Publishing House
AKAPIT

COMPUTER METHODS IN MATERIALS SCIENCE

Informatyka w Technologii Materiałów

Vol. 8, 2008, No. 4

APPLICATION OF THE CYCLIC PLASTICITY HARDENING LAW TO METAL FORMING

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Abstract

Creation of a robust numerical model of the low cyclic plastic deformation is essential for predicting inhomogeneities occurring during strain path change. Application of such model to simulate various simple plastometric tests (three point bending or tension/compression) is the main subject of the present work. To take this effect into account, a nonlinear isotropic/kinematic hardening material model is considered and all the results are compared with those obtained for the conventional isotropic hardening model. Observed differences in strain and stress distribution are discussed in details. Finally, based on this analysis, conclusions regarding the possibility to apply investigated tests in identification of the material model parameters through the inverse analysis are drawn.

Key words: cyclic hardening, bauschinger effect, finite element method

1. INTRODUCTION

The numerical modelling of material behaviour under thermo-mechanical processing conditions is widely used both in experimental research and in industrial applications. Accuracy of numerical simulations depends, to a large extent, on the correctness of description of material properties, as well as on mechanical and thermal boundary conditions. Accurate evaluation of the rheological parameters under various conditions of deformation is one of the challenges in simulation. It can be said that conventional models describe properly materials subjected to deformation in a reasonably uniform and monotonic conditions (Pietrzyk et al., 2006). However, when a process with a complex stress states (i.e. during reversal cyclic deformation) is considered, conventional rheological models fail to accurately describe the material behaviour. Cyclic deformation is particularly interesting from the industrial point of view because it may lead to a reduction in applied loads followed by the energy conservation. When material is subjected to severe axial deformation with the cyclic change in the strain path, micro shear band development is the major mechanism responsible for the mentioned effect. This observation is the basis of the proposed Structure Based Design of Metal Forming Operations (SBDMFO) (Korbel & Bochniak, 1995) and was successfully applied to industrial scale extrusion with additional cyclic oscillations of the die (Bochniak & Korbel, 2003). Development of the rheological model to simulate this kind of processes was the subject of earlier authors works (Madej et al., 2007; Madej et al., 2009) and is not discussed in the present paper. However, when material is subjected to a low cyclic oscillation, another interesting phenomenon becomes important – the Bauschinger effect (Davenport & Higginson, 2000). According to its definition, the Bauschinger effect is a transient decrease in the work hardening rate upon reversal of the loading direction. Simply, when a material is subjected to loading under tension, the yield stress (σ_t) occurs at some specific level, after that when material is reloaded under compression

 ISSN 1641-8581

(σ_c), yielding occurs at the lower stress level - as seen in figure 1.

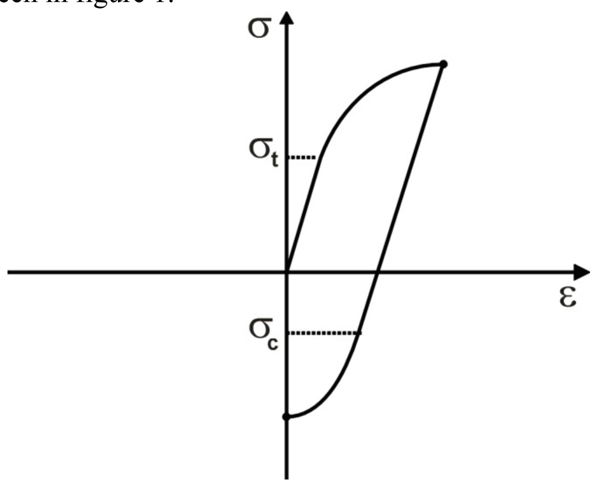

Fig. 1. Illustration of the Bauschinger effect.

Commonly used approaches to modeling metal forming processes are based on the isotropic hardening models and do not take this effect into account. During last decade several other models that predict the Bauschinger effect have been proposed, i.e. kinematic or combined hardening. Short review of their basic assumptions is presented in the following section.

