of each parameter to be selected. These cover most of the experimental investigations appearing in the literature. Table 1 shows the different process factors and the corresponding levels.

Table 1. *Process factors and corresponding levels.*

	P Values	
	Shear strain γ_{13}	Shear strain γ_{23}
Step-down size (A)	0.0622	0.01
Friction coefficient (B)	0.0776	0.0001
Tool diameter (C)	0.0003	0.001
Sheet thickness (D)	0.0001	0.0001
Strength coefficient (E)	0.594	0.351
Significant interactions	(C*D) 0.001	(B*C) 0.028 (B*D) 0.0002 (C*D) 0.032

Table 2. *Significant factors and corresponding P-values*

Factor \ Level	Low level	Intermediate level	High level
Step-down size (mm)	0.2	1.1	2
Friction coefficient	0.02	0.26	0.5
Tool diameter (mm)	10	15	20
Sheet thickness (mm)	0.4	1.7	3
Strength coefficient (MPa)	170	400	670

Response variables are also required; these are the Quality Characteristics (QC), which generally, refer to the measured results. In the present investigation, the response variables are the shear strain in a plane normal to the tool movement i.e., γ_{13} and the shear strain in a plane parallel to the tool movement i.e., γ_{23}. These were chosen as values representative of the shear deformation through the thickness. Based on the use of a design of experiment (DOE approach), the Box-Behnken design technique (Ham & Jeswiet, 2007) was used to generate a set of experiments for combinations of the five process factors which are varied over the three levels, i.e. low level, intermediate level and high level as shown in table 1. The result of running the first Box-Behnken design produced 46 different combinations of these factors. Each of these combinations is assessed through the use of a separate dual-level FE simulation.

An analysis of variance (ANOVA) was performed on the design of experiments to identify the significant factors and interactions. A significance level of 5% was used. In statistical hypothesis testing, the P-value is the probability of obtaining a result at least as extreme as the one that was actually observed, assuming that the null hypothesis is true (Lind et al., 2005). The fact that P-value are based on this assumption is crucial to their correct interpretation. A smaller P-value (less than 5%) is associated with increased importance of the factor. Table 2 shows the P-values for the significant factors and interactions. According to the R-Square and adjusted R-Square values, the Box-Behnken statistical analysis highlighted that a quadratic model provides a very good description of the evolution of the quality characteristics with respect to the working parameters. The R-Square and adjusted R-Square values for all responses did not fall below 89%.

The analysis of variance shows that the shear strain γ_{13} is affected by tool diameter, sheet thickness and the interaction between the tool diameter and sheet thickness. While, the shear strain γ_{23} is affected by step-down size, friction coefficient, tool diameter and sheet thickness. Additionally, it is affected by the following interactions; friction coefficient and tool diameter, friction coefficient and sheet thickness, tool diameter and sheet thickness. On the other hand, the strength coefficient seems to have no significant effect upon the shear deformation and hence the formability, confirming the observations of Fratini et al. (2004) and Hussain et al. (2009b).

4.1. Shear strain γ_{13}

Figure 9 shows the effect of tool diameter on the through-thickness shear strain γ_{13}. The solid line represents the quadratic model while the dotted lines represent the variation of the actual results. The results show that as tool diameter increases the shear strain γ_{13} decreases. Using a small diameter tool tends to localize the deformation underneath the tool and increase the local strains. As the tool diameter increases, the contact area increases and hence the contact pressure decreases. As a result of increasing the contact area, the deformation becomes more distributed and hence the forming forces decrease which leads to a decrease in the generated strains. The shear strain γ_{13} increases from 0.043 to 0.067 by decreasing the tool diameter from 20 mm to 15 mm and increases significantly to 0.184 by using a tool diameter of 10 mm. Similar results are found by Ham and Jeswiet (2006) and Le et al. (2008).

