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TRANSIENT THERMAL ANALYSIS OF FUNCTIONALLY GRADED SHALLOW SHELLS BY THE MLPG

JAN SLADEK[1], VLADIMIR SLADEK[1], PETER SOLEK[2]

[1]*Institute of Construction and Architecture, Slovak Academy of Sciences, 84503 Bratislava, Slovakia*
[2]*Department of Mechanics, Slovak Technical University, Bratislava, Slovakia*
Corresponding Author: sladek@savba.sk (J. Sladek)

Abstract

A meshless local Petrov-Galerkin (MLPG) method is applied to solve problems of Reissner-Mindlin shells under a transient thermal load. Functionally graded materials with a continuous variation of properties in the shell thickness direction are considered here. A weak formulation for the set of governing equations in the Reissner-Mindlin theory is transformed into local integral equations on local subdomains in the base plane of the shell by using a unit test function. Nodal points are randomly spread in the base plane of the shell and each node is surrounded by a circular subdomain to which local integral equations are applied. The meshless approximation based on the Moving Least-Squares (MLS) method is employed for the implementation.

Key words: Meshless local Petrov-Galerkin method (MLPG), Mindlin theory, orthotropic properties, transient thermal load, Laplace-transform

1. INTRODUCTION

In recent years the demand for construction of huge and lightweight shell and spatial structures is increasing. To minimize the weight of shell structures a layered profile of the shell is utilized frequently. In such a case a delaminating of individual layers may occur due to a jump change of the material properties. To remove this phenomenon the functionally graded materials (FGMs) has been introduced recently. FGMs are multi-phase materials with a pre-determined property profile, where the phase volume fractions are varying gradually in space [16]. This results in continuously nonhomogenous material properties at the macroscopic structural scale. Many linear bending studies are focused only to a lateral pressure load with assumption of uniformly distributed temperature in the whole shell. However, in shells with FGM properties the role of thermal loading is more imperative. Literature sources on this subject are poor and they are mostly restricted to analyses of plates [2, 4, 9-11].

Due to the high mathematical complexity of the boundary or initial-boundary value problems, analytical approaches for FGMs are restricted to simple geometry and boundary conditions. Most significant advances in shell analyses have been made using the finite element method (FEM). It is well known that numerical results by standard displacement-based type shell element are over stiff with yielding the shear locking phenomena in thin shells. Locking problems arise due to inconsistencies in discrete representations of the transverse shear energy and the membrane energy. The boundary element method (BEM) is a very powerful computational method if a fundamental solution is available for considered problem. However, the fundamental solution for a thick orthotropic shell wit continuously

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varying material properties is not available according to the best of the author's knowledge.

Meshless approaches for solution of problems of continuum mechanics have attracted much attention during the past decade [1,3,6]. One of the most rapidly developed meshfree methods is the Meshless Local Petrov-Galerkin method (MLPG) [1,5,7]. The solution of the uncoupled problem in the present paper is split into two tasks. In the first task the temperature distribution in the shell has to be obtained by solving the diffusion equation. The temperature distribution in shell has to be analyzed as 3-D problem. The MLPG is applied to transient heat conduction equations in a continuously nonhomogeneous solid. The Laplace transform technique is used to eliminate the time variable. Several quasi-static boundary value problems must be solved for various values of the Laplace-transform parameter. The Stehfest's inversion method is applied to obtain the time-dependent solution. In the second task, the set of governing differential equations for Reissner-Mindlin shell bending theory with Duhamel-Neumann constitutive equations is solved. Since thermal changes in solids are relatively slow with respect to elastic wave velocity, the inertial terms in Reissner-Mindlin governing equations are not considered. The problem is considered as quasi-static with time dependent thermal forces. The MLPG method is applied again to solution of that problem with the meshless Moving Least-Squares (MLS) approximation of primary field variables.

