Publishing House
AKAPIT

COMPUTER METHODS IN MATERIALS SCIENCE
Informatyka w Technologii Materiałów

Vol. 9, 2009, No. 1

A NEW METHOD FOR THE EVALUATION OF THE YIELD CRITERIA ACCURACY

LIANA PARAIANU, DOREL BANABIC

Research Centre in Sheet Metal Forming Technologies - CERTETA
Technical University of Cluj-Napoca, C. Daicoviciu 15, 400020 Cluj-Napoca, Romania
Corresponding Author: banabic@tcm.utcluj.ro (D. Banabic)

Abstract

The paper presents a test used to estimate the predictive accuracy of a few yield criteria for anisotropic materials. The performances of the models have been tested on a sort of sheet metal widely used in the industry: aluminium alloy AA3103-0. The results (predicted yield locus, distributions of the uniaxial yield stress, anisotropy coefficients) show the influence of the number of material parameters included in the identification procedure on the accuracy of the criteria.

Key words: plastic anisotropy, yield criterion, accuracy index

1. INTRODUCTION

A fair description of the plastic anisotropy of thin sheet metals plays an important role in forming operations, once material flow, springback as well as wrinkling and limit strains are significantly affected by the yield criterion used in the analysis. Such a criterion is a relationship which identifies the stress states capable to produce the transition of the material from the elastic domain to the plastic one. This relationship can be interpreted as being the equation of a geometrical entity in the stress space (the so-called yield surface [1]). In the analytical relationships that describe the yield criteria, the components of stress tensor are accompanied by material parameters. The values of the parameters are determined using experimental data. During the time, the researchers tried to find reliable formulations easy to be used in practice. It has been noticed that using more material parameters in the expression of the yield criterion will ensure better predictions. The pioneer of the development of anisotropic yield criteria was Hill [2]. He proposed in 1948 a quadratic formulation. The quadratic description has been improved by Hill himself in 1990 and 1993. Barlat defined several extensions of the isotropic yield criterion proposed by Hosford in 1951, using a linear transformation of the Cauchy stress. In a recent work, Barlat [3] introduced a new criterion based on two linear transformations of the stress tensor. The group of yield criteria proposed by Banabic [4] and improved later on by Banabic et al. [5-7] also belongs to the “Hosford family”. Using general invariants, Cazacu and Barlat [8] proposed an extension of Drucker´s isotropic yield criterion. Finally, in 2005, Aretz [9] developed a full stress state (3D) yield criterion proposing two formulations taking into account different numbers of material parameters. The most recent anisotropic yield criterion belongs to Leacock [10] and represents a development of the expression proposed by Hill in 1990. A good synthesis of the yield criteria proposed during the time can be found in [11-13]). The yield criteria developed by the authors are characterized

148 – 152 ISSN 1641-8581

by the simplicity of their mathematical form. This thing makes them efficient and, in the same time, easy to implement in finite element codes, as well as in theoretical models. The aim of the present work is to evaluate the performances of the anisotropic yield criteria. With this aim in view, the authors propose the use of a quality fit test. The testing has been done by calculating root-mean square error for the anisotropy coefficients, the uniaxial yield stresses and yield locus. The material used for testing is aluminium alloy AA3103 – 0. Due the fact that the BBC models are developments of the Hill 1948 [2] and Barlat 1989 [14] yield criteria, the predictions of these two functions are also subjected to the quality test.

2. DESCRIPTION OF THE YIELD CRITERIA SUBJECTED TO TESTING

The yield locus is described by an implicit equation, having the form:

$$\Phi\left(\bar{\sigma}, Y_{ref}\right) := \bar{\sigma} - Y_{ref} = 0 \qquad (1)$$

where $\bar{\sigma}$ is the equivalent stress and Y_{ref} is a yield parameter.