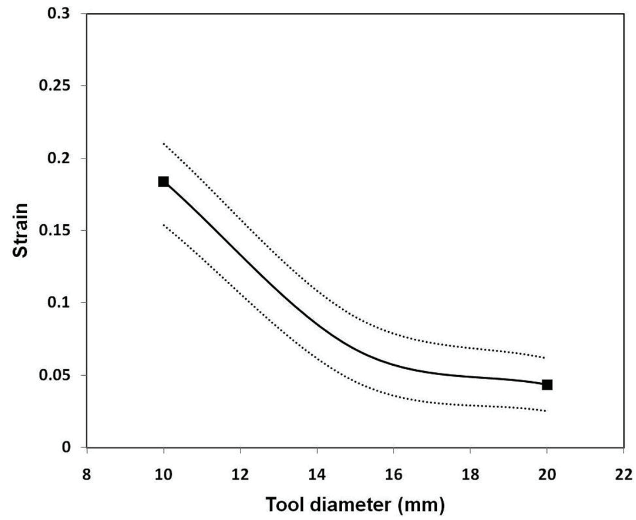

Fig. 9. Effect of tool diameter on the shear strain γ_{13}.

The effect of sheet thickness on the shear strain γ_{13} is shown in figure 10. It can be seen that the shear strain γ_{13} increases by increasing the sheet thickness. γ_{13} is the shear strain in the plane perpendicular to the tool movement and results from pushing the forming tool across the material along the wall angle. Additionally, as the sheet thickness increases, the maximum radial and thickness strains that the sheet can support without fracture and hence the maximum wall angle increase (Ham & Jeswiet, 2006). Therefore, increasing the sheet thickness will result in a significant increase in the shear strain γ_{13}.

The interaction between the sheet thickness and tool diameter has shown a significant effect on the shear strain γ_{13} as shown in figure 11. The contribution resulting from increasing the sheet thickness upon the shear strain γ_{13} becomes more significant by decreasing the tool diameter. Using a 3 mm sheet thickness and 20 mm tool diameter, the maximum shear strain that could be achieved is 0.124. For the same sheet thickness, the shear strain γ_{13} could be increased to 0.515 by decreasing the tool diameter to 10mm. This suggests that to increase the shear strain γ_{13} and hence the formability, it is recommended to minimize the tool diameter and maximize the sheet thickness. Similar findings have also been reported (Ham & Jeswiet, 2006), although not related to shear strain.

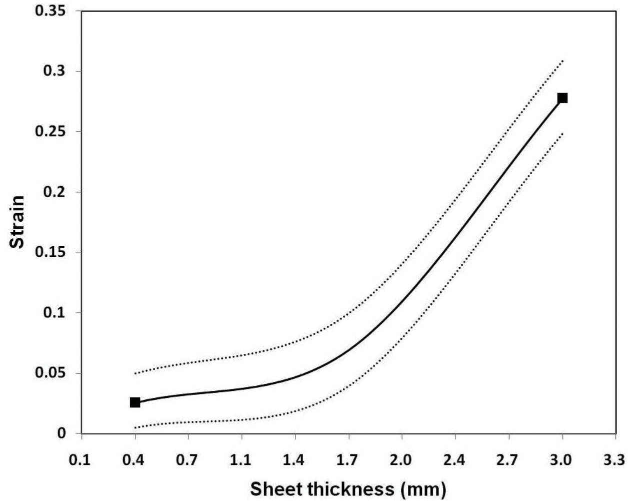

Fig. 10. Effect of sheet thickness on the shear strain γ_{13}.

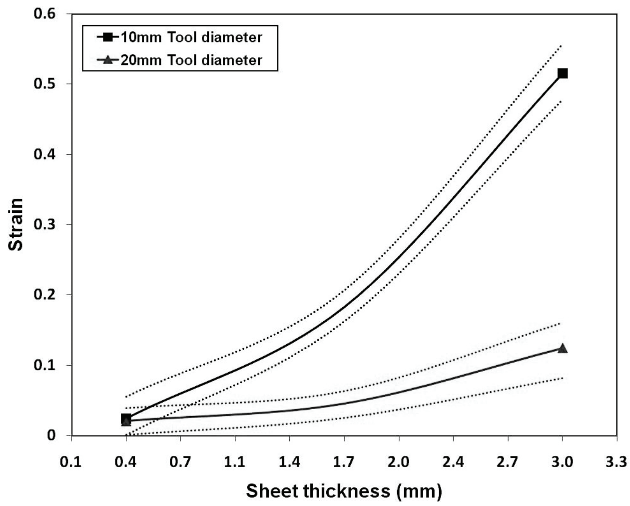

Fig. 11. Effect of the interaction between tool diameter and sheet thickness on the shear strain γ_{13}.

