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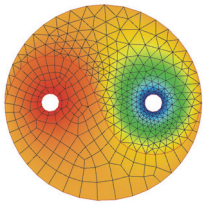

CRITICAL EVALUATION AND SENSITIVITY ANALYSIS OF RHEOLOGICAL MODELS OF HUMAN BLOOD

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Abstract

Critical analysis and classification of rheological models of human blood is the objective of the paper. In the first part of the paper main features of blood and their influence on modelling blood flow are discussed. Various models available in the scientific literature have been analysed and classified on the basis of the mathematical form of equations. Capabilities of the models to account for certain physical features of blood were evaluated. Power laws, commonly used in basic simplified simulations, have simple mathematical form and are not analysed. Among the remaining models, three groups were distinguished and selected for further analysis: Casson type models, be-exponential law and Quemada model. Sensitivity analysis was performed for these models using Morris OAT Design method. Sensitivity of the viscosity predicted by various models, with respect to the coefficients in these models and with respect to the external variable (shear rate), was determined. Importance of the investigated coefficients and their influence on models' predictions was evaluated. Suggestions concerning identification of coefficients in the models are given in conclusions.

Key words: rheological model of blood, numerical modelling, sensitivity analysis

1. INTRODUCTION

Blood is a non-Newtonian fluid with viscosity, which changes with changing shear rate. The non-Newtonian characteristic is due to the presence of various cells, the volume of which is about 45%. Thus, blood is a suspension of solid particles (cells) in liquid (plasma). When blood moves, the particles interact with each other and with plasma. Therefore, when modelling blood several aspects have to be considered: viscosity of the whole solution, viscosity of plasma, aggregation of red cells and their deformation. It makes modelling of blood flow very difficult.

Large number of publications dealing with rheological models of blood can be found in the scientific literature. Hundreds of models and equations describing the viscosity of blood have been developed during last few decades. Yilmaz & Gundogdu (2008) have presented nearly 70 models published in the second half of the last century. Good reviews of models of blood viscosity can also be found for example in (Cokelet, 1980; Zhang & Kuang, 2000). The primary analysis of the models gathered in Yilmaz & Gundogdu (2008) shows that majority of them were built by minor amendments of the existing models, while the mathematical form remains the same. It introduces confusion and difficulties with selection of the proper and realistic model for particular application. Thus, the main objective of the present work is performing the critical analysis of the rheological models of blood and classification of these models. The sensitivity analysis, which supplies information regarding impor-

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tance of different parameters in the models, was the second objective of this work.

This work is a part of the "Polish Artificial Heart" project coordinated by the Foundation of Cardiac Surgery Development. The project is a continuation of earlier research carried out in the Foundation, see (Gawlikowski et al., 2008). Thus, the perspective application of the results of the present work is simulation of blood flow through artificial heart chambers. Therefore, undoubtedly important problem of blood flow through capillary vessels is not considered in the paper.

2. KEY FEATURES OF BLOOD, WHICH INFLUENCE RHEOLOGICAL MODELLING

The features of blood, which should be taken into account in the numerical modelling of flow, are discussed below. Physiological parameters, which have an influence on the viscosity in the whole system of veins and arteries, are considered first. These parameters combine properties of plasma and red blood cells (RBCs), deformation and aggregation of RBCs and others.

2.1. Factors affecting viscosity of blood

The main factors, which determine the properties of blood, are:

- Hematocrit – volume fraction of red cells in blood. The hematocrit is expressed as the proportion of red blood cells to the total volume of blood and there is a measure of this fraction, e.g. physiological hematocrit given by Buchanan et al. (2000) is $\varphi = 0.44$ (the normal physiological range is: female: 0.37 – 0.47, male 0.42 – 0.52).
- Fibrinogen – the first factor of thrombosis. It is the protein of plasma produced in liver. Fibrinogen is involved in the final stage of thrombosis. It is transformed in the fibrinal protein (fibrin), which is a component of thrombosis. The physiological content of the fibrinogen in blood is from 2 to 4 g/l.
- Globulin – fraction of protein in plasma. Globulin is responsible for immunity mechanisms and it bonds glucose and fats. The physiological content of the globulin in blood is 0.15 – 0.3 g/l.
- TPMA – the Total Protein Minus Albumin. It is the mass of globulin and fibrinogen.

Blood is a multiphase fluid that is primarily composed of red blood cells (RBCs), white blood cells, and platelets suspended in plasma. Under normal, healthy conditions, a freely suspended RBC is a biconcave discoid with about 8 μm diameter and 2 μm thickness. RBCs constitute 40–45% of the total blood volume. Being highly deformable particles, RBCs can easily squeeze through the smallest capillaries having internal diameter less than RBCs characteristic size.

Due to the mentioned above features, blood is a fluid with a complex characteristic. With some simplifications it can be assumed that blood is a slime of elastic red blood cells (RBCs) in fluid (plasma). Red cells have a tendency to aggregation. Dimensions of aggregates can be up to 1 mm (Cokelet, 1980). Plasma is a nonuniform fluid and its composition has significant influence on the flow of blood. The particular nature of blood and the deformability of RBCs determine the overall rheological behaviour of blood. Having in mind the composition of blood, two areas of rheological modelling can be distinguished: one-phase and two-phase models. The size of the RBCs is the criterion, which distinguishes the two types of models. The following features of blood have to be accounted for when rheological model is being developed (Johnston et al., 2004):

- Decrease of the viscosity with increasing shear rate (attenuating by shearing),
- Existence of the yield stress,
- Elasto-viscoplastic effect,
- Time dependence of viscosity (thixotropy).

Beyond this, contribution of the mentioned above phenomena to the flow of blood depends on a number of parameters, such as temperature or chemical composition of blood. It means that properties of blood can change according to the state of health, time of day, age etc. Majority of these relations is difficult for identification. Evaluation of importance of various factors and phenomena in modelling is one of the objectives of this work.

2.2. Available information regarding influence of features of blood on its viscosity

Plasma remains after removing all cellular elements from blood. It can be treated as Newtonian fluid. Experiments in the wide range of shear rates (0.1 - 1200 s^{-1}) confirm lack of noticeable nonlinearities of the plasma viscosity η_F. Various experimental results concerning η_F were published, for example Buchanan et al. (2000) report the value of

η_F = 1.284 mPas as physiological viscosity of plasma.

Hematocrit φ, which represents the volume fraction of red blood cells, is among the parameters, whose influence on the blood viscosity is reasonably well investigated. It is known that the blood viscosity increases with an increase of φ, but blood remains a fluid even for φ reaching 0.95. Various phenomena are responsible for this influence for low and for high shear rates. In the latter case, when an aggregation of RBCs is negligible, the influence is due to shape of erythrocytes and their deformability, as well as to an oxidization of erythrocytes. Aggregation of RBCs is the main factor, which causes increase of the viscosity at low shear rates. Schematic qualitative illustration of contribution of plasma and hematocrit to the total blood viscosity is shown in figure 1.

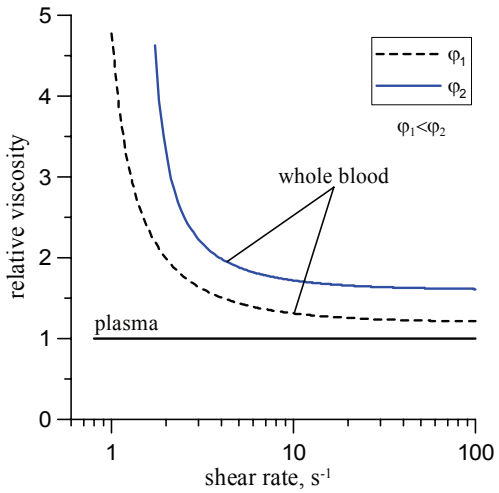

Fig. 1. *Schematic illustration of contribution of various components to the blood viscosity.*

Fahraeus-Lindquist effect, described for example in Bębenek (1999), is the next factor, which influences the blood viscosity. The decrease of the hematocrit with decreasing cross section of veins or arteries is the reason of this effect. This phenomenon is called "wall effect" and is caused by the fact, that RBCs have a tendency to move through the central part of veins or arteries, while there is lack of RBCs close to walls. It is schematically illustrated in figure 2. Increase of the blood viscosity with decreasing cross section of veins or arteries is a consequence of those phenomena. However, this effect is important only for very small cross sections of vessels. Since the long-range objective of this research is modelling of the artificial heart chamber, where the cross sections of the blood stream are large, the wall effect will not be considered in the further part of the paper.

