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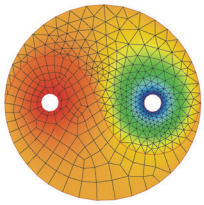

MESH ADAPTATION IN 3D NUMERICAL MODELING OF HEAT CONDUCTIVITY

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Abstract

Numerical modeling of heat conductivity in 3D is complex and time consuming issue. Mesh adaptation is one of the methods which shortens calculations time in comparison to solution on dense mesh and gives exact results, in comparison to coarsen mesh, through refinement of the mesh depending on problem type and boundary conditions. In this paper mesh discretization error is evaluated and authors present the influence of the mesh adaptation process on modeling on the example of heat conductivity. Experimental results show that mesh adaptation leads to more accurate and suited models of the physical phenomenon.

Key words: mesh adaptation, tetrahedron bisection, FEM, numerical model, simulation

1. INTRODUCTION

Since the fifties of the 20th century, the finite element method (FEM)[1] has become one of the most popular and well-established numerical techniques in engineering. It is employed in great number of commercial packages as well as in non-commercial academic software. Although commercial finite element software suites offer very rich functionality, the users sometimes face new problems that are not covered by the packages and require customizing the computations, e.g. by programming new material models.

An example of academic software is the NSC library, short for Numerical Simulations and Computations, which is a collection of C++ classes and functions organized in a library which has been developed since 1997 in the Institute of Computer and Information Sciences at Czestochowa University of Technology [2]. Initially it has solved only 2D problems and starting from a year 2004 the library was reorganised in order to be applicable to 3D problems, nowadays a mainstream in numerical computations. As a pre and post processor GiD [3] was used.

In all computations different types of errors arise. They may be divided into three categories: (i) user errors such as incorrect boundary condition or material properties, (ii) model error due to wrong assumptions or simplifications, (iii) errors caused by insufficient discretization (inadequacy or coarseness of the mesh) which are subject of presented paper.

Basing on assumptions that if the overall mesh is too coarse, the model will not capture the exact solution, in the areas of bumping changes the gradients will not be accurately predicted giving inaccurate results. In order to bear out above ideas authors investigated and measured benefits from using coarse mesh as a base for further automatic refinements dependent on the physical phenomenon on example of heat conductivity. Such an approach not only gives more accurate results but also we believe that

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it can shorten overall simulation time for large meshes although this problem was not tested here.

The paper is organized as follows: the following section provides a brief description of methods used in mesh adaptation and to partitioning mesh elements. In section 3 experimental results are presented and discussed. The last section concludes and outlines open issues and shows directions of future work.

2. MESH ADAPTATION

Mesh adaptation in general can be grouped into three different methods, depending on the level of mesh modifications. These groups are named *r*-, *p*-, and *h*-methods. In the *r*-method, the mesh connectivity is unchanged. Instead, node relocation is used to move the mesh nodes, either by means of weighted barycentric smoothing based on the location and the weight of the nodes in some neighborhood, or by means of element distortion. The criteria (weights) governing these operations are obtained by analyzing the local mesh quality not only in a geometric sense, also rules for different discretization schemes can be used. The *p*-method approach is based on an invariant mesh (in terms of points and elements) and adjusts the degree (in terms of the interpolation functions) of the used discretization scheme constructed on the mesh elements as a function of the current solution analysis. Adaptation by the *h*-method is defined in terms of local or global mesh enrichment by means of refining (by partitioning) or coarsening selected elements or all elements in a mesh [4,5].

Authors concentrate only on adaptation by *h*-method including many factors which influence on accuracy of obtained results. One of them is quality of mesh and size of the elements. This criteria is defined in different ways depending on the type of elements but generally we might say that cap or needle elements are source of problems for solvers and contribute to inappropriate results. The similar is for size of mesh but from another point of view bigger mesh increases solution time and often is unnecessary in the whole area of subject i.e. where physical phenomenon changes are small. Although it is possible to generate well suited mesh under some limited conditions, authors used approach which consists of changing the initial mesh during each time step if necessary.

The first step in such a mesh refinement process is selection of the elements which should be partitioned. This step should have been taken carefully because has strong influence both on number of elements selected to refinement and density of mesh in the next steps. The advantage is that it also strictly depends on characteristic of physical phenomenon.

In this article authors propose measuring a discretization error using approach as follows, for each face in an element mean value of temperature is calculated, then if the percentage difference between mean value and exact value of temperature from each face node is greater than it was assumed by the user the element is marked to refinement. It is obvious that a strong variation of the temperature field inside element leads to marking such ones elements to partitioning, from basis assumptions of this error estimator it detects regions containing elements with higher variation of the temperature field. The last step divides elements using method described in subsection 2.1. The additional step which is performed here is marking the adjacent element to this face to exclude it from checking in one of next iterations which brings small overall performance improvement. Since marking and splitting elements are independent this process can be easily parallelized by distributing group of element indices among set of processors.

