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AKAPIT

COMPUTER METHODS IN MATERIALS SCIENCE
Informatyka w Technologii Materiałów
Vol. 9, 2009, No. 1

USING FUZZY NUMBERS IN FATIGUE RELIABILITY ESTIMATION

TOMASZ BEDNAREK[1,2], PIOTR PROKOPOWICZ[2], WŁODZIMIERZ SOSNOWSKI[1,2]

[1] Institute of Fundamental Technological Research, PAN, Warsaw, Poland
[2] Kazimierz Wielki University, Bydgoszcz, Poland
Corresponding Author: bednarek@ippt.gov.pl (T. Bednarek)

Abstract

Increasing safety requirements of structures induces necessity of taking into consideration the random nature of variables in the fatigue design. Fatigue analysis of real structures in connection with probabilistic analysis needs a big computational effort. In this paper a new approach and methodology to fatigue analysis of structures using fuzzy numbers are presented. The method has been developed. A new efficient tool of fatigue analysis was created.

The calculation of fatigue life using fuzzy numbers set is illustrated with analysis of hydraulic cylinders. Local fatigue approach is used at critical locations on hydraulic cylinders.

The purpose of the probabilistic design is to estimate the probability of failure for critical locations on hydraulic cylinder.

Key words: fuzzy numbers, fatigue analysis, fatigue reliability

1. FUZZY NUMBERS - INTRODUCTION

In common real-life problems often both parameters and data used in mathematical modelling are vague. For example if we do not have thermometer at the moment we think about temperature in terms: cold, slightly cold, warm, etc, and concerning that information we set up (often also in vague terms like: a bit more) heater (or ventilator). In mathematical modelling vagueness can be described by a fuzzy sets and numbers. Idea of the fuzzy sets was started in 1965 by Zadeh [1]. Fuzzy set theory has its well-known achievements in many branches of science and technology. Fuzzy concept have been introduced in order to model such vague terms, as observed values of some physical or economical terms, like pressure values or stock market rates, that can be inaccurate, can be noisy or can be difficult to measure with an appropriate precision because of technical reasons. In our daily life there are many cases that observations of objects in a population are fuzzy.

In modern complex and large-scale systems it is difficult to treat the systems using only exact data. In this case also, it is inevitable to treat non-exact data involving human vagueness. In order to use imprecise data effectively we must have tools for processing them. The problem of adding vague information is present and solved in many versions of the calculus on fuzzy sets and numbers.

Fuzzy numbers and fuzzy arithmetic were introduced in 1975 by Zadeh [2] to analyze and manipulate approximate numerical values. In most approaches for numerical handling of fuzzy quantities the so-called *extension principle* is of fundamental importance. It gives a formal apparatus to carry on operations (arithmetic or algebraic) from sets to fuzzy sets. Unfortunately, the use of the extension principle results has some serious drawbacks, espe-

 ISSN 1641-8581

cially in case of performing the whole sequences of operations repeatedly.

A number of attempts to introduce non-standard operations on fuzzy numbers have been made, for example [3].

Most popular model of fuzzy numbers is called convex fuzzy numbers [4].

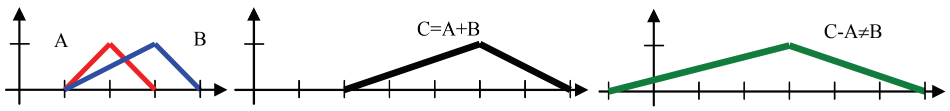

Fig. 1. *Example of standard calculations on fuzzy numbers which presents drawback.*

The main drawbacks of the existing models of operations on fuzzy numbers, regarded as a part of the theory of fuzzy sets defined on the real axis are: non-existence of natural neutral elements of addition and multiplication, non-existence of a unique solution X of the equation $A + X = C$, where A and C are fuzzy numbers. The second drawback implies that having two fuzzy numbers A and C it may not exist a fuzzy number B for equation $A+B$ is equal to C (figure1).