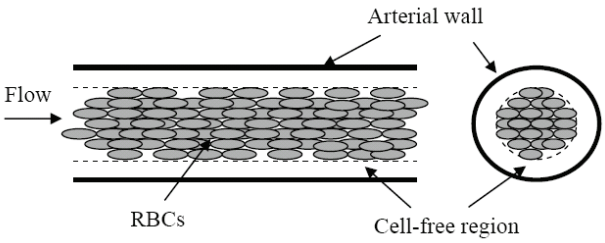

Fig. 2. *Schematic illustration of the wall effect.*

Elastic modulus is an important parameter, which influences flow of blood. RBCs are the components of blood, which are subjected to elastic deformation. It is reported by Picart et al. (1998) that, since the RBCs structure is broken by shear rate and they loose their elasticity, the elastic modulus depends on the shear rate. The modulus increases slightly with increasing shear rate and this relation is close to linear in logarithmic scale. When the shear rate is sufficiently low, the RBCs can be considered purely elastic. The fibrinogen, as well as the hematocrit, are other factors which have influence on the elastic modulus. The increase of both these factors causes the increase of the elastic modulus.

Analysis of the published results on relation between the elastic modulus and blood parameters leads to the conclusion that presented relations were determined indirectly, by measurement of the external parameters of the flow. It means that searching for more advanced models, which are based on the multiscale analysis and which will be capable to account explicitly for the elasticity of RBCs, is fully justified.

Due to their elasticity RBCs are deformed, and this deformation influences the blood viscosity. This effect is well known not only for blood. Slime with rigid solid state particles exhibits the largest increase of the viscosity comparing to the pure liquid. The more deformable are the solid state particles, the smaller increase of viscosity is observed, see figure 3. When volume fraction of rigid particles increases above about 0.6, the viscosity increases rapidly and the slime loses its viscous properties. As it has already been mentioned, such behaviour is not typical for blood, which remains viscous even for hematocrit close to 0.98.

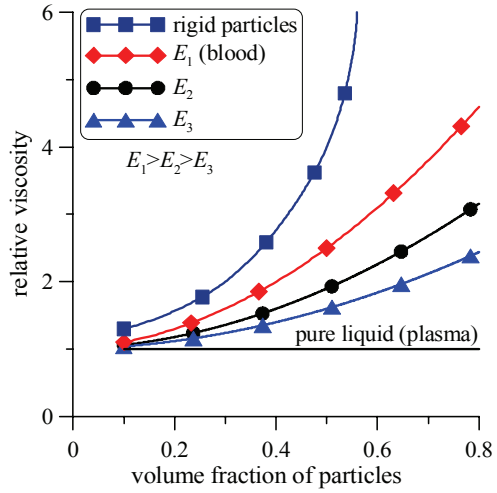

Fig. 3. *Schematic illustration of the influence of particles elastic modulus E on the mixture viscosity. When modelling blood, pure liquid is considered to be plasma.*

Aggregation of RBCs is the next important feature, which influences the blood viscosity. When RBCs gather together they aggregate and the resulting aggregates are called rouleaux. Formation of rouleaux depends strongly on the content of the fibrinogen and globulin (TPMA) in blood. Occurrence of rouleaux is the main reason of the general character of the viscosity vs. shear rate relation. The general tendency is that RBCs aggregate for small shear rates (size of rouleaux increases) and they split for high shear rates (size of rouleaux decreases). In consequence, the blood viscosity decreases with increasing shear rate and blood flows easily at high shear rates. Since RBCs are reasonably soft, under intensive shearing healthy RBCs behave almost as drops of a liquid. It is the effect known as thixotropy – softening by shearing.

In thixotropic materials, after changing of the shear rate the viscosity changes and it stabilizes at a new equilibrium point. Certain delay in reaction on a change of the shear rate is one of the features of the thixotropy. After rapid change of the shear rate certain, reasonably long time is needed to reach the new equilibrium.

Results of literature research on blood thixotropic properties are not always consistent. Thus, Huang et al. (1975) report that when the shear rate increases from near to zero to 10 s^{-1}, the viscosity decreases for about 20 s until it reaches the new stationary value. The same authors noticed that much shorter time is needed to reach equilibrium when shear rate changes from 10 to 100 s^{-1}. Other researchers (Gaspar-Rosas & Thurston, 1988) report that when shear rate drops from 500 to close to 0s^{-1}, the viscosity increases at about 50 mPas until it reaches a new stationary value. These authors have also noticed that much shorter time (few seconds) is needed to reach equilibrium when shear rate increases from low to high value. Recapitulating, the effect of thixotropy is larger when the shear rate decreases from high to low value than for the opposite direction of the change of the shear rate.

It is well known that temperature influences the viscosity of a majority of liquids. It can be expected that it influences the blood viscosity, as well. The general tendency is that the viscosity decreases with increasing temperature. The blood viscosity is usually measured at 37°C, which is the temperature of a human body. It is known from the published data that for each C degree of the temperature drop, the viscosity increases by less than 2% (Barbee, 1973).

As it has been mentioned in the introduction, the perspective application of the present work will be in simulations of blood flow through the artificial heart chamber, therefore, the effect of the temperature will be evaluated from that perspective. Authors of the artificial heart chamber (Gawlikowski et al., 2008) stated that actually used pneumatic prosthesis POLVAD has not an effect on blood temperature. Thermodynamic changes of gas supplied to the pneumatic bowl do not cause changes of the fluid temperature (blood, as well as water used in laboratory testing) in the bowl. The global change of fluid temperature in the pneumatic heart prosthesis (temperature was measured in outlet connector of chamber) has never been noticed in laboratory testing.

In rotational heart prosthesis (axial centrifugal) the influence of chamber working on blood temperature has not been yet investigated. It is known, that the hydrodynamic power at the outlet connector is of the order of 2 W and the electric power supplied to engine is 5 W (efficiency is about 40%). In earlier constructions of rotational prosthesis the electric power could reach about 10 W and even more. A part of that electric energy can be transferred into heat but there is still no literature about the experimentally proved connection between the dissipated energy and heating the flowing fluid. Since the rotational prosthesis is also supposed to be the result of the Polish Artificial Heart Program and a great effort will be made to limit dissipated energy, the noticeable changes of the temperature are not expected and the effect of the temperature can be neglected in modelling of the blood flow through the artificial heart chamber.

Contrary to a number of liquids, blood shows the yield stress effect. Several liquids, like for example

water or some oils, can move under theoretically infinitesimally small loads. Several non-Newtonian fluids, including blood, require some threshold of the load to initiate motion. This phenomenon in blood is due to the existence of RBCs, which act one on each other with Van der Waals forces and Coulomb forces. Therefore, there is a certain finite load needed to destroy these bonds. The yield stress of the human blood reported in the literature is in the wide range of 0.2 – 50 mPa and it almost does not depend on the temperature. Selected from the literature values of the yield stress are: 0.2 mPa (Tu & Deville, 1996), 5 mPa (Picart et al., 1998), 10.82 mPa (Neofytou & Drikakis, 2003), 46 mPa (Buchanan et al., 2000). The average value of 5 mPa was chosen as a basis for further investigations in the present work.

2.3. Conclusions regarding features of blood, which have to be accounted for in evaluation of its viscosity

Performed analysis allows to conclude, which features of blood affect its viscosity and should be analysed. The following suggestions regarding further research can be made:

- Plasma is the main component affecting the whole blood viscosity. The viscosity of plasma is constant in a wide range of shear rates (0.1–1200 s^{-1}). η_F = 1.2 mPas is the most commonly used the physiological viscosity of plasma.
- Blood shows the yield stress effect. The yield stress of the human blood is about 5 mPa.
- Elastic deformation of RBCs influences the blood viscosity and should be considered in modelling.
- Blood shows thixotropic properties, which means that due to aggregation of RBCs the viscosity increases with decreasing shear rates. Change of the viscosity is delayed with respect to the change of the shear rate. This delay is larger when the final shear rate after the change is lower.
- Blood shows so called wall effect, which causes the increase of the blood viscosity with the decrease of the cross section of veins or arteries. This is due to the fact, that RBCs have a tendency to move through the central part of veins or arteries, while there is lack of RBCs close to the walls. This effect is important for small cross sections of vessels and it will not be considered in this paper.
- Temperature dependence of the blood viscosity is small within the practical range of temperatures, therefore, this effect can be omitted in modelling.