Presented solution can be easily adapted to different type of problems such as strain stress problems, solidification [6] etc.

2.1. Tetrahedron Bisection

Regarding a way where new vertex will be defined three cases can be distinguished: (i) along an edge – two new tetrahedrons will arise, (ii) on a face – three new tetrahedrons will arise and (iii) inside a tetrahedron – four new tetrahedrons will arise. Case (iii) is quite simple and do not involve any additional divides, in case (ii) through shared face an adjacent tetrahedron will be splitted in a mirrored way into three pieces. The most difficult is case (i), called tetrahedron bisection, which requires more complex approach to dividing initial tetrahedron.

Results of partitioning such tetrahedron is presented in figure 1 where from cluster of tetrahedrons constituting sample mesh element e393 have been splitted into two new ones: e585 and e586 and one new node with index 165 was inserted. Due to the fact that inserted point lied on a one of common edges (bold edges in figure 1) of two adjacent tetrahedrons they were partitioned in the same way. It can be easily seen that in most cases process of bisection involves not only one tetrahedron but from a few to even several depending on location inside

the mesh. Information about previous connections between elements is saved in data structures in order to extrapolate values of temperature into newly inserted nodes from parents of the new elements what is described in next section. At present state of development no smoothing technique is used in the result mesh.

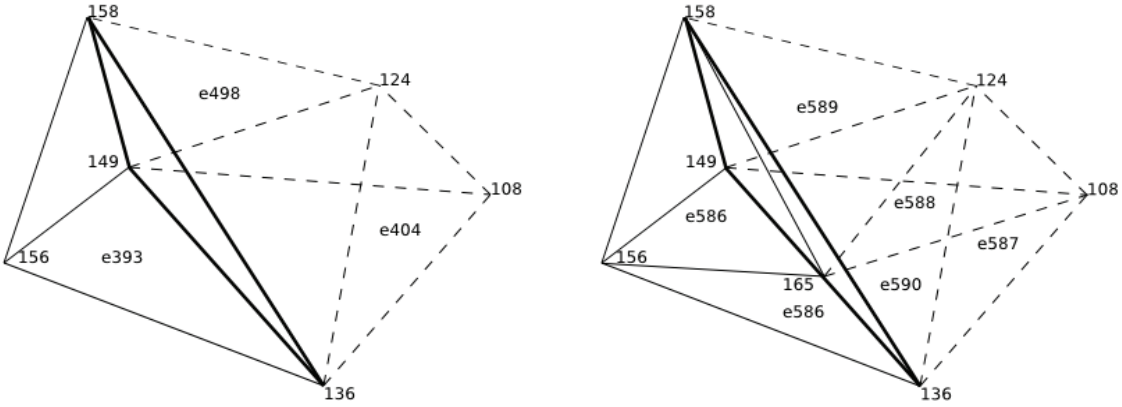

Fig. 1. *Example of partitioning tetrahedron element e393 (left side) into two new ones (right side), adjacent to bold edges tetrahedrons with dashed lines was partitioned.*

3. RESULTS

In this section description of experimental results is presented. Tests were brought off using NSC facilities for time-dependent problems i.e. the heat transfer [7].

$$-\nabla(k\nabla u) + a\dot{u} = f \quad in\ \Omega \tag{1}$$

where u is the temperature, a is the heat capacity, k is the heat conductivity and f denotes the source term with the Dirichlet, Neumann and Cauchy boundary conditions, respectively.

$$u = u_0 \quad on\ \Gamma_D \tag{2}$$

$$k\nabla u = g \quad on\ \Gamma_N \tag{3}$$

$$k\nabla u + \beta u = g \quad on\ \Gamma_C \tag{4}$$

Standard derivation leads to the following system of equations for a finite element E

$$\left(K^{(E)} + M^{(E)}\right)u^{(E)} = b^{(E)} \tag{5}$$

where

$$k_{ij}^{(E)} = \int_{\Omega_E} k\nabla\phi_i \cdot \nabla\phi_j dV \tag{6}$$

$$m_{ij}^{(E)} = \int_{\Omega_E} a\phi_i\phi_j dV + \int_{\partial\Omega_E \cap \Gamma_C} \beta\phi_i\phi_j ds \tag{7}$$

$$b_i^{(E)} = \int_{\Omega_E} f\phi_i dV + \int_{\partial\Omega_E \cap (\Gamma_C \cup \Gamma_N)} \phi_i g ds \tag{8}$$