3.2. Shear strain γ_{23}

Figure 12 shows the effect of the step-down size on the shear strain γ_{23}. A decrease in the incremental step results in an increase in the shear strain. As the step-down size increases, the normal stresses in sheet the material at the forming tool increases and thus the sheet fracture takes place earlier. Additionally, a large step-down size results in a large loss in the material thickness i.e., sheet thinning, which leads to a reduction in the shear deformation that takes place along the sheet thickness. An alternative explanation for decreasing the shear strain γ_{23} could have been the following; as a result of using a large incremental step-down size, the material is partially deformed by pulling instead of by the forming tool. This leads to a decrease in the localised deformation and thus the shear strain γ_{23} decreases. This agrees with the results reported by Ham and Jeswiet (2006).

It can be seen that the friction coefficient has a significant impact on the shear strain γ_{23} as shown in figure 13. This is the shear strain in a plane parallel to the tool movement and results from the friction between the forming tool and sheet surface. Therefore, as the friction coefficient increases, the through-thickness shear strain γ_{23} increases. This agrees with previous results found by Allwood et al. (2007) and Jackson and Allwood (2009). They reported that the formability in SPIF could be increased by increasing the friction between the forming tool and sheet surface. However, Hussain et al. (2008) suggested that if the friction is too high, an unacceptable surface roughness will develop.

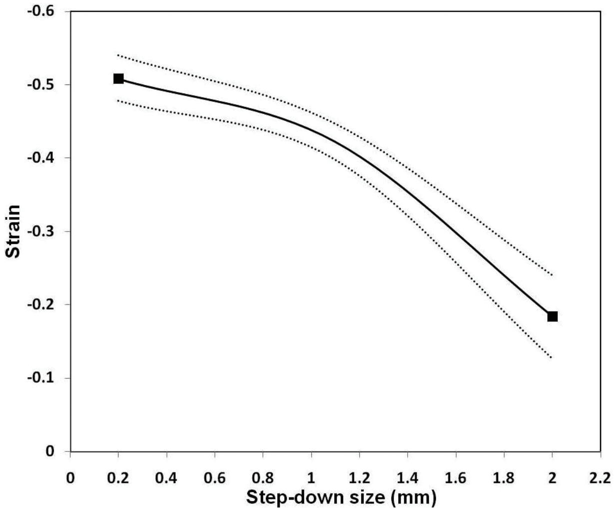

Fig. 12. *Effect of step-down size on the shear strain* γ_{23}.

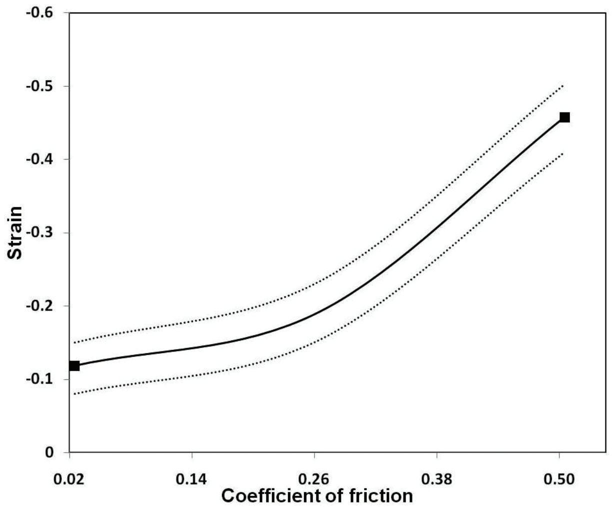

Fig. 13. *Effect of coefficient of friction on the shear strain* γ_{23}.

The effect of the tool diameter on the shear strain γ_{23} is shown in figure 14. Similar to the effect of tool diameter on the shear strain γ_{13}, shear strain γ_{23} increases by decreasing the tool diameter. As mentioned above, γ_{23} results from the friction between the forming tool and sheet surface and increases by increasing the friction coefficient. Therefore, as the tool diameter decreases the friction per unit area between the forming tool and sheet surface increases. This leads to an increase in the shear strain γ_{23}.