It is seen from the discussion in this chapter that there are several factors affecting flow of blood and development of a model, which accounts for all those phenomena and parameters, is very difficult. An attempt to classify rheological models of blood and to point out the most important independent variables of models is presented below.

3. NUMERICAL MODEL

The problem of simulation of blood flow through the artificial heart chamber is governed by the incompressible flow equations for viscous fluid. The dynamic viscosity coefficient η, which is constant for Newtonian flows, is a function of the shear rate $\dot{\gamma}$ in the case of non-Newtonian flows. The flow of blood belongs to the latter group. The dynamic viscosity coefficient η will be referred to as viscosity in the further part of the paper.

The shear rate is written in the following tensorial form:

$$\dot{\boldsymbol{\gamma}} = 2\dot{\mathbf{D}} = \nabla\mathbf{v} + \nabla\mathbf{v}^T \tag{1}$$

where: $\mathbf{v} = \{v_1, v_2, v_3\}$ – velocity vector, $\dot{\boldsymbol{\gamma}}$ – shear rate tensor, $\dot{\mathbf{D}}$ - rate of deformation tensor.

In the case of the generalized non-Newtonian behaviour, the general relation between shear stress tensor $\boldsymbol{\tau}$ and shear rate tensor $\dot{\boldsymbol{\gamma}}$ is:

$$\boldsymbol{\tau} = \eta\left(\dot{\bar{\gamma}}\right)\dot{\boldsymbol{\gamma}} \tag{2}$$

where: $\dot{\bar{\gamma}}$ – effective shear rate.

The expression for $\dot{\bar{\gamma}}$ involves the second invariant of the rate of deformation tensor:

$$\dot{\bar{\gamma}} = \sqrt{2\,\mathrm{tr}\,\dot{\mathbf{D}}^2} \tag{3}$$

In the present form equation (2) does not account directly for elasticity, stress relaxation or memory effects. Influence of these factors can be accounted for by introduction relevant function $\eta\left(\dot{\bar{\gamma}}\right)$ in equation (2).

The viscosity η, which is a function of the shear rate will be further referred to as the rheological model of blood. More advanced models, which are also discussed, account for the yield stress of blood and its elastic deformation. In some rheological

models coefficients are correlated with some physiological and pathological parameters of blood.

4. RHEOLOGICAL MODEL OF BLOOD

In the literature different models are written in a different mathematical form. It means that Authors used different equations with many parameters to fit the particular experimental data. The shear stress vs. shear rate relation is often written in a scalar form. In the present work all models are adapted to the tensorial form (2). In consequence, the models of blood will be written as a relation between viscosity and internal and external variables of the flow $\eta\left(\dot{\bar{\gamma}},.....\right)$. In the first part of research we focused on one-phase models which, however, account indirectly for the two-phase structure of blood in calculations of the viscosity. More precisely, those models neither allow to predict explicitly behaviour of the two components of blood nor can be used for the analysis of blood structure. Those models allow to determine the blood viscosity accounting indirectly for the influence of plasma and RBCs. The general form of the one-phase model is:

$$\eta = \eta(\dot{\bar{\gamma}}, a_1, ..., a_m) \tag{4}$$

where: a_i (i = 1, …., m) – parameters of the model, m – number of parameters in the model.

Review of the most commonly used one-phase models is presented, among others, by Zhang & Kuang (2000), Johnston et al. (2004), Marcinkowska-Gapińska (2007) or by Yilmaz & Gundogdu (2008). Based on the information in the scientific literature, the most representative groups of rheological models of blood can be distinguished. These groups are discussed below.

4.1. Power law

The general power law has the form:

$$\eta = k\dot{\bar{\gamma}}^{n} \tag{5}$$

where: k, n – coefficients.

The coefficient k represents consistency of a fluid. The larger the consistency is, the more viscous is a fluid. The coefficient n is a measure of a non-Newtonian behaviour. n = 0 represents Newtonian fluid. The closer to one this coefficient is, the more non-Newtonian are the properties of blood. Liquids with n > 0 show hardening by shearing. Blood with n < 0 represents softening by shearing. Typical values of the coefficients in equation (5) for the healthy human blood and for the body temperature of 37°C are: k = 1.2 ÷ 1.4 mPas and n = -0.45 ÷ -0.15. Several extended modifications of the power law can be found in the literature and two of them are given below. Values of coefficients in these models, taken from the literature, are given as well. The coefficients are recalculated so that the viscosity is always in mPas. Few examples of models, which are assigned to the group of power laws, are presented below:

– Carreau model and upgraded Carreau model, given also in publications Johnston et al. (2004) and Chen et al. (2006)

$$\eta = \eta_\infty + \left(\eta_0 - \eta_\infty\right)\left(1 + \dot{\bar{\gamma}}^2\right)^{(q-1)/2} \tag{6}$$

$$\eta = \eta_\infty + \left(\eta_0 - \eta_\infty\right)\left(1 + \lambda\dot{\bar{\gamma}}^{n}\right)^{\frac{q-1}{n}} \tag{7}$$

where: η_∞ – asymptotic viscosity of blood when shear rate tends to infinity, η_0 – viscosity of blood when shear rate approaches zero, where: λ, n, q – coefficients. Various values of these coefficient can be found in the literature, e.g. Chen et al. (2006) report λ = 0.11 s, n = 0.644, q = 0.392, η_∞ = 2.2 mPas and η_0 = 22 mPas while Johnston et al. (2004) propose λ = 3.313 s, n = 2, q = 0.3568, η_∞ = 3.45 mPas and η_0 = 56 mPas.

– Walburn and Schneck (1976)

$$\eta = C_1 \exp\left(C_2\varphi\right)\exp\left(C_4\frac{TPMA}{\varphi^2}\right)\left(\dot{\bar{\gamma}}\right)^{-C_3\varphi} \tag{8}$$

According to Johnson et al. (2004): C_1 = 0.0797 mPa, C_2 = 6.08, C_3 = 0.5, C_4 = 0.0014581 l/g, $TPMA$ = 25.9 g/l. This equation takes into account the influence of the hematocrit φ and the combined influence of fibrinogen and globulin ($TPMA$).

The power law is the simplest rheological model of blood. This model does not account for the majority of factors, which affect the properties of blood and which are mentioned above in this paper. The first step towards elimination of these drawbacks is searching for the model, which can be related to RBCs aggregation and deformability. Considering that effect the apparent blood viscosity decreases rapidly with RBCs aggregation at lower shear rates and decreases slowly with RBCs deformability at high shear rates, the model may be represented by the bi-exponent equation:

$$\eta = \eta_\infty + \eta_D \exp\left(-\sqrt{t_D \dot{\bar{\gamma}}}\right) + \eta_A \exp\left(-\sqrt{t_A \dot{\bar{\gamma}}}\right) \quad (9)$$

where: η_∞, t_D, t_A, η_D, η_A, – parameters. The first and the second exponent in equation (9) are related to RBCs aggregation and RBCs deformability, respectively. The following values of these coefficients are proposed by Zhang & Kuang (2000): η_∞= 4.24 mPa, η_D = 2.756 mPa, η_A = 41.425 mPa, t_D = 0.14 s, t_A = 4.04 s.

The physical meaning of parameters in equation (9) is explained by Zhang & Kuang (2000). η_∞ and η_D/η_∞ - 1 are the viscosity indexes, which are related to the RBCs deformability. t_D and t_A represent the sensitivity of the RBCs deformability and aggregation with respect to the shear rate. η_A/η_∞ - 1 is the viscosity index, which is related to the RBCs aggregation. This correlation between model coefficients and aggregation of the RBCs is claimed by the authors to be the advantage of the model and the first step towards elimination of disadvantages of the power law. Analysis of equation (9) shows, however, that there is no direct correlation between coefficients and the physics of the blood flow. The relation of the blood viscosity with respect to the shear rate is monotonic. This relation presented in the logarithmic scale has two intervals with different slopes and it is supposed to reflect the physical phenomena of fast changes of aggregation at low shear rates and slow changes of aggregation at high shear rates. This approach is artificial. The overall relationship can be reproduced by proper selection of coefficients in the equation but there is no phenomenological base for this model.

