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IDENTIFICATION OF CAST STEEL LATENT HEAT ON A BASIS OF THERMAL AND DIFFERENTIAL ANALYSIS

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Abstract

Thermal and differential analysis (TDA) constitutes a very popular and effective tool of quantitative estimation of solidification process parameters, such as border temperatures, evolution of latent heat etc. TDA registering system is connected to the thermocouple located at a central point of sampling casting. Using the adequate program of automatic data processing one obtains the information concerning the cooling curve and its time derivative at the point corresponding to sensor position. In the paper the solution of inverse problem consisting in the identification of cast steel latent heat is discussed. The additional information necessary to solve the problem results both from a knowledge of cooling curve T(t) and a knowledge of its time derivative T'(t). The criterion determining the optimum value of latent heat bases on the input data concerning not only T(t) but also T'(t). On a stage of numerical computations the finite difference method has been used. In a final part of the paper, the examples of identification process are shown.

Key words: identification methods, solidification process, numerical simulation

1. INTRODUCTION

We consider 2D non-homogeneous domain casting-mould oriented in cartesian coordinate system (as in figure 1). At the point x_i the sensor is placed. The position of sensor is not accidental, the point x_i is a centre of area for which the sensitivity of temperature field with respect to latent heat perturbations achieves the maximum [3].

It assures the possibility of most 'valuable' data use to identify an unknown thermohysical parameter. The problems of sensitivity analysis application in the thermal theory of foundry processes are, with full particulars, presented in [11]. At the point considered both cooling curves and cooling rates are measured (using the TDA techniques). In this paper the information concerning $T_i(t)$ and $T_i'(t)$ have been found using the numerical solution of direct problem, the results obtained were, in a certain way, randomly disturbed in order to simulate the errors of real measurements [1]. The identification of volumetric latent heat Q has been done by gradient

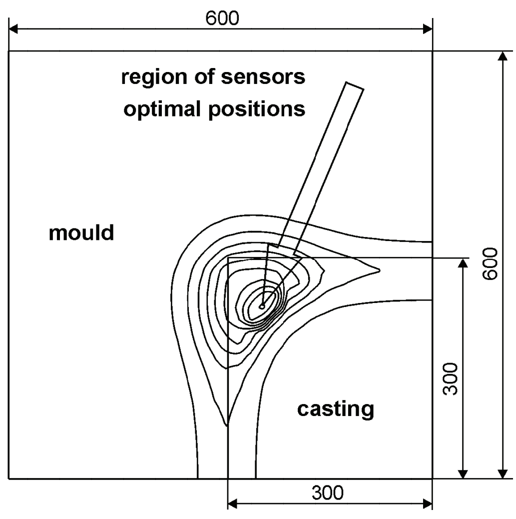

Fig. 1. *The domain and distribution of sensitivity function Z*

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method. The criterion of optimum solution (least squares criterion) contains not only temporary temperatures, but also the temporary cooling rates.

2. GOVERNING EQUATIONS

The casting-mould-environment system is considered. The transient temperature field in casting sub-domain is described by the following equation

$$x \in \Omega: \quad C(T)\frac{\partial T(x,t)}{\partial t} = \nabla\left[\lambda(T)\nabla T(x,t)\right] \quad (1)$$

where $\lambda(T)$ is the thermal conductivity, $C(T)$ is the substitute thermal capacity [7, 9], $T(x, t)$, x, t denote temperature, spatial co-ordinates and time, respectively.

A temperature field in a mould sub-domain describes the equation of the form

$$x \in \Omega_m: \quad c_m \frac{\partial T_m(x,t)}{\partial t} = \lambda_m \nabla^2 T_m(x,t) \quad (2)$$

where λ_m and c_m are the mould thermal conductivity and volumetric specific heat, respectively. On the contact surface between casting and mould the continuity condition

$$x \in \Gamma_c: \begin{cases} -\lambda(T)\,\mathbf{n}\cdot\nabla T(x,t) = -\lambda_m\,\mathbf{n}\cdot\nabla T_m(x,t) \\ T(x,t) = T_m(x,t) \end{cases} \quad (3)$$

is assumed.

