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A COMPARISON OF MESHLESS AND FINITE ELEMENT APPROACHES TO DUCTILE DAMAGE IN FORMING PROCESSES

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Abstract

In this work the ability and versatility of a meshless method, namely the Reproducing Kernel Particle Method, is assessed in the modelling of ductile damage in forming processes. The performance is compared with a traditional finite element approach. In particular the pathological behaviour associated with the size and orientation of the discretization, typical in finite elements, is investigated. The damage model is based on the Lemaitre model and includes an enhancement that distinguishes damage evolution for local tension and compression states. Damage is coupled with a plastic deformation model based on the "flow formulation". Local and non-local damage models, of integral and gradient (explicit and implicit) types, were implemented and compared.

Key words: RKPM, meshless method, damage, non-local models, finite element method

1. INTRODUCTION

In metal forming processes the damage associated with large deformations is a phenomenon that should be minimized or simply avoided as it usually leads to flawed parts. The initiation of plasticity and damage is caused by movement and accumulation of dislocations in metals but their nature and evolution is different. Ductile damage evolution in metals is usually associated with the initiation and growth of micro cracks and cavities, resulting in a progressive material softening. Damage growing influences indirectly the plastic behaviour by locally reducing the elementary area of resistance. Therefore plasticity and damage should be coupled at the constitutive level. In the theory of Continuum Damage Mechanics the damage is represented by internal variables (of scalar, vectorial or tensor type) which give a measure of the deteriorated state at each representative volume of the material. This variable may then be used to define the effective stress state.

Another important aspect is related with the fact that ductile damage localization is similar to that associated with plastic strain. These physical phenomena are characterised by the accumulation of damage and large deformations within narrow bands. In experiments these localization zones display a finite width which may be related to the micro structure of the material.

Classical theories of plasticity and damage mechanics, based on internal variable approaches, are local theories and do not include size effects associated to a characteristic dimension of the material. Their implementation in a finite element setting shows a pathologic effect of spatial mesh dependence because the constitutive models are unable to capture the limitation of the localization upon mesh refinement. In fact, the original hypothesis of homogeneous continuous models does not take into account large changes in the internal variables, like plastic strain and damage, in the localization zone.

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The aforementioned effect can be adequately explained by micro mechanical theories but their numerical implementation is still rather expensive.

Non-local models have been proposed to bridge the gap between classical continuum theories and the micromechanical ones (Peerlings et al.,1996; Peerlings et al.,2001, César de Sá et al., 2006). In these models the evolution of some internal variables describing strain and damage in a specific point is also determined by the history of the surrounding material by including in the formulation averages or gradients of part or all of them. Some of theses models have proved to be effective when implemented in a finite element framework.

Some claims have been made that the new class of computational methods, i.e. meshless methods, could be more effective when dealing with localization problems. Typically these new methods use a set of points and local support functions to represent the problem domain with no need of an additional mesh. The use of these local support functions could then be broadened for the evaluation of the evolution of the internal variables, giving a non-local character to the solution. Therefore, in this work, an incursion was made into the application of these methods to this particular type of problems in order to investigate how meshless methods deal with ductile damage phenomena, if unacceptable discretization dependences, known to be associated with the implementation in the Finite Element Method (FEM), are also present and to assess how effectively the non-local and gradient models work in those frameworks.

The chosen meshless method was the Reproducing Kernel Particle Method (RKPM), proposed by W. K Liu (1995). The deforming material is treated as an incompressible viscous fluid of a non-Newtonian kind (Zienkiewicz & Godbole, 1974). The procedure is usually known as the "flow formulation" and is formally identical to that of pure viscoplastic theory. The material model was extended in order to include ductile damage effects by coupling the viscoplastic constitutive law with the damage evolution equations proposed by Lemaitre (1996). Non-local and related gradient (explicit and implicit) models were also implemented using the RKPM.