2. CYCLIC DEFORMATION MODELS

As it has been mentioned, the isotropic model is commonly used when material hardening is described in finite element (FE) simulations. This model can be explained as an expansion of the yield surface without any change in the position of its centre, as it is seen in figure 2. From the material point of view, work hardening occurs primarily on active slip systems, however, hardening can also occurs on inactive slip systems. This is known as the latent hardening. It can result in an anisotropy of the yield surface. Nevertheless, due to its simplicity the isotropic hardening approximation is still used. It is also a good representation of material properties in the case of proportional hardening, i.e. when the representative stress vector maintains a constant direction in the stress space.

Thus, the yield surface in the isotropic hardening model can be described by a simple exponential law:

$$\sigma = \sigma_0 + A\left(1 - e^{-b\varepsilon}\right) \tag{1}$$

where: σ_0 – stress at zero plastic strain, ε – equivalent plastic strain, A, b – material parameters.

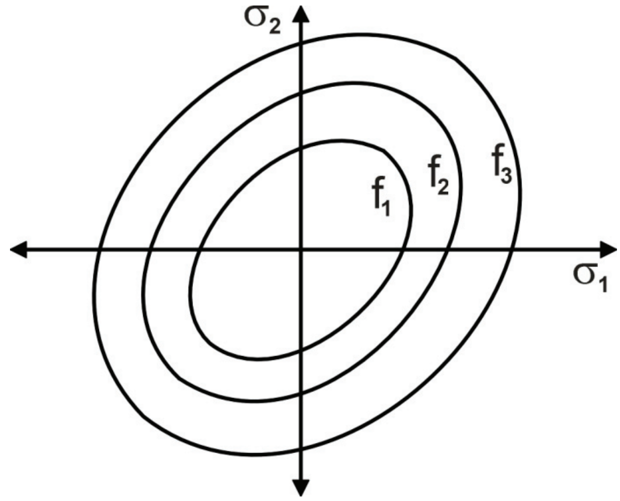

Fig. 2. Yield surface evolution in the isotropic hardening model.

Different approach to describe the evolution of the yield surface is proposed in the kinematic hardening model. It is often assumed that character of this approach is linear, i.e. the dimension of the yield surface maintains constant but its centre moves along the stress space (figure 3).

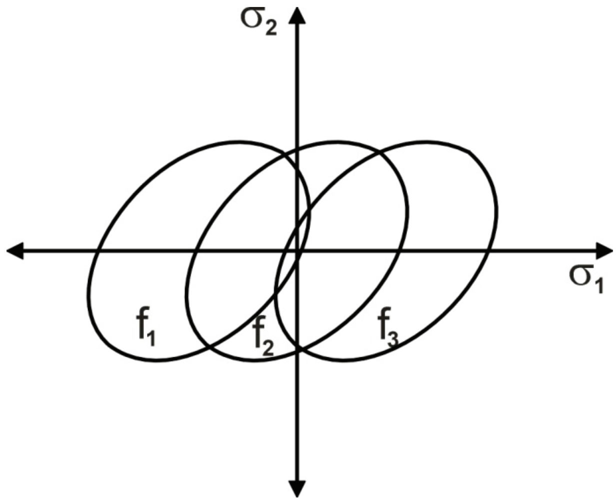

Fig. 3. Yield surface evolution in kinematic hardening model.

The mathematical form of this law is:

$$\dot{\alpha} = C\frac{1}{\sigma}\left(\boldsymbol{\sigma} - \boldsymbol{\alpha}\right)\dot{\varepsilon} + \frac{1}{C}\boldsymbol{\alpha}\dot{C} \tag{2}$$

where: $\boldsymbol{\alpha}$ - back stress tensor used to define translation of the yield surface centre ($\boldsymbol{\sigma}$-$\boldsymbol{\alpha}$), σ - size of the yield surface as a function of temperature only, C – temperature dependent parameter.