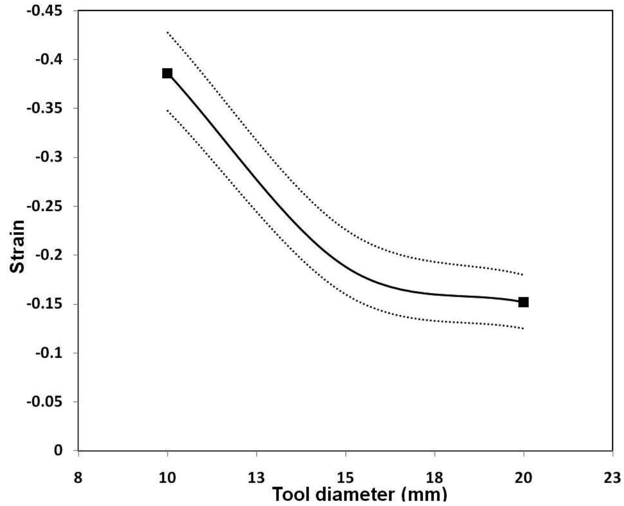

Fig. 14. Effect of tool diameter on the shear strain γ_{23}.

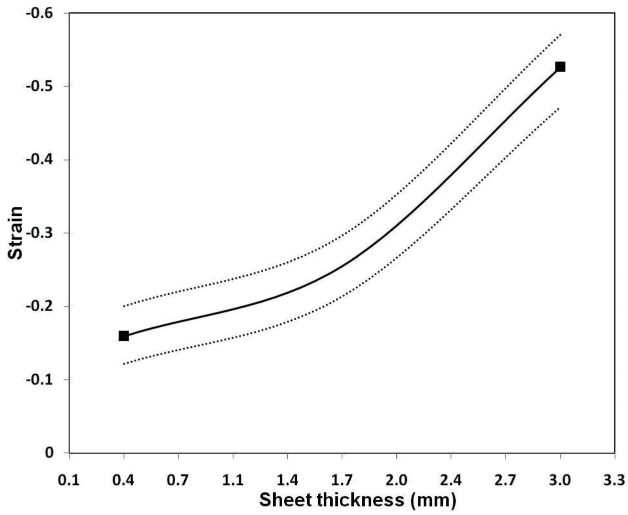

Fig. 15. *Effect of sheet thickness on the shear strain* γ_{23}.

Figure 15 shows the effect of the sheet thickness on the shear strain γ_{23}. The figure shows an increase in the shear strain γ_{23} by increasing the sheet thickness. In the results reported by Jackson and Allwood (2009), it was concluded that the shear strain γ_{23} increases as a result of overlapping contact area of the tool on successive passes. This overlapping contact area is expected to increase as a result of increasing the sheet thickness and decreasing the step-down size. Therefore, a large sheet thickness has a significant effect on increasing the shear strain γ_{23}.

The effect of the interaction between tool diameter and coefficient of friction on the shear strain γ_{23} is shown in figure 16. At a low coefficient of friction (0.02), the effect of the tool diameter on the shear strain γ_{23} is not clear. However, at a high coefficient of friction (0.5), the absolute value of the shear strain γ_{23} increased to approximately double that by decreasing the tool diameter from 20 mm to 10 mm. It means that the impact of the tool diameter upon the shear deformation becomes highly significant at higher values of the coefficient of friction. This suggests that the shear strain γ_{23} could be further increased by using a small tool diameter and a large coefficient of friction.

Figure 17 shows the effect of the interaction between sheet thickness and coefficient of friction on the shear strain γ_{13}. At 0.4mm sheet thickness and 0.02 coefficient of friction, the absolute value of shear strain γ_{23} is 0.238. This represents a small amount of shear strain compared to that of 1.433 at a 3 mm sheet thickness and 0.5 coefficient of friction. This suggests that using a large value of sheet thickness combined with a large value of coefficient of friction would increase the through-thickness shear strain γ_{23}.

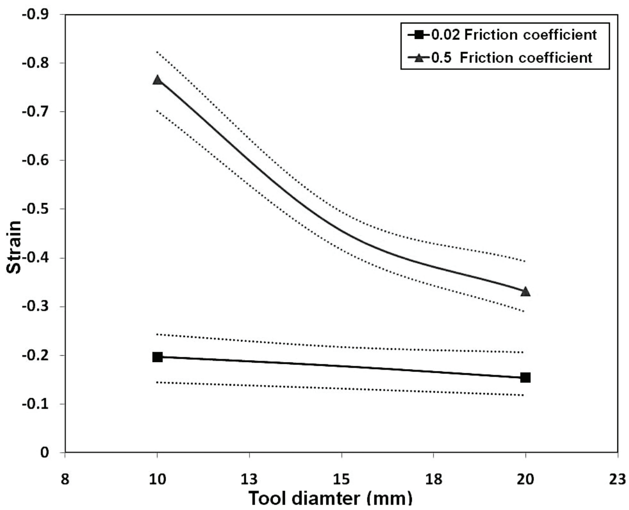

Fig. 16. Effect of the interaction between tool diameter and coefficient of friction on the shear strain γ_{23}.