For the outer surface of the system the no-flux condition can be accepted

$$x \in \Gamma_0: \quad -\lambda_m\,\mathbf{n}\cdot\nabla T_m(x,t) = 0 \quad (4)$$

For the moment $t = 0$ the initial temperature distribution is known, namely

$$t = 0: \quad T(x,0) = T_0 \;, \quad T_m(x,0) = T_{m0} \quad (5)$$

Substitute thermal capacity of casting material (cast steel – figure 2) for interval of temperatures $[T_s, T_L]$ corresponding to mushy zone sub-domain is approximated by a polynomial of 4th degree, this means

$$C(T) = \begin{cases} c_L, & T > T_L \\ c_1 + c_2 T + c_3 T^2 + c_4 T^3 + c_5 T^4, & T_S \le T \le T_L \\ c_S, & T < T_S \end{cases} \quad (6)$$

where c_L , c_S are the constant volumetric specific heats of liquid and solid state, respectively, c_e , $e = 1$, 2, ..., 5 are the coefficients. The coefficients c_e can be found on the basis of conditions assuring the continuity of C^1 class and physical correctness of approximation resulting from the polynomial integration from T_S to T_L .

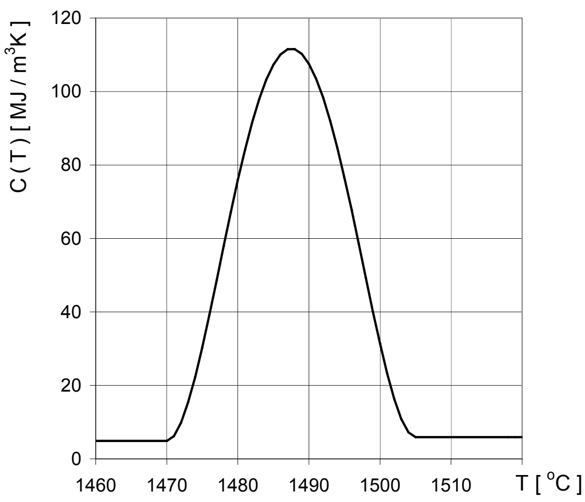

***Fig. 2.** Substitute thermal capacity*

The coefficients c_e are determined by the following formulas [6]

$$c_1 = \frac{c_S T_L - c_L T_S}{T_L - T_S} + \frac{(c_L - c_S) T_L T_S (T_L + T_S)}{(T_L - T_S)^3} + \frac{30 T_L^2 T_S^2 Q}{(T_L - T_S)^5}$$

$$c_2 = -\frac{6(c_L - c_S) T_L T_S}{(T_L - T_S)^3} - \frac{60 T_L T_S (T_L + T_S) Q}{(T_L - T_S)^5}$$

$$c_3 = \frac{3(c_L - c_S)(T_L + T_S)}{(T_L - T_S)^3} + \frac{30(T_L^2 + 4 T_L T_S + T_S^2) Q}{(T_L - T_S)^5}$$

$$c_4 = -\frac{2(c_L - c_S)}{(T_L - T_S)^3} - \frac{60(T_L + T_S) Q}{(T_L - T_S)^5}$$

$$c_5 = \frac{30 Q}{(T_L - T_S)^5} \quad (7)$$

One can see that c_1, c_2, ..., c_5 are the functions of latent heat Q.

The thermal conductivity of casting material is defined as follows

$$\lambda(T) = \begin{cases} \lambda_L, & T > T_L \\ \dfrac{\lambda_L + \lambda_S}{2}, & T_S \le T \le T_L \\ \lambda_S, & T < T_S \end{cases} \quad (8)$$

where λ_L, λ_S are the constant thermal conductivities of liquid and solid state, respectively.