Finally, the most accurate model that is a combination of the isotropic and kinematic approach can be defined. There are various methods to define the combined approach (Mroz, 1967; Chaboche et al., 1979; Lemaitre & Chaboche, 1990). General idea of these methods is that the size of the yield surface increases and the centre of the yield surface moves at the same time in the stress space, as shown in figure 4. The combined model is the most complicated but, at the same time, the most accurate in the description of material behaviour during deformation. A combined isotropic and kinematic hardening rule can be used in the case when the size of the yield surface is interpolated between the size of the initial yield surface and the size of the corresponding isotropic hardening yield surface. Within the context of the combined isotropic and kinematic hardening rule, the centre of the yield surface moves in the stress space.

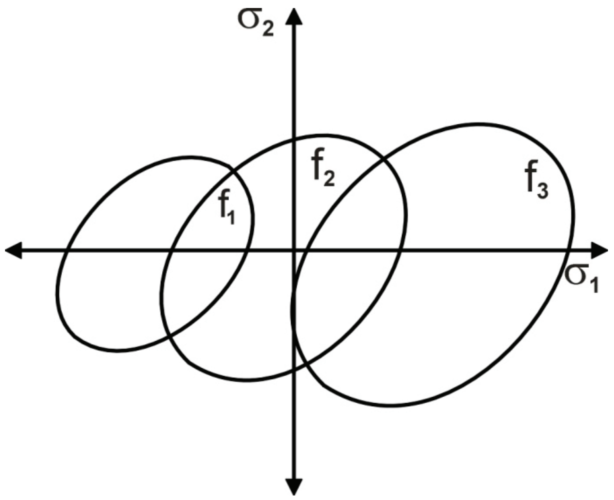

Fig. 4. *Yield surface evolution in combined hardening model.*

A very complex approach to this idea is proposed in (Mroz, 1967), where instead of having a single yield surface, a large number of concentric yield surfaces is defined (multiyield surface model). These yield surfaces can move freely as rigid bodies and touch each other, but it is assumed that they cannot intersect. So, the movement of the yield surfaces can then be used to describe material hardening. These surfaces become mutually tangent to each other at the current stress point. As the stresses increase to the plastic region, the current stress point will first encounter the smallest yield surface, which is the initial yield surface. This yield surface is then moved until it meets the next larger yield surface. These two yield surfaces are then moving together and the entire process repeats. Since the yield surface is permitted only rigid body translation, its size or the flow stress may be used as a parameter to determine the work-hardening modulus, which is equivalent for each of the yield surfaces. When there are multiple yield surfaces tangent one to another at the current stress point, the work-hardening modulus for continued plastic loading is determined by the largest yield surface in contact, the so called active yield surface. The smaller yield surfaces can become active again whenever the unloading/reloading takes place. While the material element is under the initial loading condition, the active yield surface expands isotropically as the inner yield surfaces move as rigid bodies with the current stress point. In this model, the yield surface F is described as:

$$F\left(\sigma_{i,j},\overline{\sigma}\right)=f\left(\sigma_{i,j}\right)-\overline{\sigma}=0 \tag{3}$$

where: f – function of the stress tensor $\sigma_{i,j}$, $\overline{\sigma}$ – generalized effective stress representing the current size of the yield surface F.

Another interesting approach is proposed in Chaboche et al. (1979) and in Lemaitre and Chaboche (1990). This model is based on the superposition of kinematic hardening, isotropic hardening (to describe cyclic hardening/softening) and time recovery (thermal recovery) of the hardening. It is based on the hierarchical form of the hardening rules:

– non-linear kinematic (initially proposed by Armstrong & Frederick, 1966),
– superposition of an isotropic rule to take into consideration cyclic hardening (or softening) of material
– a strain memory variable (Chaboche et al., 1979) to describe the complex cyclic hardening effects in material.

The time dependency is described in terms of the unified viscoplasticity incorporating time recovery of hardening and several additional effects.

In this model, the evolution of the size of the yield surface is defined as a function of strain ε, temperature T, and internal variables θ:

$$\sigma=f\left(\varepsilon,T,\theta\right) \tag{4}$$

i.e.

$$\sigma=\sigma_0+\exp\left(a\varepsilon\right)^n \tag{5}$$

where: σ_0 – stress at the beginning of plastic deformation, ε – strain, n – coefficient of sensitivity of flow stress to strain, a - hardening coefficients.