2. MESHLESS LOCAL INTEGRAL EQUATIONS FOR HEAT CONDUCTION PROBLEMS IN SHALLOW SHELLS

The thermal problem should be solved first, in order to determine the temperature distribution. Material properties are assumed to be continuously variable along the shell thickness. The heat conduction problem in a continuously nonhomogeneous anisotropic medium is described by

$$\rho(\mathbf{x})c(\mathbf{x})\frac{\partial\theta}{\partial t}(\mathbf{x},t)=\left[\lambda_{ij}(\mathbf{x})\theta_{,j}(\mathbf{x},t)\right]_{,i}+Q(\mathbf{x},t) \quad (1)$$

where $\theta(\mathbf{x},t)$ is the temperature field, $Q(\mathbf{x},t)$ is the density of body heat sources, λ_{ij} is the thermal conductivity tensor, $\rho(\mathbf{x})$ is the mass density and $c(\mathbf{x})$ the specific heat.

Let the analyzed domain of the shallow shell is denoted by Ω with the top and bottom surfaces being S^+ and S^-, respectively. Arbitrary temperature or heat flux boundary conditions can be prescribed on all considered surfaces. The initial condition is assumed $\theta(\mathbf{x},t)\big|_{t=0}=\theta(\mathbf{x},0)$ in the analyzed domain Ω.

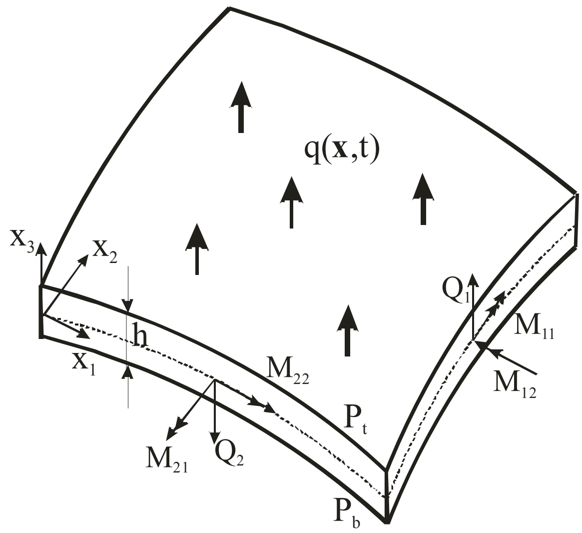

Fig. 1. *Sign convention of bending moments and forces for FGM shallow shell*

Applying the Laplace transformation to the governing equation (1), one obtains

$$\left[\lambda_{ij}(\mathbf{x})\bar{\theta}_{,j}(\mathbf{x},s)\right]_{,i}-\rho(\mathbf{x})c(\mathbf{x})s\,\bar{\theta}(\mathbf{x},s)=-\bar{F}(\mathbf{x},s) \quad (2)$$

where

$$\bar{F}(\mathbf{x},s)=\bar{Q}(\mathbf{x},s)+\theta(\mathbf{x},0)$$

is the redefined body heat source in the Laplace transform domain, with the inclusion of the initial boundary condition for the temperature and s is the Laplace transform parameter.

The weak form is constructed over local subdomains Ω_s, which is a small sphere taken for each node inside the global domain:

$$\int_{\Omega_s^a}\left[\left(\lambda_{ij}(\mathbf{x})\bar{\theta}_{,j}(\mathbf{x},s)\right)_{,i}-\rho(\mathbf{x})c(\mathbf{x})s\,\bar{\theta}(\mathbf{x},s)+\bar{F}(\mathbf{x},s)\right]\theta^*(\mathbf{x})\,d\Omega=0 \quad (3)$$

where $\theta^*(\mathbf{x})$ is a weight (test) function.

Applying the Gauss divergence theorem to equation (3) and choosing a unit test function on each subdomain, one obtains the local integral equation

$$\int_{\partial\Omega_s^a} \bar{q}(\mathbf{x},s)d\Gamma - \int_{\Omega_s^a} \rho(\mathbf{x})c(\mathbf{x})s\bar{\theta}(\mathbf{x},s)d\Omega = -\int_{\Omega_s^a} \bar{F}(\mathbf{x},s)d\Omega, \tag{4}$$

where $\partial\Omega_s^a$ is the boundary of the local subdomain and

$$\bar{q}(\mathbf{x},s) = \lambda_{ij}(\mathbf{x})\bar{\theta}_{,j}(\mathbf{x},s)n_i(\mathbf{x}) .$$

The MLS approximation of the heat flux $\bar{q}(\mathbf{x},s)$ is assumed as

$$\bar{q}^h(\mathbf{x},s) = \lambda_{ij}n_i \sum_{a=1}^{n} \phi_{,j}^a(\mathbf{x})\hat{\theta}^a(s).$$