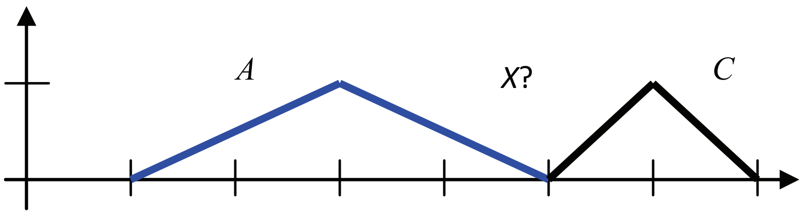

Fig. 2. *A+X=C.*

2. ORDERED FUZZY NUMBERS

Few years ago the ordered fuzzy numbers idea arose [5,6]. For constructing those numbers the concept of the membership function of a fuzzy set, introduced by L. Zadeh [1] as a fundamental concept of the fuzzy (multivalued) logic, has been weakened by requiring a mere membership relation; consequently a fuzzy number arises as an ordered pair of continuous real functions defined on the interval [0,1]. Four algebraic operations: addition, subtraction, multiplication and division of such fuzzy numbers have been constructed in a way that renders them algebra.

DEFINITION 1.

An **Ordered Fuzzy Number** (OFN) A is an ordered pair of two continuous functions,

$$A = (x_{UP}, x_{DOWN}) \tag{1}$$

called the *up-part* and the *down-part*, respectively, both defined on the closed interval [0,1] with values in real axis.

With some conditions (if the both functions x_{UP}, x_{DOWN} are monotonic then they are invertible and possess the corresponding inverse functions defined on real axis with values in [0,1]) we can transform shape from figure 2 into shape on figure 3.

Interpretation of such model is slightly extended what was discussed i.e. [7].

In result we deal with objects as presented below with an arrow as mark of order from *up part* to *down part*.

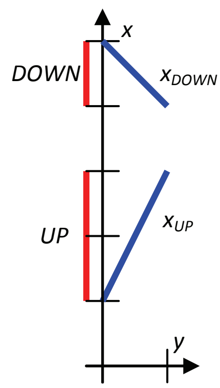

Fig. 3. *Ordered fuzzy number.*

We may define in simply way arithmetic operations on ordered fuzzy numbers. They are introduced in the following definition.

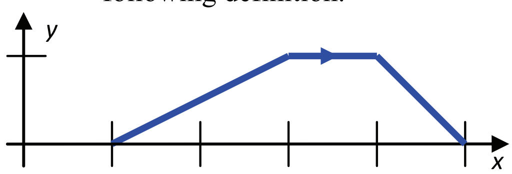

Fig. 4. *Short notation of direction in the OFN.*

DEFINITION 2.

Let $A=(x^A_{UP}, x^A_{DOWN})$, $B=(x^B_{UP}, x^B_{DOWN})$ and $C=(x^C_{UP}, x^C_{DOWN})$ are ordered fuzzy numbers. The sum $C=A+B$, subtraction $C=A-B$, product $C=A\cdot B$, and division $C=A/B$ are defined by formula for $s \in$ [0,1]

$$\begin{aligned} x^C_{UP} &= x^A_{UP}(s) * x^B_{UP}(s) \\ x^C_{DOWN} &= x^A_{DOWN}(s) * x^B_{DOWN}(s) \end{aligned} \tag{2}$$

where "*" stands for "+", "-", "•", and "/", respectively, and where *A/B* is defined, if zero does not belong to intervals *UP* and *DOWN* of *B*.

Notice that the subtraction of *B* is the same as addition of the opposite of *B*, i.e. the number *(-1) B*. Moreover, for any ordered fuzzy number *B* the difference *B-B* is a crisp (real) zero. On the other hand, if for $A=(x^A{}_{UP},\ x^A{}_{DOWN})$ we define its *complement* $A^- =(-x^A{}_{DOWN},\ -x^A{}_{UP})$ (please note that $A^- \neq (-1)\cdot A$), then the sum $A+A^-$ gives a fuzzy zero $0=(x^A{}_{UP} - x^A{}_{DOWN},\ -(x^A{}_{UP} - x^A{}_{DOWN}))$ in the sense of the classical fuzzy number calculus: the *complementary number* plays the role of the opposite number in the sense of the Zadeh's model, since the sum of the both gives a fuzzy zero, non-crisp, in general.