2.1. Hill 1948 yield criterion

In 1948, Hill proposed an anisotropic generalization of von Mises criterion. In the case of thin sheet metal under plane – stress condition, the expression of equivalent stress is [2]:

$$\bar{\sigma} = \left[G\hat{\sigma}_{11}^2 + F\hat{\sigma}_{22}^2 + H\left(\hat{\sigma}_{11} - \hat{\sigma}_{22}\right)^2 + N\hat{\sigma}_{12}^2 + N\hat{\sigma}_{21}^2\right]^{1/2} \qquad (2)$$

The components $\sigma_{\alpha\beta}$ $(\alpha, \beta = 1, 2)$ are expressed in the system of plastic orthotropy axes (1 is the rolling direction – RD, 2 is the transverse direction – TD, while 3 is the normal direction – ND). The parameters and are constant quantities. For their identification, the value of the yield stress σ_0^{exp} must be known, as well as the anisotropic coefficients corresponding to the directions inclined at 0°, 45°, 90° with respect to RD ($r_0^{exp}, r_{45}^{exp}, r_{90}^{exp}$). Using the convention $Y = \sigma_0$, in the equation of yield surface 1, the determination of coefficients from the criterion is straightforward. The simplicity and coherence of the identification procedure are the most important advantages of Hill 1948 model. In addition, the experiments needed for calculating the coefficients are usually performed in the industrial laboratories. Anyway, this criterion has a few drawbacks. The most important one is the fact that the criterion cannot model the "anomalous" behaviour of first degree.

2.2. Barlat 1989 yield criterion

In the particular case of thin sheets under plane stress conditions, Barlat 1989 [14] yield criterion has the following formulation:

$$\bar{\sigma} = \left[a\left|\hat{K}_1 + \hat{K}_2\right|^m + a\left|\hat{K}_1 - \hat{K}_2\right|^m + (1-a)\left|2\hat{K}_2\right|^m\right]^{\frac{1}{m}} \qquad (3)$$

where

$$\hat{K}_1 = \frac{\hat{\sigma}_{11} + h\hat{\sigma}_{22}}{2}; \hat{K}_2 = \sqrt{\left(\frac{\hat{\sigma}_{11} + h\hat{\sigma}_{22}}{2}\right)^2 + p^2\hat{\sigma}_{12}} \qquad (4)$$

and p and h are material parameters used to describe the anisotropy of the yield surfaces, is a material constant that takes values in the [0,1] range. The exponent m is an integer value. Its value is chosen according to the crystallographic structure of the material: $m = 6$ for BCC alloys, and $m = 8$ for FCC alloys. To identify the a, h and p material parameters, we need the experimental values of the anisotropy coefficients r_0^{exp}, r_{45}^{exp} and r_{90}^{exp}. This time, the identification procedure has to be done using numerical methods.

2.3. BBC2003 yield criterion

Following [6], the equivalent stress used in equation 1 has the form:

$$\bar{\sigma} = \left[a\left(\Gamma + \Psi\right)^{2k} + a\left(\Gamma - \Psi\right)^{2k} + (1-a)\left(2\Lambda\right)^{2k}\right]^{\frac{1}{2k}} \qquad (5)$$

where the functions Γ, Ψ şi Λ are defined as:

$$\begin{aligned} \Gamma &= \frac{\hat{\sigma}_{11} + M\hat{\sigma}_{22}}{2} \\ \Psi &= \sqrt{\frac{\left(N\hat{\sigma}_{11} - P\hat{\sigma}_{22}\right)^2}{4} + \left(Q\hat{\sigma}_{12}\right)^2} \\ \Lambda &= \sqrt{\frac{\left(R\hat{\sigma}_{11} - S\hat{\sigma}_{22}\right)^2}{4} + \left(T\hat{\sigma}_{12}\right)^2} \end{aligned} \qquad (6)$$

The coefficient k has the same meaning as for Hosford yield criterion. The other coefficients M, N, P, Q, R, S and T are also material parameters. They are calculated using the following experimental data: $\sigma_0^{exp}, \sigma_{45}^{exp}, \sigma_{90}^{exp}, \sigma_b^{exp}, r_0^{exp}, r_{45}^{exp}, r_{90}^{exp}, r_b^{exp}$.