Substituting the MLS-approximations into the local integral equation (4), the system of algebraic equations is obtained

$$\sum_{a=1}^{n}\left(\int_{L_s+\Gamma_{sp}} \mathbf{n}^T \Lambda \mathbf{P}^a(\mathbf{x})d\Gamma - \int_{\Omega_s} \rho c s \phi^a(\mathbf{x})d\Gamma \right)\hat{\theta}^a(s) = -\int_{\Gamma_{sq}} \tilde{\bar{q}}(\mathbf{x},s)d\Gamma - \int_{\Omega_s} \bar{F}(\mathbf{x},s)d\Omega \tag{5}$$

at interior nodes as well as at the boundary nodes on $\partial\Omega_N$, where $\partial\Omega_N$ is the part of the global boundary surface $\partial\Omega$ on which the heat flux is prescribed and $\Gamma_{sq} = \partial\Omega_s \cap \partial\Omega_N$. In equation (5), we have used the notations

$$\Lambda = \begin{bmatrix} \lambda_{11} & \lambda_{12} & \lambda_{13} \\ \lambda_{12} & \lambda_{22} & \lambda_{23} \\ \lambda_{13} & \lambda_{23} & \lambda_{33} \end{bmatrix}, \quad \mathbf{P}^a(\mathbf{x}) = \begin{bmatrix} \phi_{,1}^a \\ \phi_{,2}^a \\ \phi_{,3}^a \end{bmatrix},$$

$$\mathbf{n}^T = (n_1, n_2, n_3), \tag{6}$$

$$L_s \cup \Gamma_{sp} = \partial\Omega_s, \; \Gamma_{sp} = \partial\Omega_s \cap \partial\Omega_D,$$

$$\partial\Omega = \partial\Omega_D \cup \partial\Omega_N .$$

The time dependent values of the transformed quantities can be obtained by an inverse Laplace-transformation. In the present analysis, the Stehfest's algorithm [15] is used.

3. ANALYSES OF ORTHOTROPIC FGM SHALLOW SHELLS UNDER A THERMAL LOAD

Consider a linear elastic orthotropic shallow shell of constant thickness *h* and with its mid-surface being described by $x_3 = f(x_1, x_2)$ in a domain S with the boundary contour Γ in the base plane $x_1 - x_2$. The Reissner-Mindlin bending theory [8, 12] is used to describe the shell deformation. The material properties are graded along the shell thickness.

The bending moments $M_{\alpha\beta}$ and shear forces Q_α for α,β=1,2, can be expressed in terms of rotations, lateral displacements of the orthotropic plate and temperature

$$M_{\alpha\beta} = D_{\alpha\beta}\left(w_{\alpha,\beta} + w_{\beta,\alpha}\right) + C_{\alpha\beta}w_{\gamma,\gamma} - H_{\alpha\beta}$$

$$Q_\alpha = C_\alpha\left(w_\alpha + w_{3,\alpha}\right), \tag{7}$$

where

$$H_{\alpha\beta} = \int_{-h/2}^{h/2} x_3 \gamma_{\alpha\beta} \theta(\mathbf{x}, x_3, t)dx_3 .$$

For a general variation of material properties through the shell thickness:

$$D_{11} = \int_{-h/2}^{h/2} x_3^2 E_1(x_3)\frac{1-\nu_{21}}{e}dx_3 ,$$

$$D_{22} = \int_{-h/2}^{h/2} x_3^2 E_2(x_3)\frac{1-\nu_{12}}{e}dx_3 ,$$

$$D_{12} = \int_{-h/2}^{h/2} x_3^2 G_{12}(x_3)dx_3 ,$$

$$C_{11} = \int_{-h/2}^{h/2} x_3^2 E_1(x_3)\frac{\nu_{21}}{e}dx_3 ,$$

$$C_{22} = \int_{-h/2}^{h/2} x_3^2 E_2(x_3)\frac{\nu_{12}}{e}dx_3 ,$$

$$C_\alpha = \kappa\int_{-h/2}^{h/2} G_{\alpha 3}(x_3)dx_3$$

where $\kappa = 5/6$ in the Reissner plate theory.