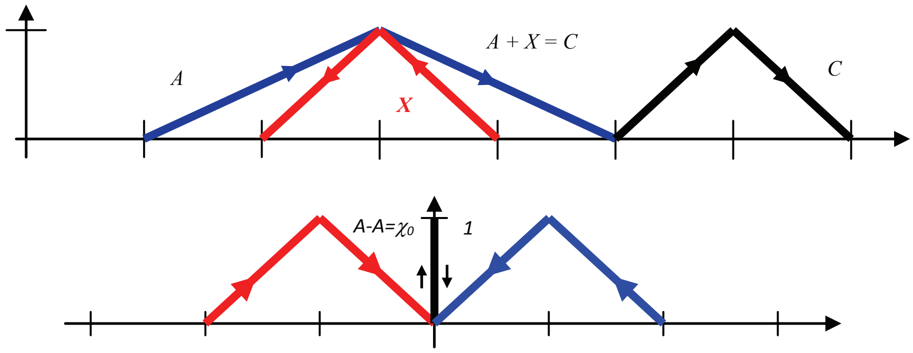

Fig. 5. *Interesting results of arithmetical operations on OFN.*

Going further in such direction we can easy propose the more complicated operations as *logarithm* and a *power* of number. Of course we need to consider restrictions which are following from the same operations on real numbers.

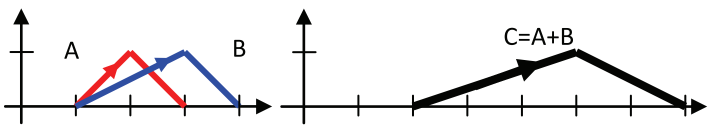

Fig. 6. *Results can be the same like in classical fuzzy numbers (see figure 1).*

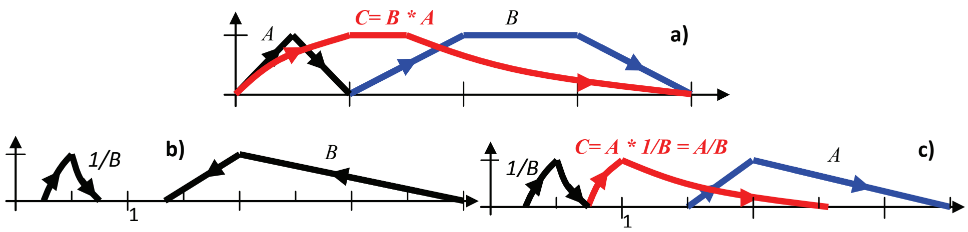

Fig. 7. *Examples of OFN counting: a) multiplications, b) calculating reverse for c) calculation result of division.*

With such definitions of operations we can find clear-cut result of equation $A+X=C$, also result of subtraction $A–A$ is a crisp zero (neutral element of addition).

Important fact is that if we restrict our analysis of operations on ordered fuzzy numbers to numbers with the same orientation then results will be the same or very similar to classical operations on convex fuzzy numbers.

Additionally, by using ordered fuzzy numbers we obtain simple algorithms of all algebraic operations to make counting's results obvious. We just calculate functions values.

In summary, using ordered fuzzy numbers we can overcome drawbacks from classical fuzzy numbers model. A method of realization of arithmetical operations allows for calculations in the same way like calculations with real numbers. Thanks to easiness of calculating on ordered fuzzy numbers we also can in quite simply way programming them to use in different applications, where is difficult to get precise data.

As the fatigue is a parameter, which is really hard to determine precisely, so using here the idea of fuzzy numbers looks like the direction worth to investigate. In case of good and easy for implementation algorithms of calculations on ordered fuzzy numbers the new model of fuzzy numbers were used in this paper in fatigue analysis of structures.