The following expression is used to describe evolution of the kinematic part of the model:

$$\dot{\alpha}=C\frac{1}{\sigma}\left(\boldsymbol{\sigma}-\boldsymbol{\alpha}\right)\dot{\varepsilon}-\gamma\boldsymbol{\alpha}\dot{\varepsilon}+\frac{1}{C}\boldsymbol{\alpha}\dot{C} \tag{6}$$

where: $\boldsymbol{\alpha}$ - back stress tensor used to define translation of the yield surface centre ($\boldsymbol{\sigma}$-$\boldsymbol{\alpha}$), $\boldsymbol{\sigma}$ - size of the yield surface and C, γ – temperature dependent parameters, γ = 0 stands for the linear-kinematic rule (Ziegler law, Ziegler, 1959).

The second term of equation (6) $\gamma \boldsymbol{\alpha} \dot{\varepsilon}$ introduces the nonlinearity in the evolution law. The main advantages of the non-linear rule are: the explicit integrability under proportional loading, the non-linearity of the stress-strain evolutions, the non-uniquiness of the relation between $\boldsymbol{\alpha}$ and $\dot{\varepsilon}$, the corresponding modelling of the Bauschinger effect, the description of ratchetting and mean stress relaxation effects under non-symmetrical cyclic loadings (i.e. tension-compression and/or tension-torsion).

In order to show the importance of the proper hardening model selection for the FE simulation, different plastometric tests where reversal straining plays significant role, were simulated. Comparison of results obtained from the isotropic and combined isotropic/kinematic models is presented in the following section.

3. NUMERICAL SIMULATION

The tension/compression test is a typical example of the process where Bauschinger effect plays an important role during subsequent straining in compression and tension (T/C). Tested material is low carbon steel. Finite element models were prepared in ABAQUS Standard commercial code. The response of the materials under cyclic loading was simulated using combined isotropic/kinematic hardening model (Chaboche et al., 1979), and - for comparison - using simple isotropic hardening model. In the case of combined isotropic/ kinematic hardening, the material parameters were taken from (Doghri, 1993). The values of C = 25500 MPa and γ = 81 were used. They define initial hardening modulus and the rate at which this modulus decreases with increasing plastic strain, respectively. Results obtained from FE simulation after ten cycles of T/C are shown in figure 5 and figure 6.

Additionally, simulations of the 2D three point bending test were performed using the same hardening models. This test was chosen due to its short calculation time, that allows its application for the determination of the parameters of the hardening models, using optimization techniques (e.g. inverse analysis) based on the data from the real experiments. Results obtained from the FE simulation after 5 cycles of 3 point bending, using different hardening rules, are presented in figure 8 and figure 9.

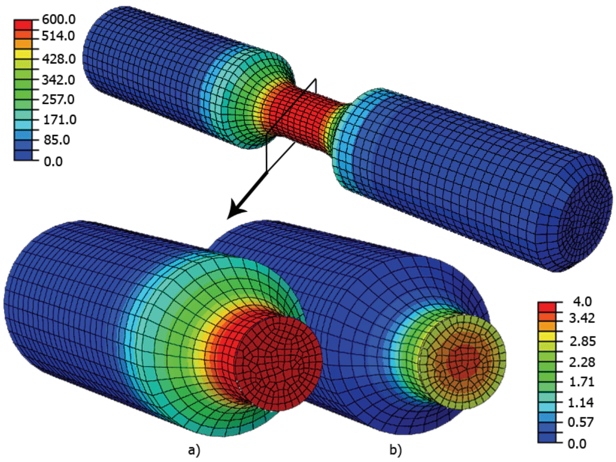

Fig. 5. Calculated stresses (a) and strains (b) at the end of the tension/compression test obtained with combined isotropic-kinematic model.