Similarly the normal forces $N_{\alpha\beta}$ (α, β=1, 2) are given as

$$\begin{bmatrix} N_{11} \\ N_{22} \\ N_{12} \end{bmatrix} = \mathbf{P}\begin{bmatrix} u_{1,1} \\ u_{2,2} \\ u_{1,2}+u_{2,1} \end{bmatrix} + \begin{bmatrix} Q_{11} \\ Q_{22} \\ 0 \end{bmatrix} w_3 - \begin{bmatrix} \theta_{11} \\ \theta_{22} \\ 0 \end{bmatrix}, \tag{8}$$

where

$$\theta_{\alpha\beta} = \int_{-h/2}^{h/2} \gamma_{\alpha\beta}\theta(\mathbf{x}, x_3, t)dx_3 ,$$

$$\mathbf{P}=\begin{bmatrix} E_1^*/e & E_1^*\nu_{21}/e & 0 \\ E_2^*\nu_{12}/e & E_2^*/e & 0 \\ 0 & 0 & G_{12}^* \end{bmatrix},$$

$$\begin{bmatrix} Q_{11} \\ Q_{22} \\ 0 \end{bmatrix}=\begin{bmatrix} (k_{11}+k_{22}\nu_{12})E_1^*/e \\ (k_{11}\nu_{12}+k_{22})E_2^*/e \\ 0 \end{bmatrix}, \tag{9}$$

$$E_\alpha^* \equiv \int_{-h/2}^{h/2} E_\alpha(x_3)dx_3 = \begin{cases} E_{\alpha t}=E_{\alpha b}\ , & n=0 \\ (E_{\alpha b}+E_{\alpha t})/2, & n=1\ , \\ (2E_{\alpha b}+E_{\alpha t})/3, & n=2 \end{cases}$$

$$G_{12}^* \equiv \int_{-h/2}^{h/2} G_{12}(x_3)dx_3 = \begin{cases} G_{12t}=G_{12b}\ , & n=0 \\ (G_{12b}+G_{12t})/2, & n=1\ , \\ (2G_{12b}+G_{12t})/3, & n=2 \end{cases}$$

and $k_{\alpha\beta}$ are the principal curvatures of the shell in x_1 - and x_2 -directions.

Using the Reissner's linear theory of shallow shells [12], the quasi-static form of the equations of motion may be written as

$$\begin{gathered} M_{\alpha\beta,\beta}(\mathbf{x},t)-Q_\alpha(\mathbf{x},t)=0\,, \\ Q_{\alpha,\alpha}(\mathbf{x},t)-k_{\alpha\beta}N_{\alpha\beta}(\mathbf{x},t)=0\,, \\ N_{\alpha\beta,\beta}(\mathbf{x},t)=0\,,\ \ \mathbf{x}\in S\ . \end{gathered} \tag{10}$$

Thermal changes in solids are relatively slow with respect to elastic wave velocity. Therefore, inertial terms are not considered in governing equations. Moreover, mechanical loads are considered to be vanishing too.

The MLPG method constructs the weak-form over local subdomains S_s

$$\int_{\Omega_s^i}\left[M_{\alpha\beta,\beta}(\mathbf{x},t)-Q_\alpha(\mathbf{x},t)\right]w_{\alpha\gamma}^*(\mathbf{x})d\Omega=0\,, \tag{11}$$

$$\int_{\Omega_s^i}\left[Q_{\alpha,\alpha}(\mathbf{x},t)-k_{\alpha\beta}N_{\alpha\beta}(\mathbf{x},t)\right]w_3^*(\mathbf{x})d\Omega=0\,, \tag{12}$$

$$\int_{\Omega_s^i}\left[N_{\alpha\beta,\beta}(\mathbf{x},t)\right]u_{\alpha\gamma}^*(\mathbf{x})d\Omega=0\,. \tag{13}$$