3. ANALYTICAL FORM OF S–N FUNCTION

An analytical form of the fatigue strength function with respect of the number of cycles *N* and the stress ratio *R* is proposed. This work extends the work of Luccioni, Chaboche and Oller [8,9,10].

$$SN(R,N) = S^{th}(R) + \left[S^u - S^{th}(R)\right]\cdot 10^{\left[-\alpha_t(R)\cdot\log_{10}(N)^{\beta}\right]}, \quad (3)$$

where, for $|R| \leq 1$

$$S^{th}(R) = S^e + (S^u - S^e)\cdot(0.5 + 0.5\cdot R)^{\gamma}$$
$$\alpha_t(R) = \alpha + (0.5 + 0.5\cdot R)\cdot\delta \qquad (4a)$$

and for $|R| > 1$

$$S^{th}(R) = S^e + (S^u - S^e)\cdot(0.5 + \frac{1}{2R})^{\gamma}$$
$$\alpha_t(R) = \alpha + (0.5 + \frac{1}{2R})\cdot\delta . \qquad (4b)$$

S^u is the material tensile strength, $\alpha, \beta, \gamma, \delta$ are the material parameters, S^{th} is the fatigue threshold function depending on the endurance fatigue stress S^e and the stress ratio R.

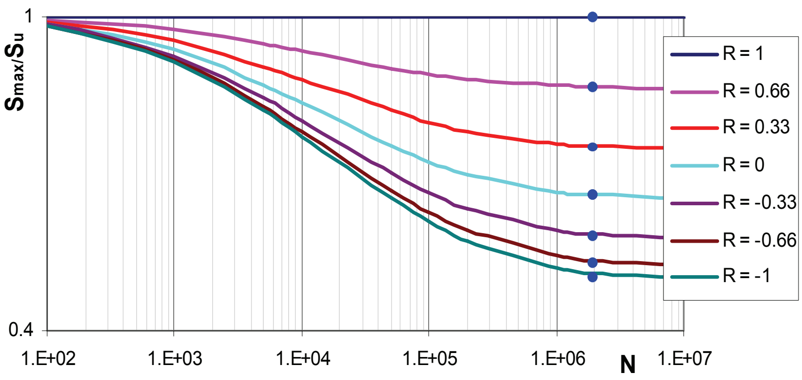

Fig. 8. *The proposed S–N curve for different R values.*

A modification of the $S - N$ function formulas developed by Oller [9] is done in order to lead to a full agreement of the presented method with the classical stress methods. The introduced correction also takes into account an influence of mean stress value σ_{med} and the stress ratio R on fatigue life of the structure in agreement with Goodman's concept [11].

The proposed $S - N$ function is presented in figure 8. The influence of the stress ratio R is shown. The bold points indicate fatigue threshold values calculated by Goodman's concept. After transformation of equation (3) and substitution of the stress analysis result (i.e. FEM analysis introduced by [11]) as $\bar{\sigma} = SN(R,N)$, the fatigue life of the structure is obtained. Fatigue life is expressed as a number of cycles to failure.

$$N_{cyc}(R) = 10^{\left[\left(\frac{\log\left(\frac{S^u - S^{th}(R)}{\bar{\sigma} - S^{th}(R)}\right)}{\alpha_t(R)}\right)^{\frac{1}{\beta}}\right]} \qquad (5)$$

The $S - N$ curve determined by equation (3) and equation (4a) or (4b) allows to determine fatigue life of the structure where the load is characterized by constant amplitude and constant stress ratio. It is observed that in fatigue crack propagation consecutive material points are damaged (separated) along a crack. In order to describe evolution of fatigue crack authors introduce a material strength reduction function, $f_{red}(\bar{\sigma}, R, N) \equiv 1 - d$ [1,3].

$$f_{red}(\bar{\sigma}, R, N) = 10^{\frac{\log\left(\frac{\bar{\sigma}}{S^u}\right)}{\log(N_{cyc})^{\beta m}}\cdot\log(N)^{\beta m}} \qquad (6)$$

where exponent m is the material brittleness measure. In figure 9 reduction of material strength function for different parameters m is presented.