3. SENSITIVITY ANALYSIS

To determine the sensitivity coefficients appearing in algorithm of inverse problem solution (see: next chapter), the sensitivity model concerning the latent heat perturbation should be constructed. In this paper the direct variant of sensitivity analysis is applied [2, 4, 11]. So, the governing equations should be differentiated with respect to Q and then the following additional boundary–initial problem appears

$$\begin{aligned}
& x\in\Omega:\ C(T)\frac{\partial Z(x,t)}{\partial t}=\nabla\left[\lambda(T)\nabla Z(x,t)\right]+\nabla\left[\frac{\partial\lambda(T)}{\partial Q}\nabla T(x,t)\right]-\frac{\partial C(T)}{\partial Q}\frac{\partial T(x,t)}{\partial t}\\
& x\in\Omega_m:\ c_m\frac{\partial Z_m(x,t)}{\partial t}=\lambda_m\nabla^2 Z_m(x,t)\\
& x\in\Gamma_c:\ \begin{cases}-\dfrac{\partial\lambda(T)}{\partial Q}\,\mathbf{n}\cdot\nabla T(x,t)-\lambda(T)\,\mathbf{n}\cdot\nabla Z(x,t)=-\lambda_m\,\mathbf{n}\cdot\nabla Z_m(x,t)\\ Z(x,t)=Z_m(x,t)\end{cases}\\
& x\in\Gamma_0:\ -\lambda_m\,\mathbf{n}\cdot\nabla Z_m(x,t)=0\\
& t=0:\quad Z(x,0)=0\ ,\quad Z_m(x,0)=0
\end{aligned}\tag{9}$$

where

$$Z(x,t)=\frac{\partial T(x,t)}{\partial Q}\quad,\quad Z_m(x,t)=\frac{\partial T_m(x,t)}{\partial Q}\tag{10}$$

It should be pointed out that $\partial\lambda(T)/\partial Q=0$, while for $T>T_L$ and $T<T_S$: $\partial C(T)/\partial Q=0$ (c.f. equation (6)), at the same time for $T_S\le T\le T_L$ one has

$$\frac{\partial C(T)}{\partial Q}=\frac{\partial c_1}{\partial Q}+\frac{\partial c_2}{\partial Q}T+c_2\frac{\partial T}{\partial Q}+\frac{\partial c_3}{\partial Q}T^2+2c_3T\frac{\partial T}{\partial Q}+\frac{\partial c_4}{\partial Q}T^3+3c_4T^2\frac{\partial T}{\partial Q}+\frac{\partial c_5}{\partial Q}T^4+4c_5T^3\frac{\partial T}{\partial Q}\tag{11}$$

or

$$\frac{\partial C(T)}{\partial Q}=\frac{\partial c_1}{\partial Q}+\frac{\partial c_2}{\partial Q}T+\frac{\partial c_3}{\partial Q}T^2+\frac{\partial c_4}{\partial Q}T^3+\frac{\partial c_5}{\partial Q}T^4+\frac{\mathrm{d}C(T)}{\mathrm{d}T}Z\tag{12}$$

The successive derivatives result directly from formulas (7).

The boundary initial problem (9) is coupled with the basic one, because in order to find its solution, the time derivative $\partial T(x,t)/\partial t$ should be known.

The basic problem for the assumed value of Q and the additional one connected with the sensitivity function Z computations have been solved using the explicit scheme of finite difference method [10].

4. INVERSE PROBLEM

To solve the inverse problem the least squares criterion in the following form is applied

$$S(Q)=\frac{w}{MF}\sum_{i=1}^{M}\sum_{f=1}^{F}\left(T_i^f-T_{di}^f\right)^2+\frac{1-w}{MF}\sum_{i=1}^{M}\sum_{f=1}^{F}\left(U_i^f-U_{di}^f\right)^2=\min.\tag{13}$$

where T_{di}^f and $T_i^f=T(x_i,t^f)$ are the measured and estimated temperatures, respectively, w is a tapering function $w\in[0,1]$, and

$$U_i^f=\left(\frac{\partial T}{\partial t}\right)_i^f\quad,\quad U_{di}^f=\left(\frac{\partial T}{\partial t}\right)_{di}^f\tag{14}$$

are the measured and estimated cooling rates, respectively.

The estimated temperatures and cooling rates are obtained from the solution of the direct problem (see: chapter 2) by using the current available estimate for the unknown parameter.