Similar idea of the division of the viscosity vs. shear rate relation into two intervals was proposed by Ree-Eyring (Marcinkowska-Gapińska, 2007):

$$\eta = \eta_\infty + \eta_D \frac{t_D \dot{\bar{\gamma}}}{\sinh^{-1}\left(t_D \dot{\bar{\gamma}}\right)} + \eta_A \frac{t_A \dot{\bar{\gamma}}}{\sinh^{-1}\left(t_A \dot{\bar{\gamma}}\right)} \quad (10)$$

Since the idea of this equation is the same as in the bi-exponential model, equation (10) is not analysed in this paper. Visco-plastic models, which are discussed in the next section, give better reproduction of the real behaviour of blood in comparison with power laws.

4.2. Casson model

This model is considered a fundamental approach in modelling blood flow. Various upgrades of this model, which consider additional phenomena affecting the flow, are proposed in the scientific literature. Thus, a motivation and basic principles of the model are presented below.

As it has also been mentioned, blood shows the yield stress. Thus, the constitutive law, which accounts for the flow stress and viscoplastic properties of blood, has to be used instead of equation (2). The simplest constitutive law for blood is that proposed by Bingham (see for example Tu & Deville, 1996):

$$\boldsymbol{\tau} = \tau_y + \eta\left(\dot{\bar{\gamma}}\right)\dot{\boldsymbol{\gamma}} \quad (11)$$

where: τ_y – yield stress in shear.

The Bingham law was originally proposed in one-dimensional form, but it can be easily written in a tensorial form, as shown in equation (11). Assuming relation of the viscosity on the shear rate given by a power-law (5), there are three coefficients in equation (11): τ_y, k, n.

The model developed by Casson introduces the constitutive law in the following form:

$$\sqrt{\tau} = \beta\sqrt{\dot{\gamma}} + \sqrt{\tau_y} \quad (12)$$

where: β – coefficient, which as it is seen below is equal to η_∞.

This law was originally also proposed in one-dimensional scalar form and for the 2D or 3D FE solutions it has to be adapted to the constitutive law in tensorial form (2). The following relation for the viscosity is obtained after adaptation:

$$\eta = \left(\sqrt{\eta_\infty} + \sqrt{\frac{\tau_y}{\dot{\bar{\gamma}}}}\right)^2 \quad (13)$$

There are several upgrades of the Casson equation. Four of them are given below, all in the form of the viscosity model, which can be implemented into tensorial constitutive law (2). Values of coefficients found in the literature are given for each model.

– Herschel-Bulkley (see for example Tu & Deville, 1996)

$$\eta = \eta_H \dot{\bar{\gamma}}^n + \frac{\tau_y}{\dot{\bar{\gamma}}} \quad (14)$$

where: η_H – parameter. According to Valencia et al. (2006): η_H = 8.9721 mPas^{1+n}, τ_y = 17.5 mPa, n = -0.1399.

– Quemada (1981)

$$\eta = \eta_F \left(1 - \frac{1}{2} \frac{k_0 + k_\infty \sqrt{\hat{\dot{\gamma}}}}{1 + \sqrt{\hat{\dot{\gamma}}}} \varphi \right)^{-2} \qquad \hat{\dot{\gamma}} = \frac{\dot{\gamma}}{\dot{\gamma}_c} \qquad (15)$$

where: $\dot{\gamma}_c$ – critical shear rate, above which the elastic effect can be neglected, φ – hematocrit, k_∞, k_0 - coefficients. According to Neofytou & Drikakis (2003): η_F = 1.2 mPas, k_∞ = 2.07, k_0 = 4.33, $\dot{\gamma}_c$ = 1.88 s^{-1}. In the Quemada (1981) model, when $\dot{\gamma} \Rightarrow \infty$, viscosity $\eta \Rightarrow \eta_F \left(1 - \frac{k_\infty \varphi}{2} \right)^{-2}$

According to Buchanan et al. (2000) k_∞ = 5, φ = 0.45, η_F = 4 mPas.

– Papanastasiou (1987)

$$\eta = \left\{ \sqrt{\eta_\infty} + \sqrt{\frac{\tau_y}{\dot{\gamma}}} \left[1 - \exp\left(-\sqrt{q\dot{\gamma}} \right) \right] \right\}^2 \qquad (16)$$

where: q – coefficient. According to Charm (1964): $\eta_\infty = 3.1 \times 10^{-3}$ mPas, τ_y = 10.82 mPa, q > 100s.

– Luo & Kuang (1992)

$$\eta = \eta_F \left(a_1 + a_2 \frac{1}{\sqrt{\dot{\gamma}}} \right) + \frac{\tau_y}{\dot{\gamma}} \qquad (17)$$

where: a_1, a_2 – coefficients. Authors of this model claim that the physical meaning is added to some of the coefficients, see Luo & Kuang (1992).

The rheological models proposed by Cross (1965) and Wang & Stoltz (1994) are also based on the Casson concept.

Relation between the viscosity and the shear rate obtained from various models is presented in figure 4. Values of coefficients used to calculate plots in this figure are given in table 1. Value of hematocrit φ = 0.45 was used in all considered models. It is seen in figure 4 that character for the relation is similar for all Casson type models and differences between those models are insignificant. There is essential difference between Casson type models and models based on the power law. The latter models predict certain, finite value of the viscosity when shear rate reaches zero. Contrary, Casson type models predict infinite viscosity for shear rate reaching zero.

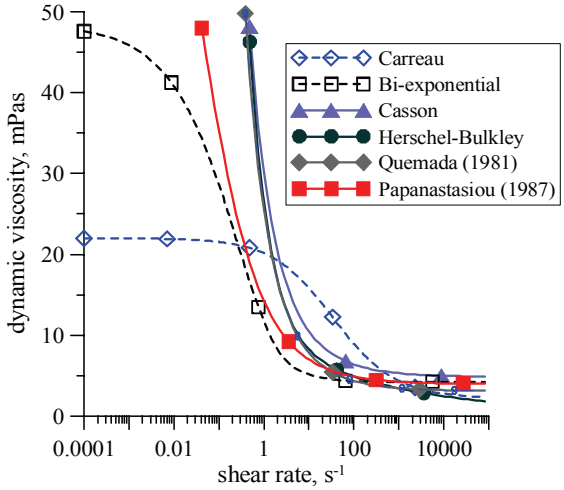

Fig. 4. *Viscosity vs. shear rate relation obtained from various models.*

Beyond the models described above, which are the most commonly used rheological models for blood, there is also a group of models treating blood as an elasto-viscoplastic fluid, as well as advanced models accounting explicitly for the multi phase structure of blood. These models are not considered in the present work and they will be an objective of further research.

5. SENSITIVITY ANALYSIS

Primary analysis of various rheological models of blood allows to conclude that, in general, these models can be divided into 4 groups: power laws, Casson type models (visco-plastic models), elasto-viscoplastic models and multi-phase models. The models within one group differ only in the values of coefficients, the mathematical form of equations is the same. Therefore, the sensitivity analysis was performed to evaluate all models and to select those adequate for the particular applications. The objec-

Table 1. *Values of coefficients used to calculate plots in figure 4.*

Model	Coefficients
Carreau	η_∞ = 2.2 mPas; η_0 = 22 mPas; λ = 0.11 s$^{1/n}$; n = 0.644; q = 0.392
Bi-expotential equation (9)	η_∞ = 4.24 mPas; η_D = 22 mPas; η_A = 41.425 mPas; t_D = 0.14 s; t_A = 4.04 s
Casson	η_∞ = 2.2 mPas; τ_y = 10.82 mPa
Herschel-Bulkley	η_H = 8.9721 mPas^{1+n}; τ_y = 17.5 mPa; n = -0.1399
Quemada (1981)	η_F = 1.2 mPas; k_∞ = 2.07; k_0 = 4.33; $\dot{\gamma}_c$ = 1.88 s^{-1}
Papanastasiou (1987)	η_∞ = 3.1 mPas; τ_y = 10.82 mPa; q = 120 s

tives of the sensitivity analysis are twofold. The first objective is supplying information for evaluation of models, from the point of view of mathematical form of equations and importance of parameters in these equations. Evaluation of sensitivity of simulation results with respect to the models parameters and external variables of the blood flow is the second objective. Both controlled and not controlled external variables (physiological and pathological factors) are considered.