The strength reduction function is identified with damage of material point $f_{red}(\bar{\sigma}, R, N) \equiv 1 - d$. The constitutive tensor of damaged material C^d_{ijkl} is given

$$C^d_{ijkl} = f_{red}(\bar{\sigma}, R, N) C^0_{ijkl} \text{ and}$$
$$\sigma_{ij} = f_{red}(\bar{\sigma}, R, N)\sigma^o_{ij} \qquad (7)$$

where C^0_{ijkl} and σ^0_{ij} are constitutive tensor and is stress tensor in undamaged material respectively. Besides

$$S^{ud} = f_{red}(\bar{\sigma}, R, N) S^u \qquad (8)$$

where S^{ud} is the damaged material strength.

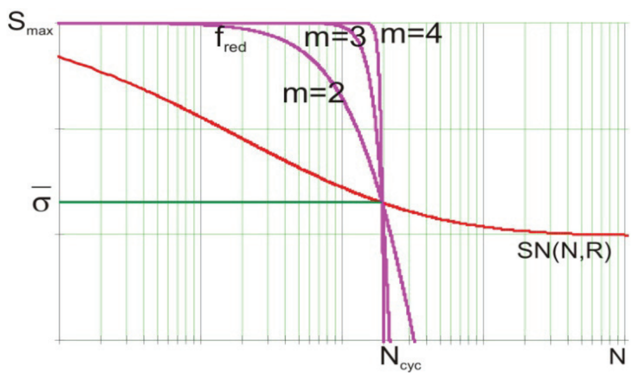

Fig. 9. *Proposed strength reduction function.*

4. CALCULATION OF FATIGUE RELIABILITY WITH FUZZY NUMBERS

Classical fatigue reliability analysis of structures in connection with probabilistic analysis needs a big computational effort. This allows only small academical problems to be considered. The fuzzy numbers approach to reliability analysis allows estimating stochastic model parameters. In consequence a fatigue life of structure can be treated as imprecise term which is modeled by fuzzy number. Analysis of fuzzy fatigue life could be an estimation of reliability.

Imprecision in fuzzy number idea have a different interpretation than probabilistic however probabilistic data can be used to determine a fuzzy number.

The calculation of fatigue life using fuzzy numbers set is illustrated with analysis of the hydraulic cylinder presented in figure 10. Local fatigue approach is used at critical locations on hydraulic cylinders. Linear elasticity material behavior is assumed.

Seven critical points in oil port connection zone are chosen (see figure 11). Points 1 to 6 correspond to oil port weld and point 7 corresponds to hole in cylinder tube. The von Mises stress field of hydraulic cylinder port is presented in figure 11. The critical point, corresponding to a hole in the cylinder tube is marked as number 7 (370 MPa with internal pressure 30.8 MPa). Fatigue experiments prove that fatigue crack is initiated in this point. In further analysis we will consider only the critical point 7.

Fuzzy numbers idea was applied to define material strength S_u and value of internal oil pressure p. Material *S-N* curve is obtained using equation (3) and it is also fuzzy function. Graphical interpretation of the scatter of material strength and internal oil pressure were presented in figure 12. Most of all cylinders are made from steel with ultimate stress 540 MPa, but also there exist some cylinders made from steel with ultimate stress 520 MPa and 650 MPa. The distribution of material ultimate stress S_u corresponds to statistical data. Similarly, most of all cylinders work with internal pressure about 30 MPa, but other cylinders in some cases work with internal pressure 15 MPa or 45 MPa.

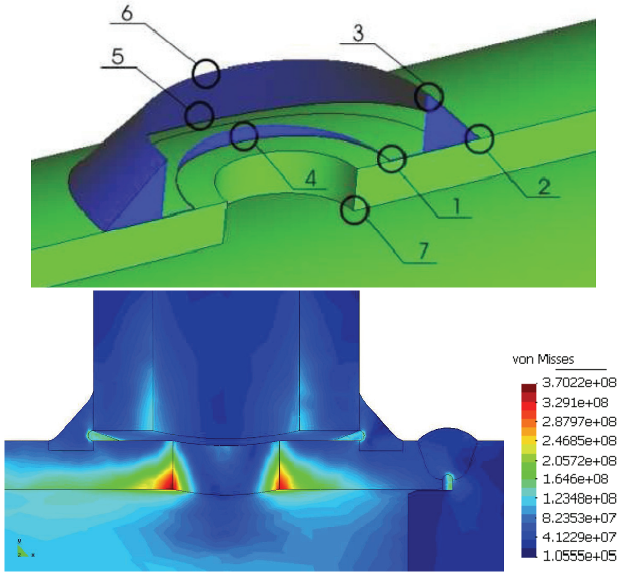

Fig. 11. *Critical points in hydraulic cylinder fatigue resistance (left hand side) and von Mises stress field in oil port connection zone (right hand side).*