Due to the fact that the determination of equibiaxial coefficients is time-consuming and needs special machines and special specimens, some particular formulations of the yield criterion have been defined. These are reliable and offer good predictions even if they need less parameters.

2.3.1. BBC2003 yield criterion with 6 coefficients

In this case the following restrictions are imposed:

$$N = R, \quad M = P \tag{7}$$

The identification procedure needs the experimental determination of the following mechanical characteristics of the sheet metal: uniaxial yield stresses and uniaxial anisotropic coefficients associated to the directions defined by the angles $0^0, 45^0$ and 90^0 from RD ($\sigma_0^{exp}, \sigma_{45}^{exp}, \sigma_{90}^{exp}, r_0^{exp}, r_{45}^{exp}, r_{90}^{exp}$). To obtain those experimental data only uniaxial tensile tests are needed.

2.3.2. BBC2003 yield criterion with 7 coefficients

By using only the first condition from equation 7, the BBC2003 yield criterion with 7 parameters is obtained. Thus, besides the mechanical parameters used at §2.3.1., the equibiaxial yield stress (σ_b^{exp}) is needed.

2.3.3. BBC2003 yield criterion with 8 coefficients

In this case, six coefficients are established using the uniaxial material properties, described as above. The other two coefficients ($\sigma_b^{exp}, r_b^{exp}$) are determined using a biaxial tension machine and cruciform specimens. The experimental procedure and the tension test machine are described in [5].

3. DEFINING THE ACCURACY COEFFICIENTS

In order to test the accuracy of the yield criteria in a more systematic manner, a quality test is proposed. The test determines root-mean square errors for yield stresses:

$$\Delta\sigma = \sqrt{\frac{\sum_{i=1}^{n} y_i^2}{n-1}} \Big/ \sigma_{med} \tag{8}$$

where the residual y_i has been calculated using the formula:

$$y_i = \frac{n-1}{n}\left|\sigma_i^{exp} - \sigma_i^{teor}\right| - \frac{1}{n}\sum_{\substack{i=1\\ j\neq i}}^{n}\left|\sigma_j^{exp} - \sigma_j^{teor}\right| \tag{9}$$

n is the number of the pairs of data subjected to comparison, σ^{exp} and σ^{teor} are the experimental and theoretical uniaxial yield stresses obtained at certain angles, and σ_{med} is the average value of the yield stress:

$$\sigma_{med} = \frac{\sigma_0^{exp} + 2\sigma_{45}^{exp} + \sigma_{90}^{exp}}{4} \tag{10}$$

A similar function was also developed to determine the distribution of the root-mean square errors for the anisotropy coefficient:

$$\Delta r = \sqrt{\frac{\sum_{i=1}^{n} y_i^2}{n-1}} \Big/ r_{med} \tag{11}$$

where the residual y_i has the following form:

$$\mathrm{y_i} = \frac{\mathrm{n-1}}{\mathrm{n}}\left|\mathrm{r_i^{exp} - r_i^{teor}}\right| - \frac{1}{\mathrm{n}}\sum_{\substack{\mathrm{i=1}\\ \mathrm{j\neq i}}}^{\mathrm{n}}\left|\mathrm{r_j^{exp} - r_j^{teor}}\right| \tag{12}$$

n is the number of the pairs of data compared, r^{exp} and r^{teor} are the experimental and theoretical anisotropy coefficients obtained at certain angles, and r_{med} is the average value of the anisotropy coefficient:

$$r_{med} = \frac{r_0^{exp} + 2r_{45}^{exp} + r_{90}^{exp}}{4} \tag{13}$$

To measure the accuracy of the theoretic yield locus the following expression is proposed:

$$\Delta\sigma_{sp} = \sqrt{\frac{\sum_{i=1}^{n}\left(y_{1i}^2 + y_{2i}^2\right)}{n-1}} \Big/ \sigma_{med} \tag{14}$$

where the residuals $\mathrm{y_{1i}}$ and $\mathrm{y_{2i}}$ are calculated as follows:

$$y_{1i} = \frac{n-1}{n}\left|\sigma_{1i}^{exp} - \sigma_{1i}^{teor}\right| - \frac{1}{n}\sum_{\substack{i=1\\ j\neq i}}^{n}\left|\sigma_{1j}^{exp} - \sigma_{1j}^{teor}\right| \tag{15}$$

$$y_{2i} = \frac{n-1}{n}\left|\sigma_{2i}^{exp} - \sigma_{2i}^{teor}\right| - \frac{1}{n}\sum_{\substack{i=1\\ j\neq i}}^{n}\left|\sigma_{2j}^{exp} - \sigma_{2j}^{teor}\right| \tag{16}$$

n is the number of the pairs of the data compared, σ^{exp} and σ^{teor} are the experimental and theoretical yield stresses from the principal directions 1 and 2, obtained at certain angles and σ_{med} is average value of the yield stress determined with the help of the relation 10.

4. EVALUATION OF THE BBC MODELS

To evaluate the accuracy of the yield criteria, a sort of aluminium alloy is used, namely AA3103-0. The mechanical parameters are presented in table 1. The identification of the yield criteria coefficients have been done following the authors' suggestions.

Figure 1 shows the distribution of the *r* – coefficient in the plane of the sheet, corresponding to the AA3103 – 0 alloy. One may notice that the distributions are coincident, and the root - mean square error is situated between 0.57% in the case of Hill 1948 and 0.75% in the case of BBC2003 yield criterion. The range between those limit values is very low, due to the fact that the identification procedure used to calculate the parameters of the yield criteria is based on three experimental values of the *r* – coefficient (r_0, r_{45} and r_{90}).

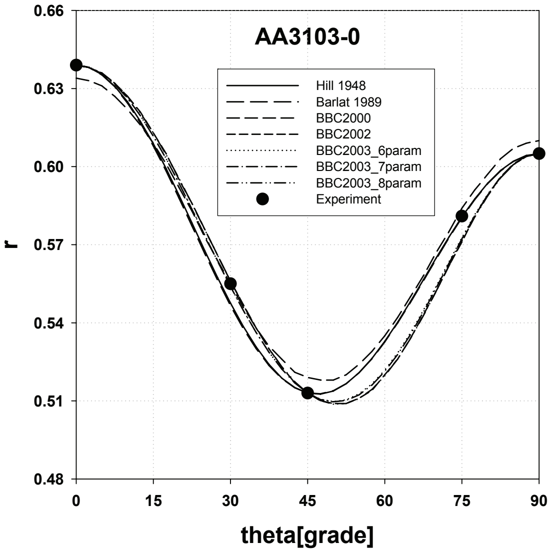

Fig. 1. Distribution of the r-coefficient in the plane of the sheet.

Table 1. Mechanical parameters of the material AA3103-0.

MECHANICAL PARAMETERS	σ_0^{exp}	σ_{45}^{exp}	σ_{90}^{exp}	σ_b^{exp}	r_0^{exp}	r_{45}^{exp}	r_{90}^{exp}	r_b^{exp}
AA3103-0	55	58	61	60	0.513	0.810	0.605	1.604

Table 2. Root - mean square errors of the yield stresses and anisotropy coefficients introduced by the yield criteria for AA3103-0.

YIELD CRITERION	HILL 1948	BARLAT 1989	BBC2003_6 PARAM	BBC2003_7 PARAM	BBC2003_8 PARAM
σ %	5.84	5.79	0.43	0.42	0.40
r %	0.57	0.64	0.63	0.67	0.75