In the case of typical gradient method application [5, 7, 8] the criterion (13) is differentiated with respect to the unknown parameter Q and next the necessary condition of optimum is used. Finally one obtains the following equation

$$\frac{\mathrm{d}S(Q)}{\mathrm{d}Q}=\frac{2w}{MF}\sum_{i=1}^{M}\sum_{f=1}^{F}\left(T_i^f-T_{di}^f\right)\left(Z_i^f\right)^k+\frac{2(1-w)}{MF}\sum_{i=1}^{M}\sum_{f=1}^{F}\left(U_i^f-U_{di}^f\right)\left(W_i^f\right)^k=0\tag{15}$$

where

$$\left(Z_i^f\right)^k = \left.\frac{\partial T_i^f}{\partial Q}\right|_{Q=Q^k} \quad , \quad \left(W_i^f\right)^k = \left.\frac{\partial U_i^f}{\partial Q}\right|_{Q=Q^k} \qquad (16)$$

are the sensitivity coefficients, k is the number of iteration, Q^0 is the arbitrary assumed initial value of Q, while Q^k for $k > 0$ results from the previous iteration.

Functions T_i^f, U_i^f are expanded in a Taylor series about known value of Q^k, this means

$$T_i^f = \left(T_i^f\right)^k + \left(Z_i^f\right)^k \left(Q^{k+1} - Q^k\right) \qquad (17)$$

and

$$U_i^f = \left(U_i^f\right)^k + \left(W_i^f\right)^k \left(Q^{k+1} - Q^k\right) \qquad (18)$$

Putting (17), (18) into (13) one obtains

$$\sum_{i=1}^{M}\sum_{f=1}^{F}\left\{ w\left[\left(Z_i^f\right)^k\right]^2 + (1-w)\left[\left(W_i^f\right)^k\right]^2 \right\}\left(Q^{k+1} - Q^k\right) = \sum_{i=1}^{M}\sum_{f=1}^{F}\left\{\left[T_{di}^f - \left(T_i^f\right)^k\right]\left(Z_i^f\right)^k + \left[U_{di}^f - \left(U_i^f\right)^k\right]\left(W_i^f\right)^k\right\} \qquad (19)$$

from which results that ($k = 0, 1, ..., K$)

$$Q^{k+1} = Q^k + \frac{\sum_{i=1}^{M}\sum_{f=1}^{F}\left\{\left[T_{di}^f - \left(T_i^f\right)^k\right]\left(Z_i^f\right)^k + \left[U_{di}^f - \left(U_i^f\right)^k\right]\left(W_i^f\right)^k\right\}}{w\sum_{i=1}^{M}\sum_{f=1}^{F}\left[\left(Z_i^f\right)^k\right]^2 + (1-w)\sum_{i=1}^{M}\sum_{f=1}^{F}\left[\left(W_i^f\right)^k\right]^2} \qquad (20)$$

Fig. 3. Cooling curve

A start point of iterative procedure resulting from equation (20) corresponds to $Q^0 = 0$. It turned out, that for different tapering functions the iteration process is quickly convergent. In figure 5 the results of computations are shown. The different tapering functions have been taken into account (e.g. w = 0.75, w = 0.5, w = 0.25, etc.). The results show that the information concerning the cooling curve is most essential than the information concerning the cooling rate, but for all numerical experiments the identified value of Q was very close to the value assumed.

5. EXAMPLE OF COMPUTATIONS

The casting-mould system shown in Figure 1 has been considered. The following input data have been introduced: $\lambda_L = 20$ [W/mK], $\lambda_S = 40$ [W/mK], $\lambda_m = 1$ [W/mK], c_S = 4.875 [MJ/m^3 K], c_L = 5.904 [MJ/m^3 K], c_m = 1.75 [MJ/m^3 K] , latent heat Q = 1984.5 [MJ/m^3] (the identified parameter), pouring temperature $T_0 = 1550$ °C, liquidus temperature $T_L = 1505$ °C, solidus temperature $T_S = 1470$ °C, initial mould temperature $T_{m0} = 20$ °C.

The direct problem and sensitivity one have been solved by means of the explicit scheme of finite difference method [10]. The regular mesh created by 30×30 nodes with constant step h = 0.002 [m] has been introduced, time step Δt = 0.1 [s].