2. BASICS FORMULATIONS

2.1. Rigid viscoplastic model

In bulk forming processes the elastic part of the deformation is negligible when compared to the plastic permanent deformation and can therefore be neglected. This results in a significant simplification in the governing equations allowing to treat the deformation process as the flow of an incompressible viscous fluid of a non-Newtonian kind. This classical approach is know as the "flow formulation" and its basics assumptions are recovered in the next paragraphs.

The viscoplastic strain rate tensor $\dot{\boldsymbol{\varepsilon}}$ is assumed to be proportional, at any time, to the instantaneous deviatoric stress tensor $\boldsymbol{S}$

$$\dot{\boldsymbol{\varepsilon}} = \frac{1}{2\mu}\boldsymbol{S} \tag{1}$$

In (1) μ is the viscosity, that for metals may assume a general form of

$$\mu = \frac{\sigma_y + \left(\dot{\bar{\varepsilon}}/\gamma\right)^{\frac{1}{n}}}{3\dot{\bar{\varepsilon}}} \tag{2}$$

where σ_y is the yield stress, $\dot{\bar{\varepsilon}}$ is the equivalent viscoplastic strain rate and γ and n are viscoplastic material parameters.

Strain hardening effects may be included by assuming that the yield stress is dependent on the effective viscoplastic strain, $\bar{\varepsilon}$,

$$\sigma_y = \sigma_y\left(\bar{\varepsilon}\right) \tag{3}$$

As the viscoplastic deformation is assumed to be incompressible then

$$tr\left(\dot{\bar{\varepsilon}}\right) = 0 \tag{4}$$

If this condition is imposed via a Penalty Method in the weak form of the equilibrium equations it is possible to recover the local mean stress σ_m by the expression

$$\sigma_m \cong \alpha \; tr\left(\dot{\bar{\varepsilon}}\right) \tag{5}$$

where α is a local penalty parameter that may be viewed as the bulk viscosity. Therefore the Cauchy stress tensor may finally be written as

$$\boldsymbol{\sigma} = 2\frac{\sigma_y + \left(\dot{\bar{\varepsilon}}/\gamma\right)^{\frac{1}{n}}}{3\dot{\bar{\varepsilon}}}\dot{\boldsymbol{\varepsilon}} + \alpha \; tr\left(\dot{\bar{\varepsilon}}\right)\boldsymbol{I} = \boldsymbol{S} + \sigma_m \boldsymbol{I} \tag{6}$$

where $\boldsymbol{I}$ is the identity tensor.

2.2. Damage model

Assuming an homogeneous distribution of microvoids damage may be locally characterized, by an internal scalar variable, D, representing a surface density of discontinuities at any surface of a representative volume element as in (Lemaitre, 1996). Following the work of Andrade Pires et al. (2003) and Vaz Junior et al. (2005), based on the models proposed by Lemaitre, the effective stress tensor is then defined as

$$\tilde{\boldsymbol{\sigma}} = \left(1 - f\left(D\right)\right)^{-1} \boldsymbol{\sigma} = \left(1 - f(D)\right)^{-1} \left(\boldsymbol{S} + \sigma_m \boldsymbol{I}\right) = \tilde{\boldsymbol{S}} + \tilde{\sigma}_m \boldsymbol{I} \tag{7}$$

where $f\left(D\right)$ is a function of damage. In the original work of Lemaitre (1996), the effective stress tensor is written in the principal directions and, using the Macauley brackets (Lemaitre, 1996) defined as

$$\begin{cases} \langle x \rangle = x & if \ \ x \geq 0 \\ \langle x \rangle = 0 & if \ x < 0 \end{cases}, \tag{8}$$

is split in a tensile and a compressive stress tensors as

$$\tilde{\boldsymbol{\sigma}} = \begin{bmatrix} \langle \tilde{\sigma}_1 \rangle & 0 & 0 \\ 0 & \langle \tilde{\sigma}_2 \rangle & 0 \\ 0 & 0 & \langle \tilde{\sigma}_3 \rangle \end{bmatrix} - \begin{bmatrix} \langle -\tilde{\sigma}_1 \rangle & 0 & 0 \\ 0 & \langle -\tilde{\sigma}_2 \rangle & 0 \\ 0 & 0 & \langle -\tilde{\sigma}_3 \rangle \end{bmatrix} =$$