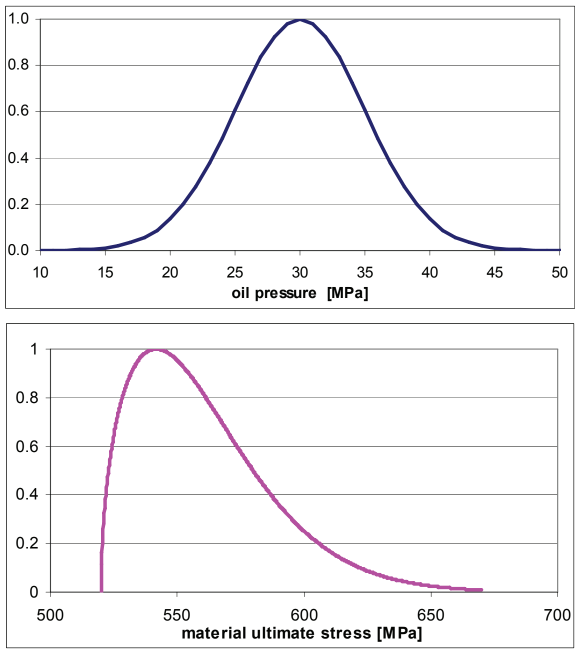

Fig. 12. *Graphical interpretation of the scatter of material strength and internal oil pressure.*

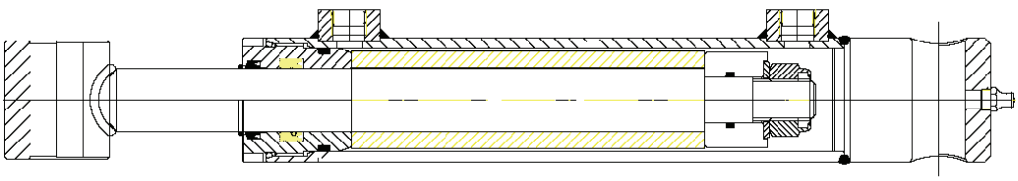

Fig. 10. *Hydraulic cylinder.*

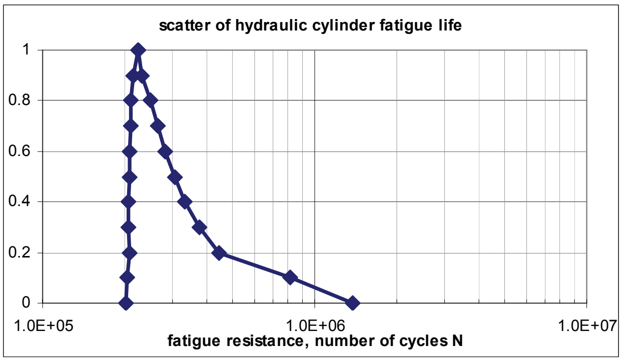

Fig. 13. *The scatter of fatigue life of the hydraulic cylinder.*

The scatter of fatigue life of the cylinder is presented in figure 13. We can see, that majority of cylinders have fatigue life grater than 220 000 cycles. The analysis of the scatter of fatigue life could make possible to estimate the fatigue reliability of the cylinder or plan service repairs. The methodology of fatigue reliability using fuzzy numbers needs further investigations and interpretations of obtained results.

5. CONCLUSIONS

- The paper deals with fatigue reliability analysis of structures. The new approach using fuzzy numbers was presented.
- The presented fatigue analysis with fuzzy numbers could decrease computer resources necessary to carry on reliability analysis with probabilistic approach.
- The result of fatigue reliability analysis of structures using fuzzy numbers is full scatter of fatigue resistance. This information gives considerably more data about reliability in opposite to classical probabilistic methods where the result is a percentage reliability with given failure criterion.
- The methodology of fatigue reliability using fuzzy numbers demand further investigations and interpretations of obtained results.
- The experimental or field tests of reliability of structures results are one of the most confidential data. Validation of presented method is difficult. In time of the conference authors will present comparison of reliability obtained by presented method and calculated by classical, probabilistic software.