In figures 3 and 4 the cooling curve and cooling rate at the point corresponding to sensor position are shown. As was mentioned, the solution of direct problem was disturbed in a random way and the final effect is visible in figures presented.

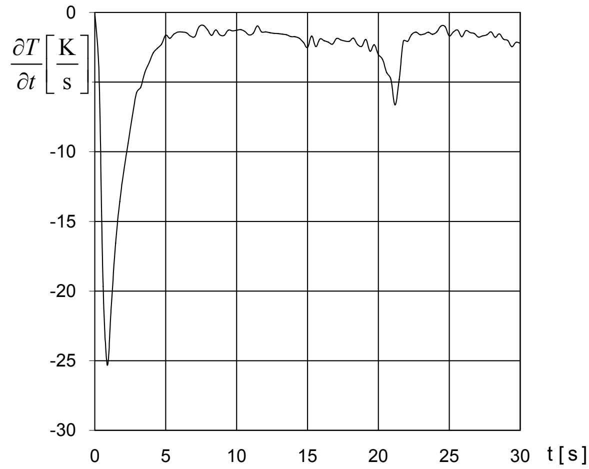

Fig. 4. Cooling rate

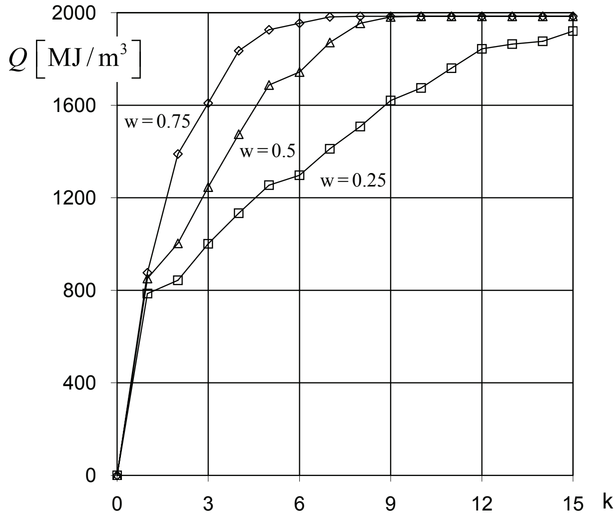

Fig. 5. Identification of Q – iteration process

6. FINAL REMARKS

The concept of cooling rate application on a stage of inverse problems solution results from the possibilities of TDA equipment. In a such case, the typical least squares criterion characteristic for gradient methods, should be supplemented by additional term concerning the differences between measured and calculated cooling rates. Introduction of $T(t)$ and $T'(t)$ to the generalized form of quality criterion seems to be profitable because the additional approachable information is used. It turned out, that the effects of this generalization are not very essential, and the basic information assuring the proper solution of inverse problem is connected with the set of temperatures creating the cooling curve. This conclusion should be verified by the solutions obtained on a basis of real measurements.

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IDENTYFIKACJA UTAJONEGO CIEPŁA KRZEPNIĘCIA STALIWA NA PODSTAWIE ANALIZY TERMICZNEJ I RÓŻNICZKOWEJ

Streszczenie

Analiza termiczna i różniczkowa (ATD) stanowi bardzo popularne i efektywne narzędzie jakościowej estymacji parametrów procesu krzepnięcia takich jak temperatury graniczne, utajonego ciepło krzepnięcia itp. System rejestrujący ATD połączony jest z termoparą umieszczoną w centralnej części odlewu próbnego. Wykorzystując odpowiedni program komputerowy automatycznego przetwarzania danych otrzymuje się informacje dotyczące krzywej stygnięcia i jej pochodnej względem czasu (szybkości krzepnięcia) w punkcie odpowiadającym położeniu sensora. W artykule przedstawiono tzw. problem odwrotny polegający na identyfikacji utajonego ciepła krzepnięcia. Dodatkową informacją niezbędną do rozwiązania tego typu zadania była znajomość przebiegu zarówno krzywej stygnięcia T(t) jak i jej pochodnej T`(t). Na etapie obliczeń numerycznych zastosowano metodę różnic skończonych. W końcowej części pracy przedstawiono wyniki identyfikacji.

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