$$= \langle \ \tilde{\boldsymbol{\sigma}} \ \rangle - \langle \ -\tilde{\boldsymbol{\sigma}} \ \rangle = \tilde{\boldsymbol{\sigma}}^{+} + \tilde{\boldsymbol{\sigma}}^{-} \tag{9}$$

The damage function may be written as

$$f(D) = \begin{cases} f_{+}(D) \\ f_{-}(D) \end{cases} = \begin{cases} 1 - h^{+} D \\ 1 - h^{-} D \end{cases} \tag{10}$$

where h^{+} and h^{-} are parameters that take different values for tension or compression states. Usually for tensile stresses $h^{+} = 1$, therefore corresponding to the standard Lemaitre model. For compressive stresses $0 \leq h^{-} < 1$ and is associated with a crack closure effect and a consequent local stiffening. Vaz Jr. et al. (2005) extend the significance and the range values of this parameter by associating it also to shear decohesion effects in compression.

2.3. Damage model coupled with the flow formulation. Damage evolution

Adopting the hypothesis of strain equivalence, where, as stated by Lemaitre (1996), the constitutive equations of the damage material are the same as the virgin material in which the stress is substituted by the effective stress, it is possible to write, as shown in Andrade Pires et al. (2003), the following constitutive relations

$$\tilde{\boldsymbol{S}} = 2 \tilde{\mu} \ \dot{\boldsymbol{\varepsilon}} \tag{11}$$

with

$$\tilde{\mu} = \left(1 - f\left(D\right)\right) \frac{\sigma_y + \left(1 - f\left(D\right)\right)^{\frac{1}{n}} \left(\dot{\bar{\varepsilon}} / \gamma\right)^{\frac{1}{n}}}{3 \dot{\bar{\varepsilon}}} \tag{12}$$

The damage evolution may be determined by recourse to the thermodynamics of irreversible processes (Lemaitre, 1996). In the hypothesis of isotropic plasticity and isotropic damage, a dissipation potential, associated to the damage process, may be assumed, (Lemaitre, 1996), as

$$\varphi_d = \frac{1}{\left(1 - D\right)} \frac{S_0}{\left(s + 1\right)} \left(\frac{-Y}{S_0}\right)^{s+1} \tag{13}$$

in which S_0 and s are material properties and $-Y$ is the thermodynamical variable associated to damage.

The damage evolution, for associated plasticity, may be evaluated from

$$\dot{D} = -\dot{\lambda} \frac{\partial \Psi}{\partial Y} = \frac{\dot{\lambda}}{\left(1 - D\right)} \left(-\frac{Y}{S_0}\right)^{s} \tag{14}$$

where $\dot{\lambda}$ is the plastic multiplier, ensuring the yielding normality condition for plasticity or the viscosity property of proportionality of strain rates and instantaneous deviatoric stresses. It is possible to identify, for both cases, (Lemaitre, 1996), this positive multiplier with

$$\dot{\lambda} = \left(1 - D\right) \dot{\bar{\varepsilon}} \tag{15}$$

allowing to recast equation (14) as

$$\dot{D} = \dot{\bar{\varepsilon}} \left(-\frac{Y}{S_0}\right)^{s} \tag{16}$$

The associated variable to damage, $-Y$, may be identified with the variation of internal elastic energy density, W^e, due to damage growth at constant stress and therefore obtained as

$$\left(-Y\right) = \frac{1}{2} \frac{dW^e}{dD}\bigg|_{\sigma} = \frac{1}{2} \frac{d\left(\tilde{\boldsymbol{\sigma}} : \boldsymbol{\varepsilon}^e\right)}{dD}\bigg|_{\sigma} = \frac{1}{2} \frac{\left(-f'(D)\right)}{f\left(D\right)} \boldsymbol{\sigma} : \boldsymbol{\varepsilon}^e \bigg|_{\sigma} \tag{17}$$

where $\boldsymbol{\varepsilon}^e$ is the elastic strain tensor.