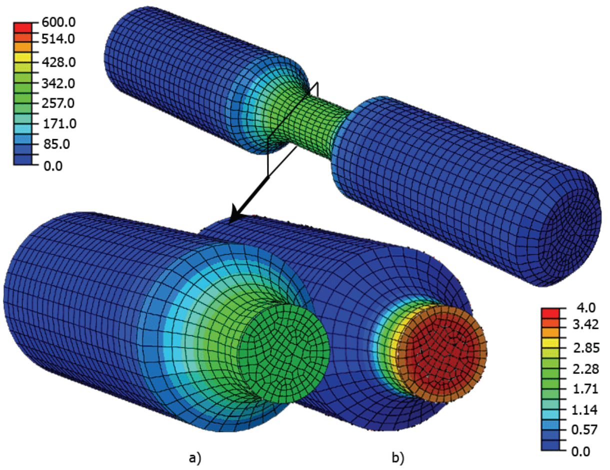

Fig. 6. Calculated stresses (a) and strains (b) at the end of the tension/compression test obtained with the isotropic model.

As it is seen in presented results, isotropic hardening model predicts much lower values of the stress and higher values of accumulated strains in comparison with the combined model. From the theoretical point of view, when parameters C and γ in equation (6) are equal to zero, combined model should provide the same response as the isotropic one. Results obtained for this case confirm this assumption, as it is seen in figure 7.

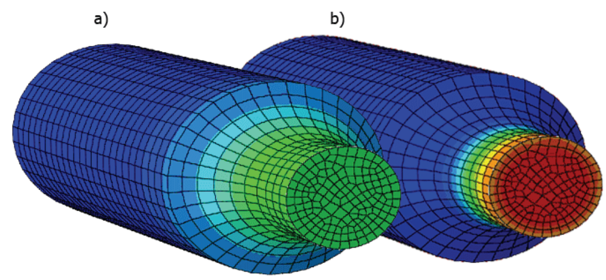

Fig. 7. Calculated stresses (a) and strains (b) at the end of the tension/compression test obtained with the combined model and assumption that C and γ in equation (5) are equal zero (scales are shown in figure 6).

Similar comparison between isotropic and combined models is made for the three point bending test (after 5 cycles), as seen in figure 8 and figure 9. Due to the symmetry of the model only half of the cross section after ten bending cycles is presented.

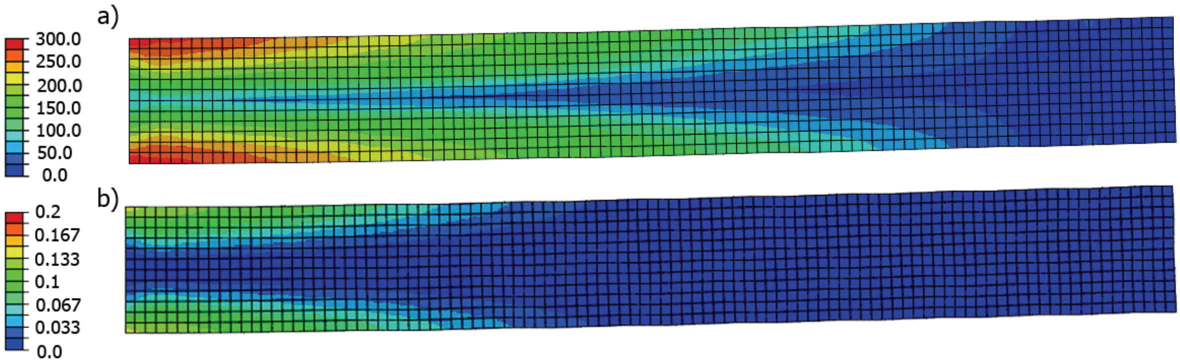

Fig. 8. Calculated stresses (a) and strains (b) at the end of the three point bending test obtained with combined isotropic-kinematic model.