Applying the Gauss divergence theorem to local weak forms and choosing the test functions as a unit step function one obtains local boundary-domain integral equations

$$\int_{\partial S_s^i} M_\alpha(\mathbf{x},t)d\Gamma-\int_{S_s^i} Q_\alpha(\mathbf{x},t)d\Omega=0 \tag{14}$$

$$\int_{\partial S_s^i} Q_\alpha(\mathbf{x},t)n_\alpha(\mathbf{x})d\Gamma-\int_{S_s^i} k_{\alpha\beta}(\mathbf{x})N_{\alpha\beta}(\mathbf{x},t)d\Omega=0 \tag{15}$$

$$\int_{\partial S_s^i} T_\alpha(\mathbf{x},t)d\Gamma=0\,. \tag{16}$$

where

$$T_\alpha(\mathbf{x},t)=N_{\alpha\beta}(\mathbf{x},t)n_\beta(\mathbf{x})\ . \tag{17}$$

The generalized displacements (two rotations and deflection) are approximated as

$$\mathbf{w}^h(\mathbf{x},t)=\mathbf{\Phi}^T(\mathbf{x})\cdot\hat{\mathbf{w}}(t)=\sum_{a=1}^{n}\phi^a(\mathbf{x})\hat{\mathbf{w}}^a(t), \tag{18}$$

Substituting the approximation (18) into the definition of the normal bending and shear forces, one obtains

$$\begin{aligned} \mathbf{M}(\mathbf{x},t)=\mathbf{N}_1\sum_{a=1}^{n}\mathbf{B}_1^a(\mathbf{x})\mathbf{w}^{*a}(t)\ + \\ \mathbf{N}_2\sum_{a=1}^{n}\mathbf{B}_2^a(\mathbf{x})\mathbf{w}^{*a}(t)-\mathbf{H}(\mathbf{x},t)\ = \\ =\mathbf{N}_\alpha(\mathbf{x})\sum_{a=1}^{n}\mathbf{B}_\alpha^a(\mathbf{x})\mathbf{w}^{*a}(t)\ -\mathbf{H}(\mathbf{x},t), \end{aligned} \tag{19}$$

$$\mathbf{Q}(\mathbf{x},t)=\mathbf{C}(\mathbf{x})\sum_{a=1}^{n}\left[\phi^a(\mathbf{x})\mathbf{w}^{*a}(t)+\mathbf{F}^a(\mathbf{x})\hat{w}_3^a(t)\right], \tag{20}$$

where matrices definitions are given in [13, 14].

The in-plane displacements are approximated by

$$\mathbf{u}^h(\mathbf{x},t)=\mathbf{\Phi}^T(\mathbf{x})\cdot\hat{\mathbf{u}}(t)=\sum_{a=1}^{n}\phi^a(\mathbf{x})\hat{\mathbf{u}}^a(t)\,. \tag{21}$$

Then, the traction vector can be expressed as

$$\begin{aligned} \mathbf{T}(\mathbf{x},t)=\mathbf{N}_1(\mathbf{x})\mathbf{P}(\mathbf{x})\sum_{a=1}^{n}\mathbf{B}^a(\mathbf{x})\mathbf{u}^{*a}(t)+ \\ \mathbf{J}(\mathbf{x})\sum_{a=1}^{n}\phi^a(\mathbf{x})\hat{w}_3^a(t)-\mathbf{\Theta}(\mathbf{x},t)\,. \end{aligned} \tag{22}$$

Furthermore, in view of the MLS approximations (19), (20) and (22) for the unknown fields in the local boundary-domain integral equations (14) - (16), we obtain their discretized forms as

$$\sum_{a=1}^{n}\left[\int_{L_s^i+\Gamma_{sw}^i}\mathbf{N}_\alpha(\mathbf{x})\mathbf{B}_\alpha^a(\mathbf{x})d\Gamma-\int_{S_s^i}\mathbf{C}(\mathbf{x})\phi^a(\mathbf{x})d\Omega\right]\mathbf{w}^{*a}(t)\ -$$

$$-\sum_{a=1}^{n}\left[\int_{S_s^i}\mathbf{C}(\mathbf{x})\mathbf{F}^a(\mathbf{x})d\Omega\right]\hat{w}_3^a(t)=$$

$$\int_{L_s^i+\Gamma_{sM}^i}\mathbf{H}(\mathbf{x},t)d\Gamma-\int_{\Gamma_{sM}^i}\tilde{\mathbf{M}}(\mathbf{x},t)d\Gamma \qquad (23)$$