5.1. Methodology

The Morris design belongs to the class of screening methods (Saltelli et al., 2000). Screening designs, as a part of the sensitivity analysis methods (Kleiber et al., 1997), deal with the question which factors of the physical model or computer simulation are really important. The factor means either parameter, which describes properties of the model or input variable, which is directly observable in the corresponding real system. Screening methods estimate qualitative statistic of the factors in order of their importance, i.e. they state that one factor is more important than another, but they do not provide the quantitative information of the factors significance. Screening designs widely use the One-At-a-Time (OAT) approach. Methods based on the OAT technique investigate the impact of the variation of each factor in turn. The OAT design developed by Morris (Morris, 1991) is called the global sensitivity analysis, because the algorithm explores the entire space over which the factors vary. In this algorithm the main effect of the factor is estimated by computing the assumed number of local measures at different points in the input space and next the average value is taken. These points are selected in such a way that each factor covers the whole interval in which it was defined. The key definitions and steps of Morris design are presented below.

Assumptions and definitions. Let $\mathbf{x}$ is the k-dimensional vector of simulation factors x_i. The components x_i, $i = 1 \ldots k$, accept p values in the set {0, 1/(p-1), 2/(p-1), …, 1}. Then the experimental space $\Omega \subset \mathfrak{R}^k$ forms k-dimensional p-level grid. Let Δ depend on p (an even number for convenience) and describe the side length of the grid element:

$$\Delta := 1/(p-1) \tag{18}$$

The elementary effect of the ith factor at a given point $\mathbf{x}$ is defined as:

$$d_i(\mathbf{x}) := \frac{y(x_1,\ldots,x_{i-1},x_i+\Delta,x_{i+1},\ldots,x_k)-y(\mathbf{x})}{\Delta} \tag{19}$$

Vector $\mathbf{x}$ is any point from Ω region such that the perturbed point $\mathbf{x} + \Delta$ is still in Ω. A finite distribution F_i of elementary effect calculated for the ith factor is found by sampling $\mathbf{x}$ in Ω. The number of F_i values of each factor is $p^{k-1}[p - \Delta(p-1)]$ for k-dimensional p-level grid.

The distribution F_i can be described by mean and standard deviation, the values which characterize the influence of the ith factor on the model output.

Algorithm of randomly selected orientation matrix $\mathbf{B}^*$.

Step 1. Select matrix $\mathbf{B}$ of $(k + 1) \times k$ dimensions with elements b_{ij} equal 0 or 1 ($i = 1 \ldots k+1$, $j = 1 \ldots k$) and every two columns of the matrix differ in only one element. In particular, $\mathbf{B}$ may be defined as a strictly lower triangular matrix with values of 1.

Step. 2. Build diagonal k dimensional matrix $\mathbf{D}^*$ such that:

$$d'_{ij} = \begin{cases} \pm 1 & i = j \\ 0 & i \neq j \end{cases} \quad \text{with equal probabilty} \tag{20}$$

Step 3. Build random permutation matrix $\mathbf{P}^*$ of $k \times k$ dimensions such that every column contains one element equal 1 and others equal 0 and there are no two columns which have values of 1 at the same position. In particular $\mathbf{P}^* = \mathbf{I}$.

Step 4. Build the matrix $\mathbf{B}^*$ following the formula:

$$\mathbf{B}^* = \left(\mathbf{J}_{k+1,1}\mathbf{x}^* + \left(\frac{\Delta}{2}\right)\left[\left(2\mathbf{B} - \mathbf{J}_{k+1,k}\right)\mathbf{D}^* + \mathbf{J}_{k+1,k}\right]\right)\mathbf{P}^* \tag{21}$$

where $\mathbf{J}_{k+1,1}$ and $\mathbf{J}_{k+1,k}$ are matrices with values 1 of dimensions appropriate $(k + 1) \times 1$ and $(k + 1) \times k$. Orientation matrix $\mathbf{B}^*$ provides a single elementary effect per factor.

Algorithm of estimation of mean and variance of the distribution F_i, $i = 1 \ldots k$.

Step 1. Run r times algorithm of randomly selected orientation matrix $\mathbf{B}^*$, each for different starting point $\mathbf{x}^*$. It provides r independent orientation matrices with different trajectories for k factors, what is equivalent to r values for distribution F_i for each factor $i = 1 \ldots k$.

Step 2. Since the trajectories are independent and they give independent estimators, estimate the mean μ and standard deviation σ for each of the

factor through the classic estimators for independent random samples.

5.2. Selection and classification of models

A number of models have been considered on the basis of the list presented in Yilmaz & Gundogdu (2008). Among the investigated models those developed by Casson (described for example by Zhang & Kuang, 2000), Walburn-Schneck (1976), Herschel – Bulkley, Luo & Kuang (1992), Papanastasiou (1987) and Quemada (1981) should be mentioned. After the primary analysis the similarity of the mathematical form of various models have been notices. Thus, the models were classified into three groups and the sensitivity analysis was performed for these three types of equations defining dynamic viscosity of human blood as a function of shear rate:

I. General equation, which includes Casson, Walburn-Schneck, Herschel – Bulkley, Luo and Kuang and Papanastasiou models,
II. Bi – exponential model (original, not transformed equation (9)),
III. Model based on Quemada equation.

I. General viscosity equation

Rheological models of blood: Casson, Walburn-Schneck, Herschel – Bulkley, Luo and Kuang and Papanastasiou models are transformed to the general equation of the form:

$$\eta = A + B\dot{\gamma}^{n} + C\dot{\gamma}^{m} \tag{22}$$

where A, B, C, n, m – parameters of the equation.

Relations of equation (22) with Casson, Walburn-Schneck, Herschel – Bulkley, Luo and Kuang and Papanastasiou models are presented in table 2. Plots of model (22) obtained for various parameters are presented in figure 5. Values of parameters in equation (22) were determined based on the values of appropriate parameters of the literature models.

Table 2. *Relations between general equation (22) and literature models of dynamic viscosity.*

A	η_F	Casson model
	0	Walburn-Schneck model
	0	Herschel – Bulkley model
	$a_1\eta_F$	Luo and Kuang model
	η_F	Papanastasiou model
B	$2\sqrt{\eta_F}\sqrt{\tau_y}$	Casson model
	$C_1 e^{C_2\varphi} e^{C_4\frac{TPMA}{\varphi^2}}$	Walburn-Schneck model
	η_H	Herschel – Bulkley model
	$a_2\eta_F$	Luo and Kuang model
	$2\sqrt{\eta_F}\sqrt{\tau_y}\left[1-\exp\left(-\sqrt{q\dot{\gamma}}\right)\right]$	Papanastasiou model
n	-1/2	Casson model
	$C_3\varphi$	Walburn-Schneck model
	n	Herschel – Bulkley model
	-1/2	Luo and Kuang model
	-1/2	Papanastasiou model
C	τ_y	Casson model
	0	Walburn-Schneck model
	τ_y	Herschel – Bulkley model
	τ_y	Luo and Kuang model
	$\tau_y\left[1-\exp\left(-\sqrt{q\dot{\gamma}}\right)\right]^2$	Papanastasiou model
m	-1	Casson model
	not present	Walburn-Schneck model
	-1	Herschel – Bulkley model
	-1	Luo and Kuang model
	-1	Papanastasiou model

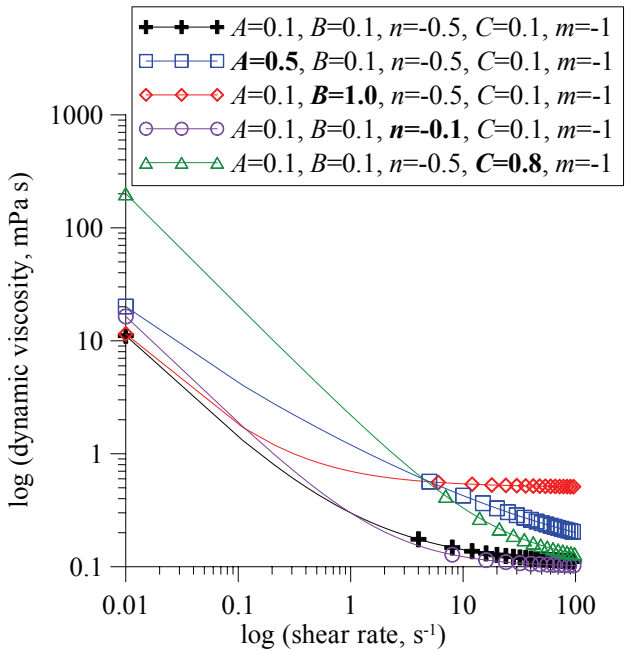

Fig. 5. Plots of the general model (22) for various values of parameters.