ACKNOWLEDGEMENTS

We thank for the financial support of the EC project PROHIPP inside the 6FP, priority 3NMP FP62002-NMP-2-SME, research area 3.4.3.1.5.

REFERENCES

1. Chaboche, J.L., Continuum damage mechanics and its application to structural lifetime prediction, Rech. Aerosp, 4, 1987, 37–54.
2. Dubois, D., Prade, H., Operations on fuzzy numbers, Int. J. System Science, 9, 1978, 576-578.
3. Klir, G.J., Fuzzy arithmetic with requisite constraints, Fuzzy Sets and Systems, 91, 1997, 165-175.
4. Kosiński, W., Prokopowicz, P., Algebra liczb rozmytych, Matematyka Stosowana: matematyka dla społeczeństwa, 5 (46), 2004, 37-63 (in Polish).
5. Kosiński, W., Prokopowicz, P., Fuzziness - representation of dynamic changes, using ordered fuzzy numbers arithmetic, New dimensions in fuzzy logic nd related technologies, vol I, Proc. of the 5th EUSFLAT Conference (eds) Martin Stepnicka, Vilem Novak, Ulrich Bodenhofer, University of Ostrava, Ostrava, 2007, 449-456
6. Kosiński, W., P. Prokopowicz, P., Ślęzak, D., Ordered fuzzy number, Bulletin of the Polish Academy of Sciences, Ser. Sci. Math., 51 (3), 2003, 327-338.
7. Luccioni, B., Oller, S., Danesi, R., Coupled plastic-damaged model. Comp. Meth. Appl. Mech. Eng., 129, 1996, 81–89
8. Marczewska, I., Bednarek, T., Marczewski, A., Sosnowski, W., Jakubczak, H, Rojek, J., Practical fatigue analysis of hydraulic cylinders and some design recommendation, Int. Journal of Fatigue, 28/12, 2006, 1739-1751.
9. Oller, S., Salomón, O., Onate, E., A continuum mechanics model for mechanical fatigue analysis, Computational Materials Science, 32(2) , 2005, 175–195.
10. Zadeh, L.A., Fuzzy sets, Information and Control, 8 (1965), 1965, 338-353
11. Zadeh, L.A., The concept of a linguistic variable and its application to approximate reasoning, Part I, Information Sciences, 8, 1975, 199-249.

LICZBY ROZMYTE W ANALIZIE NIEZAWODNOŚCI KONSTRUKCJI NARAŻONYCH NA ZNISZCZENIE ZMĘCZENIOWE

Streszczenie

Ciągle zwiększające się wymogi bezpieczeństwa wywołują konieczność wzięcia pod uwagę stochastyczną charakterystykę zmiennych w analizie zmęczeniowej konstrukcji. Analizę zmęczeniową rzeczywistych konstrukcji w połączeniu z analizą niezawodności w klasycznym probabilistycznym ujęciu cechuje niezwykle duży koszt obliczeniowy. W artykule przedstawiono nową metodę oceny niezawodności konstrukcji za pomocą zbioru ukierunkowanych liczb rozmytych. Obliczenia niezawodności konstrukcji narażonych na zmęczenie materiału przedstawiono na przykładzie analizy cylindra hydraulicznego. Zidentyfikowano miejsca narażone na pękanie zmęczeniowe. Obliczono trwałość zmęczeniową cylindra hydraulicznego za pomocą zbioru liczb rozmytych. Celem analizy zmęczeniowej z wykorzystaniem liczb rozmytych jest oszacowanie prawdopodobieństwa zniszczenia zmęczeniowego dla danej trwałości projektowej lub wyznaczenie okresów serwisowych przy zadanym poziomie niezawodności konstrukcji.

Submitted: October 7, 2008
Submitted in a revised form: November 10, 2008
Accepted: November 24, 2008