To take into account the cracking close effect under compression a new proposal, that slightly differs from previous ones, (Andrade Pires et al., 2003; Vaz Jr. et al., 2005), is here put forward. In our proposal the variation of internal elastic energy density due to damage growth is derived directly from equation (17), splitting the two parts resulting from positive principal stresses and negative principal stresses, as follows:

$$(-Y)=(-Y)_+ + (-Y)_- = \frac{1}{2}\frac{(-f'_+)}{f_+}\boldsymbol{\sigma}^+ : \boldsymbol{\varepsilon}^{+e}\Big|_{\sigma^+} + \frac{1}{2}\frac{(-f'_-)}{f_-}\boldsymbol{\sigma}^- : \boldsymbol{\varepsilon}^{-e}\Big|_{\sigma^-} \tag{18}$$

Then by using the elasticity relations it is possible to obtain:

$$(-Y)_+ = \frac{1}{2}\frac{(-f'_+)}{f_+}\left\{\frac{1}{f_+}\frac{(1+\nu)}{E}\boldsymbol{\sigma}^+ : \boldsymbol{\sigma}^+ - \right.$$
$$\left. -\frac{1}{f_+}\frac{\nu}{E}\left[\left(\mathrm{tr}\left[\boldsymbol{\sigma}^+\right]\right)^2 + \mathrm{tr}\left[\boldsymbol{\sigma}^+\right]\mathrm{tr}\left[\boldsymbol{\sigma}^-\right]\right]\right\} \tag{19}$$

$$(-Y)_- = \frac{1}{2}\frac{(-f'_-)}{f_-}\left\{\frac{1}{f_-}\frac{(1+\nu)}{E}\boldsymbol{\sigma}^- : \boldsymbol{\sigma}^- - \right.$$
$$\left. -\frac{1}{f_-}\frac{\nu}{E}\left[\left(\mathrm{tr}\left[\boldsymbol{\sigma}^-\right]\right)^2 + \mathrm{tr}\left[\boldsymbol{\sigma}^-\right]\mathrm{tr}\left[\boldsymbol{\sigma}^+\right]\right]\right\} \tag{20}$$

where E and ν are the elasticity constants.

After some algebraic manipulation the final expression is obtained as:

$$(-Y) = \frac{h^+}{(1-h^+D)^2}\left[\frac{(1+\nu)}{2E}\boldsymbol{\sigma}^+ : \boldsymbol{\sigma}^+ - \frac{\nu}{2E}\left(\mathrm{tr}\left[\boldsymbol{\sigma}^+\right]\right)^2\right] +$$
$$+\frac{h^-}{(1-h^-D)^2}\left[\frac{(1+\nu)}{2E}\boldsymbol{\sigma}^- : \boldsymbol{\sigma}^- - \frac{\nu}{2E}\left(\mathrm{tr}\left[\boldsymbol{\sigma}^-\right]\right)^2\right] -$$
$$-\frac{\nu}{2E}\left[\frac{h^+}{(1-h^+D)^2} + \frac{h^-}{(1-h^-D)^2}\right]\mathrm{tr}\left[\boldsymbol{\sigma}^+\right]\mathrm{tr}\left[\boldsymbol{\sigma}^-\right] \tag{21}$$

2.4. Non local damage models

Some non-local models (of integral and gradient type) were implemented using a meshless method (RKPM) as a numerical framework. The resulting performance was assessed, in what respects discretization dependency of the solutions, and compared with the traditional FEM. The different types of non-local models implemented are briefly described in the following sections.