Equation (1) is the first-order differential equation with respect to time, ϕ_i denotes basis functions. Partial discretization over space gives and ordinary differential equation which must be integrated over time. Many time stepping schemes can be used and here Euler backward was used.

$$(M_{n+1} + \Delta t K_{n+1})T_{n+1} = M_{n+1}T_n + \Delta t b_{n+1} \tag{9}$$

The superscript n means that a quantity is calculated at the time t_n. Generation of coarse-grained mesh, imposing boundary conditions and material properties, visualization results was done using GiD application using interface to the NSC library.

It is important to note that in each time step when mesh is changed new system of equations must be built. Another issue is a fact that in newly inserted nodes temperatures must be calculated in some way and this is done as follows. After changes in the mesh structure snapshot of all geometry data is created in order to save current state and for each element, node or edge information about their parents is available. Using such hierarchy and basis functions of newly inserted element unknown temperature can be easily approximated.

Experimental results were taken off using cuboid element with dimensions 0.141x0.141x0.200 [m]. Material properties were set as follows: conductivity 35 [W/(mK)], specific heat 644 [J/(kgK)], density 7760 [kg/m^3]. Initial temperature was set to 100 [K], simulation time was set to 5 [s] with time step equal to 0.01 [s]. Obtained results was compared with existing analytical solution [8] with such boundary conditions and an error was measured using following equation

$$B_i^N = \frac{T_i^N - T_i^A}{T_i^A} \cdot 100\% \qquad (10)$$

where T_i^N is value of temperature from numerical solution in node i and T_i^A is temperature in this node obtained from analytical solution.

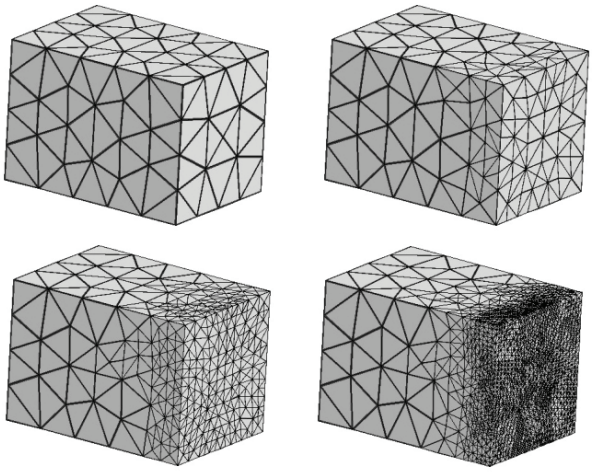

Fig. 2. *Sequence of result meshes from initial mesh (top left) through second step (top right), step 250 (bottom left) to the last step (bottom right) with refinement threshold value 10%.*