III. Model based on Quemada equation

To make the sensitivity analysis more clear, the original Quemada model (15) was transformed to the following form:

$$\eta = \left(\frac{1 + a\sqrt{\dot{\gamma}}}{b + c\sqrt{\dot{\gamma}}} \right)^2 \tag{23}$$

where:

$$a := \frac{1}{\sqrt{\dot{\gamma}_c}}, \quad b := \left(1 - \frac{1}{2}\varphi k_0\right)\frac{1}{\sqrt{\eta_F}},$$

$$c := \left(1 - \frac{1}{2}\varphi k_\infty\right)\frac{1}{\sqrt{\eta_F}}\frac{1}{\sqrt{\dot{\gamma}_c}}$$

5.3. Results

Algorithm of Morris design was performed for k = 4 parameters of general equation (22), k = 5 parameters of bi-exponential model (9), k = 3 for Quemada model (23), p = 8 grid level (see equation (18)); 20 independent trajectories were generated to estimate mean and standard deviation for each group of parameters. The model output was dynamic viscosity.

I. General equation

Intervals of parameter variability were assumed as follows: $A \in\langle 0,\ 5\rangle$ mPa s, $B \in\langle 0,\ 15\rangle$ mPa s^{1+n}, $n \in\langle -1,\ -0.01\rangle$, $C \in\langle 0,\ 5\rangle$ mPa and they were determined based on literature, e.g. (Papanastasiou, 1987; Zhang & Kuang, 2000; Neofytou & Drikakis, 2003; Obidowski, 2007). Results of sensitivity calculations are presented in figure 6.

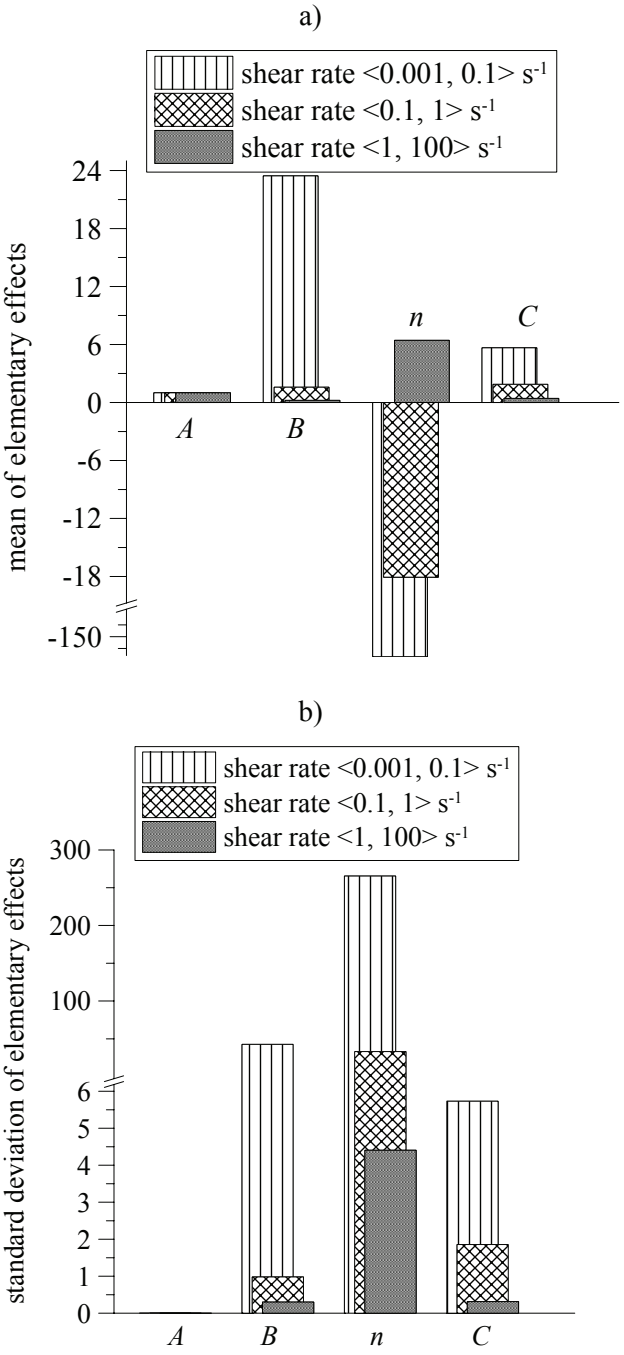

Fig.6. a) Mean and b) standard deviation of elementary effects of parameters of equation (22) for various shear rate intervals.

II. Bi–exponential model

Intervals of parameter variability were assumed as follows: $\eta_e \in\langle 0.1,\ 1\rangle$ mPa s, $\eta_D \in\langle 0.001,\ 4\rangle$ mPa s, $t_D \in\langle 0.01,\ 0.5\rangle$ s, $\eta_A \in\langle 10,\ 100\rangle$ mPa s, $t_A \in\langle 1,\ 8\rangle$ s, and they were determined on the basis of literature (Zhang & Kuang, 2000).

Two terms of bi-exponential model are similar: $\eta_D \exp\left(-\sqrt{t_D\dot{\gamma}}\right)$ and $\eta_A \exp\left(-\sqrt{t_A\dot{\gamma}}\right)$ but according to intentions of the Authors of the model, they are responsible for different phenomena occurring in blood flow (separate domains for these parameters were defined). Small values of η_D and t_D guarantee that $\eta_D \exp\left(-\sqrt{t_D\dot{\gamma}}\right)$ term has significant effect on the model output for whole shear rate domain – this term is going to zero slowly, while greater values of η_A and t_A influence the model output only for not high shear rates – $\eta_A \exp\left(-\sqrt{t_A\dot{\gamma}}\right)$ term is going to zero rapidly (figure 7).

Results of sensitivity calculations are presented in figure 8.

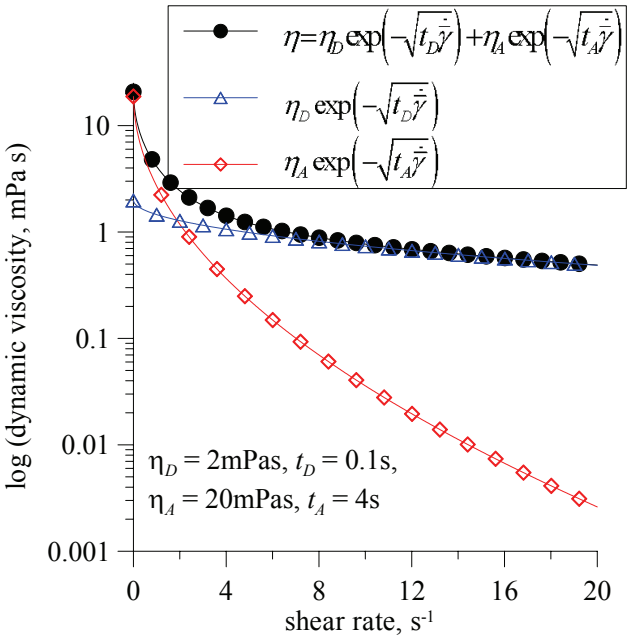

Fig. 7. *Effects of* $\eta_D \exp\left(-\sqrt{t_D\dot{\gamma}}\right)$ *and* $\eta_A \exp\left(-\sqrt{t_A\dot{\gamma}}\right)$ *terms on bi-exponential (equation (9)) dynamic viscosity model.*