Figure 18 shows the effect of the interaction between sheet thickness and tool diameter on the shear strain γ_{23}. As a result of using a large tool diameter and small sheet thickness the generated shear strain γ_{23} reduces. On the other hand, using a small tool diameter and large sheet thickness leads to a localisation of the deformation underneath the tool and an increase in the overlapping contact area of the tool on successive passes. Thus, the generated shear strain γ_{23} increases. Based on the previous results, the through-thickness shear strains and hence the process formability could be improved by increasing the sheet thickness and the coefficient of friction and decreasing the tool diameter and step-down size. With a better control of these parameters, a large amount of through-thickness shear deformation could be generated and further improvement of the process formability would be expected.

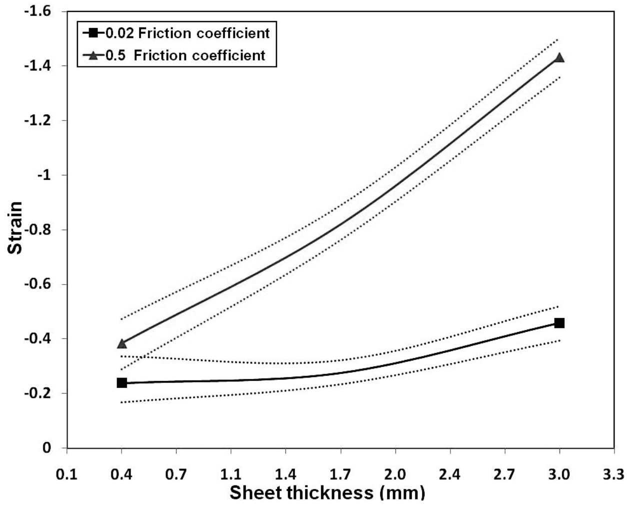

Fig. 17. Effect of the interaction between sheet thickness and coefficient of friction on the shear strain γ_{13}.

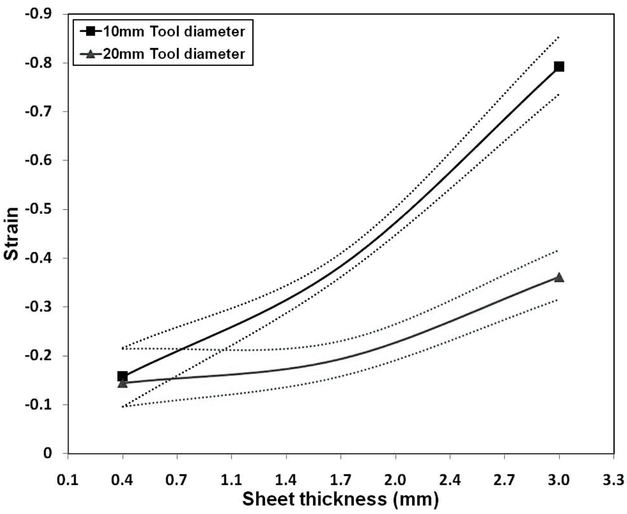

Fig. 18. Effect of the interaction between sheet thickness and tool diameter on the shear strain γ_{23}.

5. PREDICTION AND OPTIMIZATION OF THE THROUGH-THICKNESS SHEAR STRAIN

It is useful to develop an empirical model that can predict the value of each of the through-thickness shear strain components for any combination of the working parameters. As a result of using numerical factors in this study, it is possible to predict the equivalent shear strain components for any value of each parameter within the range studied even if it was not one of the pre-selected levels. Using a general second order polynomial equation, an empirical model can be developed based on the working parameters, i.e. step-down size, coefficient of friction, tool diameter, sheet thickness, strength coefficient and their interactions. Each parameter and interaction is multiplied by a coefficient as shown in equations (1) and (2). Although these empirical models do not determine the generated through-thickness shear strains at the end of deformation (only at the end of the second level FE model), they represent a ratio of the final shear deformation. Future investigations could be conducted to correlate these models to predict the through-thickness shear strains at the end of deformation.