$$\sum_{a=1}^{n}\left[\int_{\partial S_s^i}\mathbf{C}_n(\mathbf{x})\phi^a(\mathbf{x})d\Gamma\right]\mathbf{w}^{*a}(t)-$$

$$-\mathbf{K}(\mathbf{x})^{\mathrm{T}}\mathbf{P}(\mathbf{x})\sum_{a=1}^{n}\left[\int_{S_s^i}\mathbf{B}^a(\mathbf{x})d\Omega\right]\mathbf{u}^{*a}(t)+$$

$$+\sum_{a=1}^{n}\left[\int_{\partial S_s^i}\mathbf{C}_n(\mathbf{x})\mathbf{F}^a(\mathbf{x})d\Gamma-\int_{S_s^i}\mathrm{O}(\mathbf{x})\phi^a(\mathbf{x})d\Omega\right]\hat{w}_3^a(t)=$$

$$\int_{S_s^i}\left[\theta_{11}(\mathbf{x},t)k_{11}+\theta_{22}(\mathbf{x},t)k_{22}\right]d\Omega \qquad (24)$$

$$\sum_{a=1}^{n}\left[\int_{L_s^i+\Gamma_{su}^i}\mathbf{N}_1(\mathbf{x})\mathbf{P}(\mathbf{x})\mathbf{B}^a(\mathbf{x})d\Gamma\right]\mathbf{u}^{*a}(t)\;+$$

$$\sum_{a=1}^{n}\left[\int_{S_s^i}\mathbf{J}(\mathbf{x})\phi^a(\mathbf{x})d\Omega\right]\hat{w}_3^a(t)=$$

$$=\int_{L_s^i+\Gamma_{sM}^i}\mathbf{\Theta}(\mathbf{x},t)d\Gamma-\int_{\Gamma_{sP}^i}\tilde{\mathbf{T}}(\mathbf{x},t)d\Gamma. \qquad (25)$$

Recall that the discretized local boundary-domain integral equations (23)-(25) are considered on the sub-domains adjacent to the interior nodes $\mathbf{x}^i$ as well as to the boundary nodes on Γ_{sM}^i and Γ_{sP}^i. For the source point $\mathbf{x}^i$ located on the global boundary Γ the boundary of the subdomain ∂S_s^i is composed of the interior and the boundary portions L_s^i and Γ_{sM}^i, respectively, or alternatively of L_s^i and Γ_{sP}^i, with the portions Γ_{sM}^i and Γ_{sP}^i lying on the global boundary with prescribed bending moments or stress vector, respectively. Equations (23) and (25) are vector equations for the two components of rotations and in-plane displacements, respectively. Then, the set of Eqs. (23)-(25) represents 5 equations at each node for five unknown components, namely, two rotations, one out-of-plane deflection and two in-plane displacements.

4. NUMERICAL EXAMPLES

A shallow spherical shell with a square contour is investigated in the first example here. We consider simply supported boundary conditions of the shell with a side-length $a = 0.254m$ and the thicknesses $h/a = 0.05$. A thermal shock $\theta = H(t-0)$ with Heaviside time variation is applied on the top surface of the shallow shell. If the lateral ends of the shell are thermally insulated, a uniform temperature distribution on shell surfaces is given. The bottom surface is thermally insulated too. A homogeneous and isotropic medium is considered: Young's moduli $E_1 = E_2 = 0.6895\cdot10^{10}$ N/m^2, Poisson's ratios $\nu_{21} = \nu_{12} = 0.3$, and the thermal expansion coefficients $\alpha_{11} = \alpha_{22} = 1\cdot10^{-5}\deg^{-1}$, the thermal conductivity $\lambda = 100W/m\deg$, mass density $\rho = 7500kg/m^3$ and specific heat $c = 400Ws/kg\deg$.