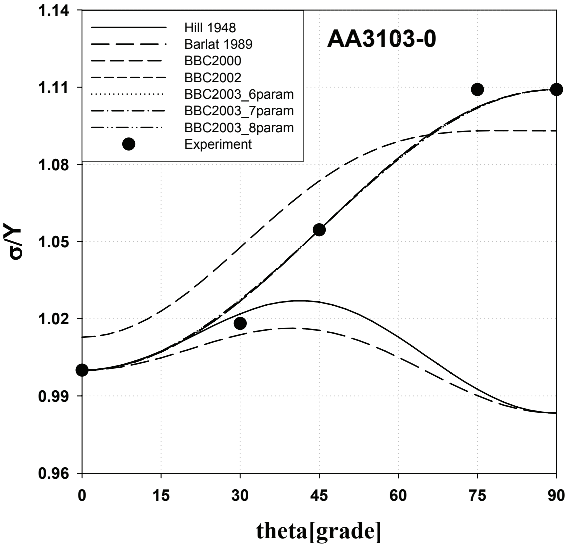

Fig. 2. Distribution of the uniaxial yield stress in the plane of the sheet.

Figure 2 evaluates the way that all the yield criteria describe the distribution of the uniaxial yield stress in the plane of the sheet. On the same diagram, the experimental points corresponding to the uniaxial yield stresses $Y_0, Y_{30}, Y_{45}, Y_{75}, Y_{90}$ for aluminium alloy have been plotted. From all the yield criteria, only BBC2003 (with all three alternatives) rigorously fit three experimental points, because they are used in the identification procedure. Hill 1948 and Barlat 1989 use in the identification only the uniaxial yield stress at the angle $\theta = 0^0$ from the RD, failing to detect the other two values. If one takes into account the fact that the representation on the ordinate axis is normalized, the bigger relative deviation does not exceed 5.84%, corresponding to the Hill 1948 predictions. The minimum value of the average error is caught by more criteria: BBC2003_6param, BBC2003_7param and BBC2003_8param (0.4%).

Figure 3 shows the accuracy of the yield criteria in relation with their capability to predict the shape of the yield locus.

Table 3 contains the root - mean square errors of the yield locus introduced by the yield criteria for aluminium alloy. The root - mean square errors are located in the range 8.86% predicted by Hill 1948 and 3.1%, value predicted by BBC2003.

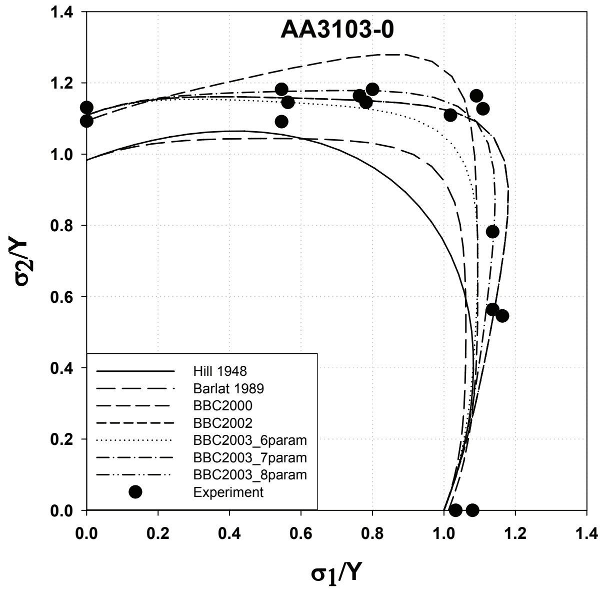

Fig. 3. Yield locus for AA3103-0.

Table 3. *Root - mean square errors of the yield locus for AA3103-0.*

HILL 1948	BARLAT 1989	BBC2003-6 PARAM	BBC2003-7 PARAM	BBC2003-8 PARAM
8.86%	6.9%	4.35%	3.58%	3.1%

5. CONCLUSIONS

Following the qualitative study on the yield criteria developed by the authors, one may emphasize that the best predictions are given by the functions that use more material parameters. Although the determination of many material constants is expensive, their use in the identification procedure considerably increases the accuracy of the models. The results of this study show that such models are in a good agreement with experimental data for aluminium alloys. The authors also prove the high flexibility of the BBC 2003 yield criterion, even if only uniaxial material data are used for calculating the coefficients.