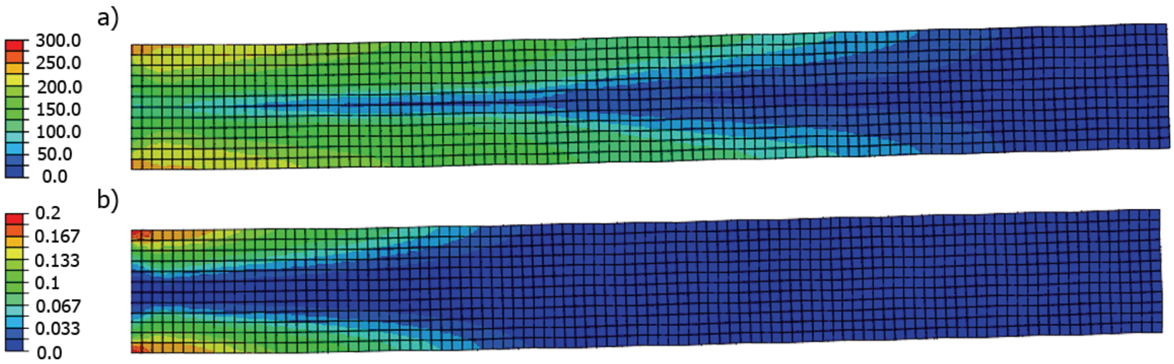

Fig. 9. Calculated stresses (a) and strains (b) at the end of the three point bending test obtained with isotropic model.

Again the same inversion in the stress and strain values obtained from the isotropic and combined models is observed. The isotropic hardening model predicts larger strain values.

4. CONCLUSIONS

In the present paper, different hardening rules specifying the evolution of the yield surface during plastic deformation, were reviewed. Simple isotropic hardening model and more complex combined isotropic/kinematic hardening were tested, using numerical models of the materials subjected to cyclic deformation (tension/compression and three point bending tests). In both cases, it was shown that isotropic hardening model predicts much lower values of the stress and higher values of accumulated strains in comparison with the combined model. The importance of proper selection of the accurate model for the FE simulation was highlighted. When processes with the strain path change are under consideration the combined model should be used during simulation.

The next step of this work will be focused on the proper determination of parameters of the cyclic hardening models for selected steel grades, based on the experimental data. Accurate determination of the C, γ, Q, and b parameters in the combined isotropic/kinematic hardening model is crucial and can be done through the inverse analysis (minimization of the error between measured and calculated results based on i.e. loading-displacement curves). The better understanding of the cyclic behaviour of the materials will enable creation of a robust numerical model of the low cyclic plastic deformation that is essential for predicting inhomogeneities occurring during strain path changes.

ACKNOWLEDGEMENTS

Financial assistance of the MNiSzW, project no. SPB 22.22.110.7014, is acknowledged.

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WYKORZYSTANIE MODELI ODKSZTAŁCEŃ CYKLICZNYCH DO SYMULACJI PROCESÓW PLASTYCZNEJ PRZEROBKI METALI

Streszczenie

Przygotowanie odpowiedniego modelu numerycznego, opisującego zachowanie materiału poddanego nisko-cyklicznym odkształceniom plastycznym jest kluczowe dla prawidłowego przewidywania niejednorodności wynikających ze zmiany drogi odkształcenia podczas rzeczywistych procesów przeróbki plastycznej metali. Głównym celem prezentowanej pracy jest dyskusja możliwości wykorzystania takich modeli do symulacji prostych testów plastometrycznych (trójpunktowego zginania i cyklicznego rozciągania/ściskania). Aby w pełni uwzględnić wpływ odkształceń cyklicznych, w symulacjach wykorzystano izotropowo-kinematyczny model umocnienia, a wyniki porównano z tymi, uzyskanymi z zastosowaniem konwencjonalnego modelu izotropowego. Szczegółowej dyskusji poddano zaobserwowane różnice w rozkładach odkształceń i naprężeń. Bazując na tej analizie, podano wnioski dotyczące możliwości wykorzystania prezentowanych testów plastometrycznych do identyfikacji parametrów modeli dla odkształceń cyklicznych z wykorzystaniem analizy odwrotnej.

Received: December 24, 2008
Received in a revised form: January 27, 2009
Accepted: January 27, 2009