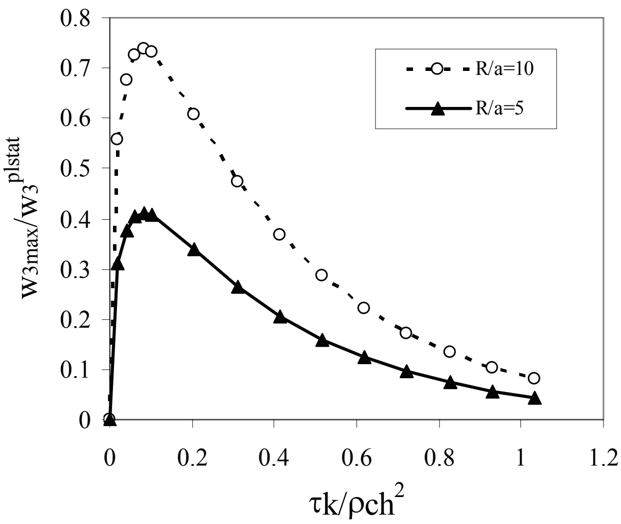

Fig. 2. Time variation of the central deflection in the shell

The numerical results for the central shell deflection are presented in figure 2. Two different shell curvatures are considered here. The deflections are normalized by the central deflection corresponding to the stationary thermal distribution with $\theta = 1\deg$ on the top plate surface and vanishing temperature on the bottom surface. For homogeneous material properties the corresponding stationary deflection is $w_3^{plstat} = 0.4829\cdot10^{-5}m$. One can observe that in the whole time interval deflections for both shells are lower than in a stationary case. The deflection is approaching to zero for a large time since thermal forces are vanishing. The temperature distribution is going to be uniform in the whole shell with increasing time.

The bending moment at the center of the plate $M_{11}^{plstat} = 0.4634Nm$ is used as a normalized parameter in figure 3. The time variations of bending moments for the shells of both curvatures are similar. The peak value of the bending moment is larger for larger curvature of the shell.

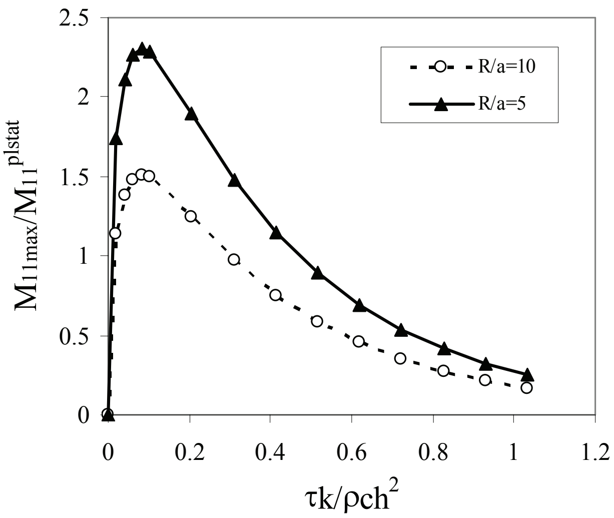

Fig. 3. Time variation of the bending moment

5. CONCLUSIONS

A meshless local Petrov-Galerkin method is applied to orthotropic shallow shells under a thermal load. Material properties are continuously varying along the shell thickness. The behaviour of the shell is described by the Reissner-Mindlin theory, which takes the shear deformation into account. The temperature distribution in shells is determined by the heat conduction equation. The Laplace-transform technique is applied to eliminate the time variable in the considered diffusion equation. The MLPG method for 3-D problem is used to solve the governing equation in the Laplace transform space. Mechanical quantities are described by quasi-static governing equation following from Reissner-Mindlin theory with time playing the role of monotonic parameter. The MLPG method is applied to solve this problem. The Moving Least-Squares (MLS) scheme is adopted for approximating the physical quantities. The main advantage of the present method is its simplicity and generality.

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ANALIZA TERMICZNA POWŁOK Z FUNKCJONALNYCH MATERIAŁÓW GRADIENTOWYCH Z WYKORZYSTANIEM METODY MLPG

Streszczenie

Tematem niniejszej pracy jest wykorzystanie lokalnej beziastkowej metody Petrova-Galerkina (MLPG) do problemu odkształceń termicznych powłok Reissnera-Mindlina. W studium wykorzystano model funkcjonalnych materiałów gradientowych z założeniem ciągłej zmiany własności na grubości elementu powłokowego. Forma słaba równań występujących w teorii Reissnera-Mindlina została przeniesiona na zbiór równań całkowych rozwiązywanych w obszarze zdefiniowanych poddomen. Cylindryczne poddomeny losowo otaczają wygenerowane punkty węzłowe. W rozwiązaniu wykorzystano beziatkową aproksymację metody Moving Least-Squares (MLS).

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