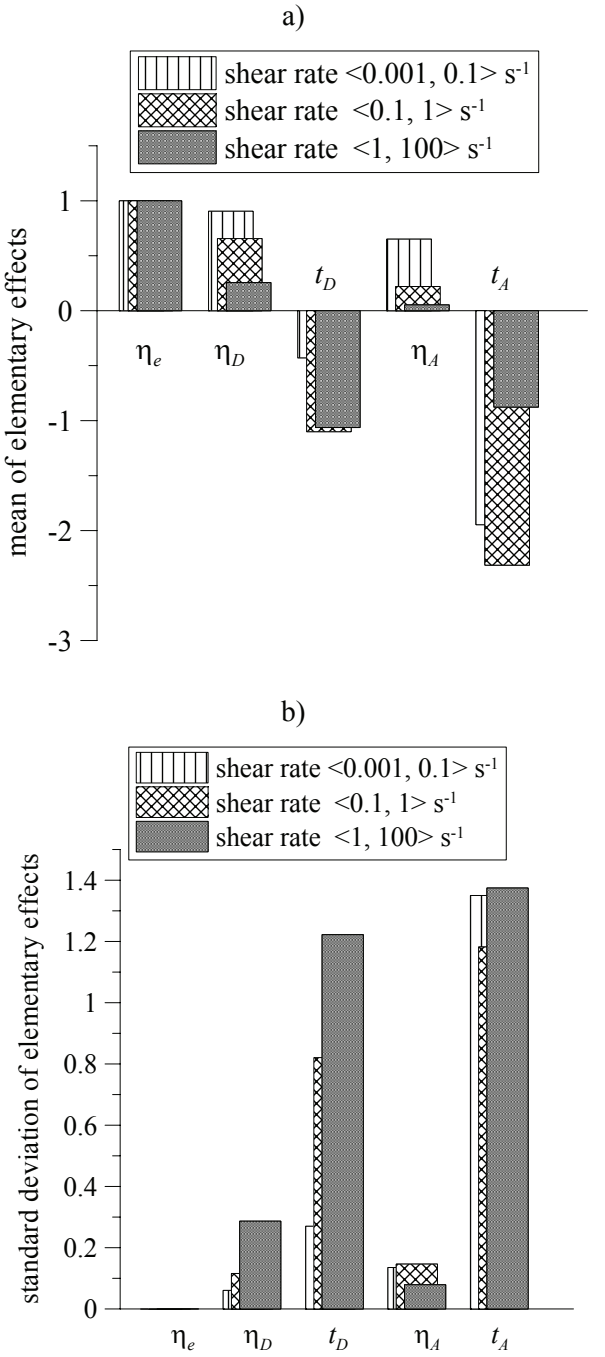

Fig. 8. *a) Mean and b) standard deviation of elementary effects of parameters of bi–exponential model (equation (9)) for various shear rate intervals.*

III. Quemada model

Intervals of parameter variability were assumed as follows: $a \in \langle 0.5, 1\rangle$, $b \in \langle 0.01, 0.1\rangle$, $c \in \langle 0.1, 1\rangle$, and they were determined taking into consideration the values of original Quemada model parameters (Quemada, 1981, Zhang & Kuang, 2000). Results of sensitivity calculations are presented in figure 9.

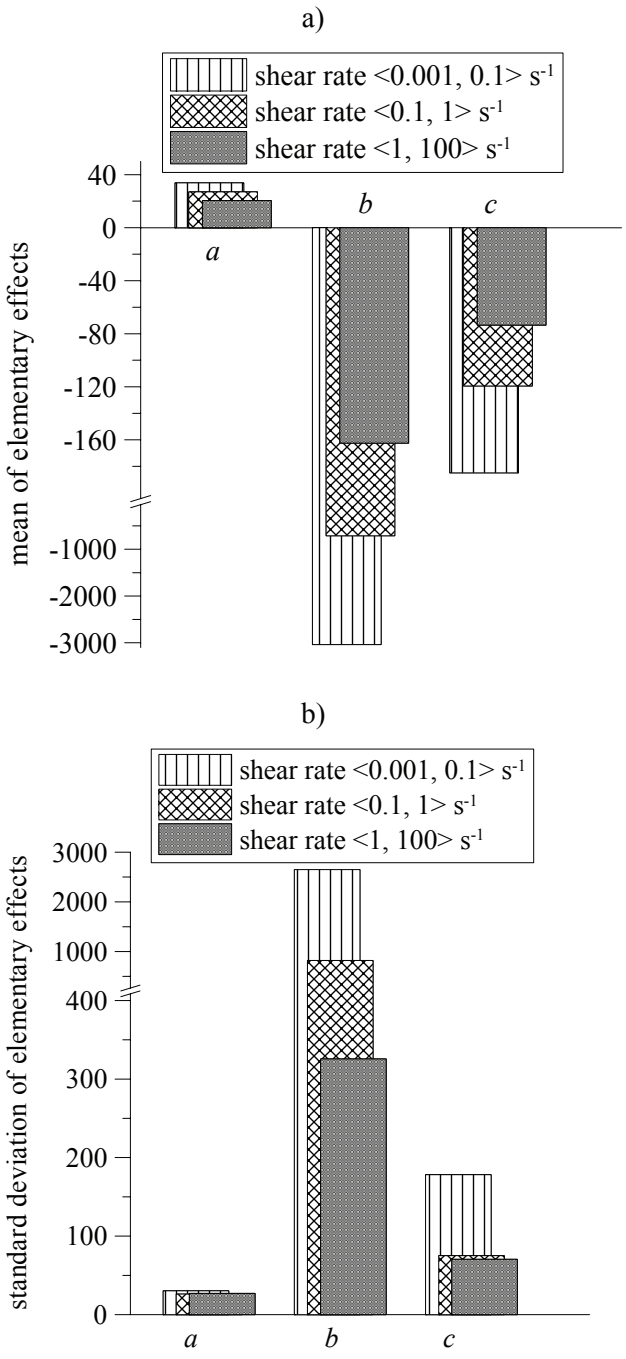

Fig. 9. *a) Mean and b) standard deviation of elementary effects of parameters of transformed Quemada model (equation (23)) for various shear rate intervals.*

6. DISCUSSION AND AREAS OF FURTHER RESEARCH

Sensitivity analysis of various rheological models of blood was performed. The models belonging to the two groups were investigated: power law models and Casson type models. The general observations from this analysis are summarized below.

6.1. Sensitivity analysis

Morris design provides qualitative information about the sensitivity of the model parameters to the model output. The mean of elementary effects describes sensitivity of the model with respect to the model parameter. The higher values of mean the more sensitivity of the model output is observed. Standard deviation is a measure of spread around the parameter mean value. High values of standard deviations mean that dependence parameter-model is nonlinear or the parameter interacts with other model parameters.

Parameter A of equation (22) shifts the curve of this model along ordinates axis and it controls the value, which the model is converging to while shear rate $\dot{\bar{\gamma}}$ is going to infinity (figure 5). The sensitivity of the model with respect to this parameter is the least (see mean of elementary effects in figure 6a). Standard deviation calculated for A is close to zero, less than 10^{-4} (figure 7b). It means that A does not interact with other model parameters. The same character has parameter η_e of bi-exponential model (9) (figure 8b), but the sensitivity on this parameter is significant comparing the other bi-exponential parameters (figure 8a).

Parameters B and C of general model (22) are amplification factors and they have an effect on the rate of model convergence to A while shear rate $\dot{\bar{\gamma}}$ is going to infinity. The model is the most sensitive to these parameters for small values of shear rate (figure 6a). Since shear rate is greater than one, to sensitivity is getting low. The standard deviation of B and C is the highest for small values of shear rate (figure 6b) and it means that B and C interact directly with this parameter. Amplification factors are parameters η_D, t_D and η_A, t_A of bi-exponential model (9), as well. All of them control convergence rate to zero while shear rate is going to infinity. The domains of these parameters were set up to get specific model outputs, that is why model is sensitive with respect to η_D, t_D for all values of shear rates and η_A, t_A influence the model for shear rates lower than $1 s^{-1}$. High values of standard deviation for t_D and t_A confirm nonlinear relationship with model output. Amplification factors are also parameters a and c in transformed Quemada model (23). Both of them control the slope of the curve generated by equation (23). The sensitivity of parameter c is much stronger than of parameter a (figure 9a), especially for low shear rates. The high values of standard deviation for c confirm nonlinear relationship with model output.