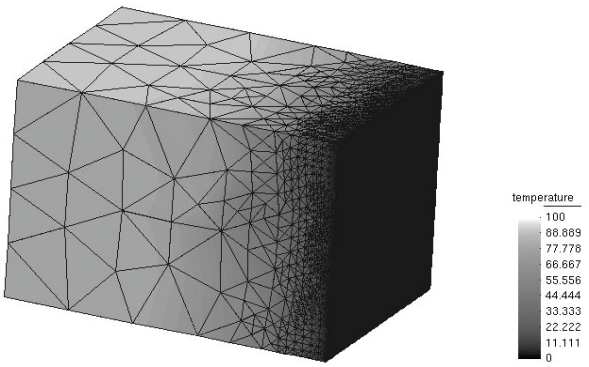

Fig. 3. *Contour fill of temperature.*

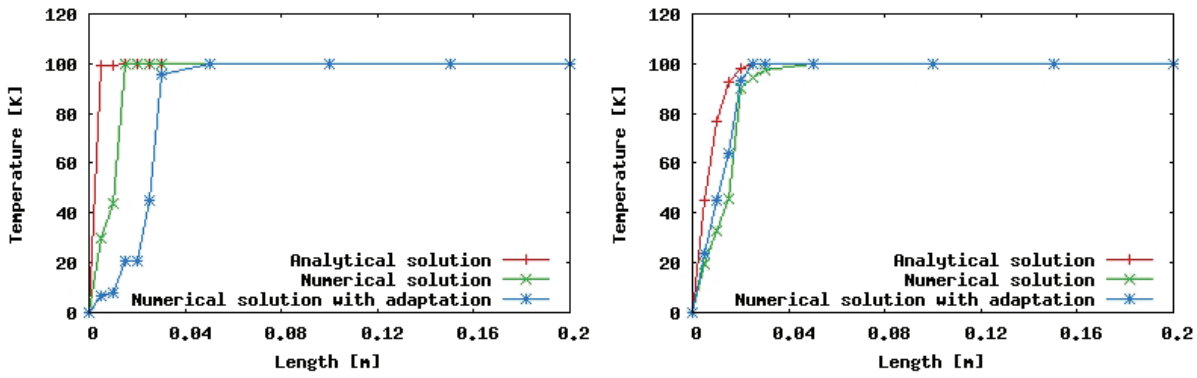

Fig. 4. *Averaged to 1D case distribution of temperature from second (left side) and the last step (right side).*

As it may be seen in figure 2 density of the mesh increases over the time in area where one can observe strong variation of field of temperature, more elements is generated in order to better coverage of physical processes. The figure 3 presents contour fill of temperature after end of simulation with adaptation. In comparison to obtained results without adaptation, what is depicted in figure 4 and 5, authors noticed that distribution of temperature field faster spreads inside the area under interest and despite bigger error in first steps of simulation next steps shows that solution converges faster with adaptation (right side of figure 5). For this example we also observed that in order to get results comparable to simulation with adaptation time need to be lengthened to about 10 seconds that is two times longer. Above considerations confirm correctness of assumptions taken by the authors and are convergent with phenomenon.

4. FINAL REMARKS

The results from previous section bears out that automatic mesh adaptation is a complex problem especially in three dimensional space and minimizes an discretization error. Although for simple case obtained results do not bring time benefits for complex geometries presented approach might improve performance and accuracy in comparison to solutions using dense uniform meshes.

Future work points to coarsening the mesh where physical phenomenon changes are small or solution is stable and might be neglected and also surface reconstruction during refinement process. There are also plans to add mesh adaptation to parallel version of the NSC library together with dynamic load balancing between nodes of parallel system. The authors believe that there is still enough unresolved issues to deal with in mesh adaptation related problems.

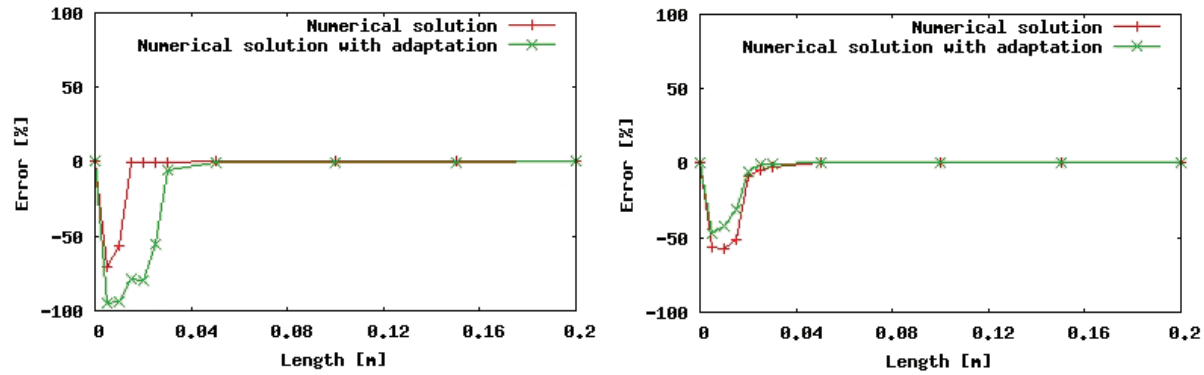

Fig. 5. Error distribution based on distribution of temperature in figure 4, second step (left side) and the last step (right side).

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ADAPTACJA SIATKI W ZAGADNIENIACH 3D NUMERYCZNEGO MODELOWANIA PRZEWODZENIA CIEPŁA

Streszczenie

Numeryczne modelowanie zagadnienia przewodzenia ciepła w przestrzeni trójwymiarowej jest złożonym i czasochłonnym zadaniem. Adaptacja siatki stanowi jedną z technik pozwalających skrócić czas symulacji numerycznych w odniesieniu do obliczeń z zastosowaniem gęstej siatki i poprawia ich dokładność poprzez proces adaptacji zależny od rozpatrywanego zagadnienia i warunków brzegowych. W pracy zawarto szacowanie błędu dyskretyzacji siatki, na podstawie którego zbadano wpływ procesu adaptacji na modelowane zagadnienie przewodzenia ciepła. Badania eksperymentalne pokazują, że zastosowanie adaptacji siatki prowadzi do uzyskania dokładniejszych wyników.

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