$$\gamma_{13} = 0.134287 + 0.041008*\mathbf{A} - 0.21807*\mathbf{B} - 0.02944*\mathbf{C} + 0.145754*\mathbf{D} + 0.000195*\mathbf{E} - 0.03227*\mathbf{A}*\mathbf{B} + 0.000671*\mathbf{A}*\mathbf{C} + 0.009768*\mathbf{A}*\mathbf{D} - 4.8E\text{-}06*\mathbf{A}*\mathbf{E} - 0.03076*\mathbf{B}*\mathbf{C} + 0.32892*\mathbf{B}*\mathbf{D} - 1.1E\text{-}05*\mathbf{B}*\mathbf{E} - 0.01911*\mathbf{C}*\mathbf{D} - 4E\text{-}07*\mathbf{C}*\mathbf{E} - 4.6E\text{-}05*\mathbf{D}*\mathbf{E} - 0.02174*\mathbf{A}^2 + 0.628904*\mathbf{B}^2 + 0.001841*\mathbf{C}^2 + 0.04736*\mathbf{D}^2 - 1.7E\text{-}07*\mathbf{E}^2 \quad (1)$$

$$\gamma_{23} = 0.523286 + 0.229111*\mathbf{A} + 0.579018*\mathbf{B} - 0.17483*\mathbf{C} + 0.17483*\mathbf{D} - 0.0002*\mathbf{E} - 0.34656*\mathbf{A}*\mathbf{B} + 0.007258*\mathbf{A}*\mathbf{C} - 0.07342*\mathbf{A}*\mathbf{D} + 1.4E\text{-}05*\mathbf{A}*\mathbf{E} - 0.10893*\mathbf{B}*\mathbf{C} + 0.786695*\mathbf{B}*\mathbf{D} - 0.00034*\mathbf{B}*\mathbf{E} - 0.01955*\mathbf{C}*\mathbf{D} + 2E\text{-}05*\mathbf{C}*\mathbf{E} - 9.9E\text{-}05*\mathbf{D}*\mathbf{E} - 0.09806*\mathbf{A}^2 + 1.874758*\mathbf{B}^2 + 0.004124*\mathbf{C}^2 + 0.059989*\mathbf{D}^2 + 3.81E\text{-}08*\mathbf{E}^2 \quad (2)$$

Where, **A** is the step-down size, **B** is the coefficient of friction, **C** is the tool diameter, **D** is the sheet thickness and **E** is the strength coefficient.

R-Square, multiple correlation coefficients indicate that the model fit did not go below 89% for both models. This suggests a very good fit for the data points. The normal distribution of the residuals is one of the measures that used to check the efficiency of an empirical model. It has been used to verify the above empirical models as shown in figure 19. The residual plot should have a linear spread close to the normal distribution in order to accept the model results. The normal plots for γ_{13} and γ_{23} show a satisfactory deviation from normality and acceptable variance distribution. The results suggest that the developed models can be used to explore the design space of the working parameters.

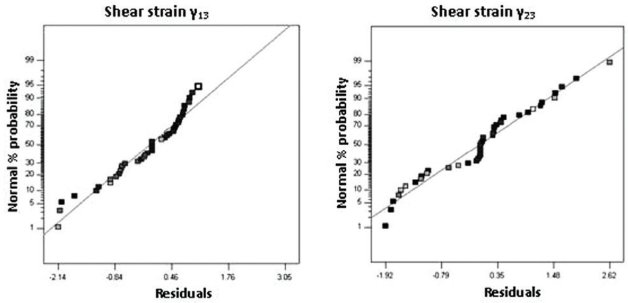

Fig. 19. *Normal plots of the residual for the γ_{13} and γ_{23} empirical models.*

Using a Min-Max optimization method (see for example, Essa & Hartley, 2010b), equations (1) and (2) are solved together to obtain the optimal setting of the working parameters that maximizes the through-thickness shear deformation. The objective function is to maximize the response variables i.e., γ_{13} and γ_{23} for a minimum coefficient of friction. This objective function helps to increase the through-thickness shear deformation and hence the formability at minimum surface roughness. All working parameters are constrained within their design space i.e., their pre-selected levels, and all strain components given the same weight i.e. same priority and importance. The optimal setting obtained is shown in table 3 while the corresponding through-thickness strains are presented in table 4. In order to validate the results, a single process simulation using the optimal working parameters is performed through the dual-level FE modelling approach. The through-thickness shear strains γ_{13} and γ_{23} are measured (observed strains) and compared to those predicted by the empirical models. The results show very good agreement between predicted and observed strains which give another check on the efficiency of the empirical models.