REFERENCES

1. Mattiasson, K., Continuum Mechanics Principles for Large Deformation in Solid and Structural Mechanics, Publication 81:6-Departement of Structural Mechanics, Chalmers University of Technology-Göteborg, 1981.
2. Hill, R., The Mathematical Theory of Plasticity, Oxford, Clarendon Press, 1950.
3. Barlat, F., Brem, J.C., Yoon, J.W., Chung, K., Dick, R.E., Lege, D.J., Pourboghrat, F., Choi, S.-H., Chu, E., Plane stress yield function for aluminium alloy sheets – part 1: theory, Int. J. of Plasticity, 19, 2003, 1297 – 1319.
4. Banabic, D., Comsa, D.S., Balan, T., A new yield criterion for orthotropic sheet metals under plane stress conditions, Proc. 7th TPR 2000 Conf. on Cold Metal Forming, ed., Banabic, D., Printek Publishing House, Cluj Napoca, 2000, 217-224.
5. Banabic, D., Kuwabara, T., Balan, T., Comsa, D.S., Julean, D., Non -quadratic yield criterion for orthotropic sheet metals under plane-stress conditions, Int. J. Mechanical Sciences, 45, 2003, 797-811.
6. Banabic, D., Aretz, H., Comşa, D.S., Părăianu, L., An improved analytical description of orthotropy in metallic sheets, International Journal of Plasticity, 21, 2005, 493-512.
7. Comsa, D.S., Banabic, D., Plane-stress yield criterion for highly-anisotropic sheet metals, Numisheet 2008, ed., Hora, P., Interlaken, 2008, 43-48.
8. Cazacu, O., Barlat, F., Generalization of Drucker's yield criterion to orthotropy, Mathematics and Mechanics of Solids, 6, 2001, 613-630.
9. Aretz, H., Barlat, F., General orthotropic yield functions based on linear stress deviator transformations, Materials Processing and Design. Modeling, Simulation and Applications, ed, Ghosh, A.K., 2004, 147-151.
10. Leacock, A. G., A mathematical description of orthotropy in sheet metals, Journal of the Mechanics and Physics of Solids, 54, 2006, 425-444.
11. Barlat, F., Cazacu, O., Zyczkowski, M., Banabic, D., Yoon, J.-W., Yield surface plasticity and anisotropy, Continuum Scale Simulation of Engineering Materials Fundamentals-Microstructures-Process Applications, eds, Raabe, D., Chen, L.Q., Barlat, F., Roters, F., Wiley-VCH, Weinheim, 2004, 145-185.
12. Banabic, D., Tekkaya, E.A., Forming Simulation, Virtual Fabrication of Aluminum Alloys: Microstructural Modeling in Industrial Aluminum Production, ed., Hirsch, J., Wiley-VCH, Weinheim, 2006, 275-303.
13. Banabic, D., Barlat, F., Cazacu, O., Kuwabara, T., Anisotropy and formability, Advances in Material Forming-ESAFORM 10 Years on, eds, Chinesta, F., Cueto, E., Springer, Heidelberg-Berlin, 2007, 143-173.
14. Barlat, F., Lian, J., Plastic Behavior and Stretchability of Sheet Metals. Part.I: A Yield Function for Orthotropic Sheet under Plane Stress Conditions, Int. J. of Plasticity, 5, 1989, 51-66.

NOWA METODA OSZACOWANIA DOKŁADNOŚCI KRYTERIUM PLASTYCZNOŚCI

Streszczenie

Artykuł przedstawia test wykonany w celu oszacowania przewidywanej dokładności kilku kryteriów plastyczności dla materiałów anizotropowych. Działanie modeli zostało przetestowane na przykładzie blachy cienkiej ze stopu aluminium AA3103-0 szeroko stosowanej w przemyśle. Wyniki (oszacowanie miejsca płynięcia materiału, jednoosiowy rozkład granicy plastyczności, współczynniki anizotropii) pokazały wpływ wielu parametrów materiału uwzględnionych w procedurze identyfikacji na dokładność analizowanych kryteriów.

Submitted: October 22, 2008
Submitted in a revised form: December 4, 2008
Accepted: December 4, 2008