Parameter n is responsible for the slope of the model (22) curve first of all for shear rates greater than $1s^{-1}$. The higher values of n are applied the steeper slope is observed (figure 5). The mean of elementary effects of n is high and the model is sensitive to this parameter (figure 6a). The standard deviation of n is also significant and it explains the relationship with shear rate $\dot{\bar{\gamma}}$ (figure 6b).

Transformed Quemada model (23) is the most sensitive to the parameter b (figure 9a), which controls the model at the beginning of the curve. The lower value of b is the greater slope is observed and the value of dynamic viscosity for shear rate equal zero is greater. The high values of standard deviation for b (figure 9b) confirm nonlinear relationship with model output. Considering that $b = \left(1 - \frac{1}{2}\varphi k_0\right) \frac{1}{\sqrt{\eta_F}}$, the original Quemada model (15) is the most sensitive to parameters k_0 and φ.

6.2. Limitations and capabilities of the considered models

It should be emphasised that not all types of models were considered in the present work. Having in mind the perspective objective of the whole project, which is simulation of blood flow through the artificial heart chamber, the models accounting for the so called wall effect were not considered. One has to realize that computational modelling of blood flow in micro vessels with internal diameter of 20–500 μm is a major challenge. It is because blood in such vessels behaves as a multiphase suspension of deformable particles. Continuum models of blood, such as considered in the present work, are not adequate if the motion of individual RBCs in the suspension is of interest. At the same time, multiple cells, often a few thousands in number, must also be considered to account for cell-cell hydrodynamic interaction. Moreover, the RBCs are highly deformable. Deformation of the cells must also be considered in the model, as it is a major determinant of many physiologically significant phenomena, such as e.g. formation of a cell-free layer. All these aspects of modelling of the blood flow will be investigated during further research.

Analysis of all the results in the present work allows to draw the conclusion that considered models give good results when simulation of the blood flow is the main objective of modelling. These models account indirectly for a majority of features of blood. The models are simple enough to be easily implemented in the FE code and show good predictive capabilities, as far as the viscosity itself is considered. Therefore, these models will be implemented into the computational fluid dynamics (CFD) finite element software for modelling of the flow through the artificial heart chamber.

6.3. Advanced modeling of blood

All the more advanced models, which account explicitly for the multi phase structure of blood, are needed when the objective of modelling is prediction of structural changes in blood. The general approach in these solutions is as follows:

- Simulation of the blood flow, usually using CFD finite element software combined with conventional rheological model.
- Application of the results of the CFD simulations to predictions of microscopic blood structure, accounting for the local flow characteristics (shear rates, locations of rigid regions, wall effect, etc.).

Large number of such advanced models can be found in the literature, some of them are listed below.

The first group of these models is definitely focussed on modelling the blood flow through small vessels and the authors put the main emphasis on accounting for the Fahraeus-Lindquist effect. E.g. Srivastava (2007) developed the two-fluid model, which consists of two layers. The first is a core region with suspension of erythrocytes in plasma and the second is a peripheral layer of a cell-free plasma. Two different rheological laws are used for these two layers. This model does not consider mesoscale discrete phenomena and it still treats blood as a continuum. But it allows to describe properly the blood flow accounting for the wall effect. The concept of distinguishing two layers in the stream of blood and applying different models to the core flow and to the surface layer, is commonly used. See for example work of Sankar and Lee (2007), who treated surface layer as a Newtonian fluid and applied Herschel-Bulkley model (equation (14)) to the core layer. Publication of Sun and Munn (2005) is another example of such an approach. In the latter work an interesting data on contribution of various phases to the overall blood viscosity are presented. The authors have used the lattice Boltzman model to explicitly account for the nature of blood.

The objective of the work of Bagchi (2007) was similar as in the previously mentioned papers, but in this work mesoscale model was introduced. Bagchi

(2007) stated that continuum modelling of blood is not adequate if the motion of individual RBCs in the suspension is of interest and he proposed model based on the immerse boundary method. RBCs were modelled as liquid capsules and their trajectories during the blood flow were determined.

Jafari et al. (2009) have also performed multiscale simulations. They used Fluent software to solve CFD problem for a continuum and in representative volume of fluid they simulated deformation of RBCs during passage through small vessels. In consequence, interesting information regarding behaviour of RBCs was obtained.

Another group of models, which should be mentioned, introduces volume fractions of phases in blood and mechanisms for momentum, heat and mass exchange between the phases. Multiphase description of blood given by Gidaspow (1994) is considered as a basis of this group of multiscale models. Jung et al. (2006) presented an example of an application of this approach to modelling of the flow through a right coronary artery (RCA). Predictive capabilities of this model include accounting for the local hydrodynamic factors such as wall shear stress and this model contributes to better understanding phenomena, which occur in blood, and which are responsible for the early stage of an atherosclerosis.

All the models discussed in this subchapter have wide predictive capabilities, as far as accounting for certain features of blood structure are considered. These models were only reviewed briefly in the present paper and they will be a subject of future research.

7. CONCLUSIONS

Analysis of rheological models of human blood allows to draw the following conclusions:

- Large number of equations describing the blood viscosity can be found in the scientific literature. Analysis of the mathematical form of these models allowed the Authors to distinguish 3 groups. Description of the viscosity when the shear rate tends to zero is the factor, which distinguishes these groups.
- The investigated models often have coefficients dependent on particular parameters of blood. It means that simulations can indirectly account for changes of these parameters. However, these models cannot predict changes of blood structure caused by flow and shear stresses.
- The most advanced models seems to be those, which simulate flow of blood using finite element solution for a continuum. These models are combined with mesoscale models, which give insight into blood structure and predict behaviour of RBCs. In the models analysed in this work there is no feedback from the mesoscale to the CFD software. It means that predicted changes in the blood structure do not influence the viscosity of blood as a continuum.
- Ranges of values of parameters for the models were determined on the basis of the sensitivity analysis and on the physiological sense of the coefficients discussed in the paper.
- Selection of rheological model of blood and stability of the numerical model is crucial for correct evaluation of the simulation results with real blood flow process. Analysis of the models has shown that while shear rate is going to zero, dynamic viscosity goes to infinity for general model, what is unrealistic, and it is equal to a fixed value for bi-exponential and Quemada models. Following this, FE numerical calculations of the blood flow based on bi-exponential or Quemada models should produce less numerical errors and the simulation should be more stable.
- Estimation of parameters of rheological models requires information of the importance of the parameters and their sensitivities to the model output. Sensitivity analysis determines the efficiency of the identification of rheological parameters on the basis of experimental data and the results of numerical simulation of experiment.
- Presented analysis focused on the models, the task of which is to predict the blood viscosity and which are expected to supply information on changes of blood structure. More advanced models, which account explicitly for the multi phase structure of blood and which are capable to predict structural changes, are reviewed in the last part of the paper. These models will be a subject of the future research.

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KRYTYCZNA OCENA I ANALIZA WRAŻLIWOŚCI REOLOGICZNYCH MODELI KRWI

Streszczenie

Celem pracy jest krytyczna analiza oraz klasyfikacja reologicznych modeli krwi. W pierwszej części artykułu omówiono główne cechy i właściwości krwi w aspekcie ich wpływu na modelowanie przepływu krwi. Analizie poddano różne modele krwi dostępne w literaturze, które sklasyfikowano w oparciu o matematyczną postać zastosowanych równań. Oceniono możliwości poszczególnych modeli w zakresie uwzględnienia wpływu fizycznych cech krwi. Prawa potęgowe, powszechnie stosowane w symulacjach przepływu krwi, mają prosty matematyczny zapis i nie były szczegółowo analizowane. Do dalszej analizy modele podzielono na trzy grupy: modele typu Cassona, prawo Biekspotencjalne i model Quemady. Dla wybranych modeli przeprowadzono analizę wrażliwości stosując metodę opartą o algorytm Morrisa. Określono współczynniki wrażliwości lepkości krwi względem współczynników modeli oraz względem zmiennej zewnętrznej prędkości ścinania. Wyznaczono wskaźniki istotności poszczególnych współczynników oraz ich wpływ na wyniki modelu. We wnioskach z pracy zawarto wskazówki dla identyfikacji współczynników analizowanych modeli.

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