Table 3. *Optimal setting of the involved working parameters.*

	Step-down size (mm)	Coefficient of friction	Tool diameter (mm)	Sheet thickness (mm)	Strength coefficient (MPa)
Optimal setting	0.41	0.3	10	3	170

Table 4. *Predicted and observed shear strains.*

	shear strain γ_{13}	shear strain γ_{23}
Predicted strains	0.535	1.055
Observed strains	0.531	1.121

6. CONCLUSIONS

A dual-level FE model for single point incremental forming has been used to explore the shear deformation through the sheet thickness. A DOE approach was adopted to study the effect of process, design and material parameters on the through-thickness shear strains. This shear strain was used as a direct indicator of formability in the SPIF process.

This study demonstrated the following;

– The dual-level FE model revealed a significant magnitude of the through-thickness shear strains on a plane perpendicular to the tool movement (γ_{13}) of 0.01 and a plane parallel to the tool movement (γ_{23}) of 0.034.
– A significant increase in the shear strain γ_{13} was noticed by decreasing the tool diameter and increasing the sheet thickness.
– A higher shear strain γ_{23} was achieved by increasing the coefficient of friction and sheet thickness and decreasing the step-down size and tool diameter.
– A mathematical model was developed to describe the through-thickness shear strains for any combination of working parameters.
– The min-max optimization method allowed the optimal setting of the working parameters that maximize the through-thickness shear deformation for a minimum coefficient of friction to be obtained.

ACKNOWLEDGMENTS

The financial support of the UK ORSAS Scheme (K Essa) is gratefully acknowledged.

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ZASTOSOWANIE DWUPOZIOMOWEGO ROZWIĄZANIA MES I ANALIZY STATYSTYCZNEJ DO BADANIA WPŁYWU PARAMETRÓW PROCESU NA ROZKŁAD ODKSZTAŁCEŃ POSTACIOWYCH NA GRUBOŚCI BLACHY W PROCESIE PRZYROSTOWEGO KSZTAŁTOWANIA

Streszczenie

Jednopunktowa przyrostowa przeróbka plastyczna (ang. Single Point Incremental Forming - SPIF) jest procesem umożliwiającym formowanie wyrobów o złożonych kształtach w narzędziach o prostej geometrii, bez potrzeby zastosowania matrycy o kształcie takim jak wyrób gotowy. Zagadnienia stanu odkształcenia na grubości blachy oraz zależności odkształceń postaciowych od parametrów technologicznych w tym procesie nadal nie są w pełni rozeznane. Celem niniejszej pracy jest, w pierwszej kolejności, zdefiniowanie najbardziej krytycznych parametrów procesu, które wpływają na rozkład odkształceń postaciowych. Drugim celem jest wyznaczenie optymalnej kombinacji tych parametrów dającej maksimum odkształceń postaciowych. Te odkształcenia są uznawane za bezpośredni wskaźnik odkształcalności w procesach SPIF. Dla badania wpływu poszczególnych parametrów zastosowano metodę projektowania doświadczeń (ang. Design Of Experiment - DOE). W szczególności analizowano wpływ wielkości kroku narzędzia, grubości blachy, wymiarów narzędzia, współczynnika tarcia oraz wskaźnika wytrzymałości materiału. Jako przykład wybrano wytwarzanie stożka ściętego metodą SPIF. Dwupoziomowy model MES zastosowano do symulacji procesu i do wyznaczenia odkształceń postaciowych odpowiadających badanym kombinacjom parametrów. Analizę wyników wykonano metodami statystycznymi (ang. the Analysis of Variance - ANOVA) i wyznaczono najbardziej krytyczne parametry procesu. Wyniki wykazują, że odkształcenia postaciowe, a co zatem idzie odkształcalność, mogą zostać zwiększone poprzez zwiększenie współczynnika tarcia i grubości blachy oraz przez zmniejszenie kroku narzędzia i średnicy narzędzia.

Received: May 28, 2010
Received in a revised form: June 20, 2010
Accepted: July 16, 2010