2.4.a. Integral non-local model

In this model a non-local damage field is defined from a weighted average of the local damage using a weight function proposed by (Pijaudier-Cabot & Bazant, 1987; Pijaudier-Cabot et al., 1988), as:

$$\bar{D}_i = \frac{\int_V \Phi(\|\mathbf{x}-\mathbf{x}_i\|)\cdot D(\mathbf{x})dV}{\int_V \Phi(\|\mathbf{x}-\mathbf{x}_i\|)dV} \tag{22}$$

in which $D(\mathbf{x})$ is the local damage value at the sampling integration points and $\bar{D}_i$ represents the non-local damage value on node i. The weight function $\Phi(\|\mathbf{x}-\mathbf{x}_I\|)$ takes the following expression

$$\Phi\left(\|\mathbf{x}-\mathbf{x}_i\|\right) = \frac{1}{(2\pi)^{3/2} l^3}\exp\left[-\frac{\|\mathbf{x}-\mathbf{x}_i\|^2}{2l^2}\right] \tag{23}$$

where a length scale parameter l, related with microstructure information, defines the support of the weighting function.

2.4.b. Explicit gradient model

Admitting a smooth damage distribution and its expansion on a Taylor series, as in the work of Peerlings et al. (1996), i.e.,

$$D(\mathbf{x}+\Delta\mathbf{x}) = D(\mathbf{x}) + \nabla D(\mathbf{x}).\Delta\mathbf{x} + \frac{1}{2}\nabla^2 D(\mathbf{x}).\Delta\mathbf{x}^2 + ... \tag{24}$$

and substituting the definition of equation (22) into equation (24) the following expression for a the non-local damage variable may be obtained:

$$\bar{D}(\mathbf{x}) = D(\mathbf{x}) + \frac{1}{2}l^2\ \nabla^2 D(\mathbf{x}) + ... \tag{25}$$

The non-local damage variable may then be obtained explicitly from the previous equation.

In the FEM solution a polynomial approximation of D, based on the Least Square Method, is built locally in a support region of radius l, in order to obtain, in each point, the spatial derivatives on equation (25).

2.4.c. Implicit gradient model

Following again the work of Peerlings et al. (1996), a truly non-local implicit gradient model can be obtained by differentiating twice equation (25) and combining it with the implicit definition of the gradient of the local variable of damage in equation (25), resulting in:

$$\left(\bar{D}(\mathbf{x}) - D(\mathbf{x})\right) - \frac{1}{2}l^2\ \nabla^2\bar{D}(\mathbf{x}) + ... = 0 \tag{26}$$

The non-local damage is, in this case, an independent variable related with the local value through the diffusion/advection equation (26), promoting an averaging of the local variable dependent on the length scale parameter.

3. NUMERICAL TEST

A standard tensile test of a pre-notched axi-symmetric specimen of an aluminum alloy, figures 1, was used to assess the discretization dependence of the meshless method (RKPM) and the performance of non-local models in the meshless framework. The material properties are listed in Table 1.

Table 1. *Material properties.*

E =	69004 N/mm^2	ν =	0.3
σ_Y =	$589(10^{-4}+\overline{\varepsilon})^{0.216}$ N/mm^2	γ =	∞
s_0 =	1.0	S_0 =	1.25 N/mm^2

had 189 points and the second one had 576 points, both shown in figure 1. In the finite element solution the meshes were constructed so that the nodes were coincident with the points used in the corresponding RKPM simulation, figure 1.

In figure 2 the damage contours for those cases are represented for the local model, and a clear discretization dependence is visible with the size of localization of damage diminishing dramatically with the increase of the number of points/nodes used. Figures 3 to 5 show how this discretization dependence is eliminated with the non-local models implemented both in RKPM and, as expected, in the FEM. It is also clear how in the explicit gradient method the size of the localization zone is much smaller and the damage value is much higher than in the other non-local models where the damage is smoothed over a zone that depends on the length scale adopted. This fact seems to be in accordance with what was stated in Peerlings et al. (2001) at this respect.

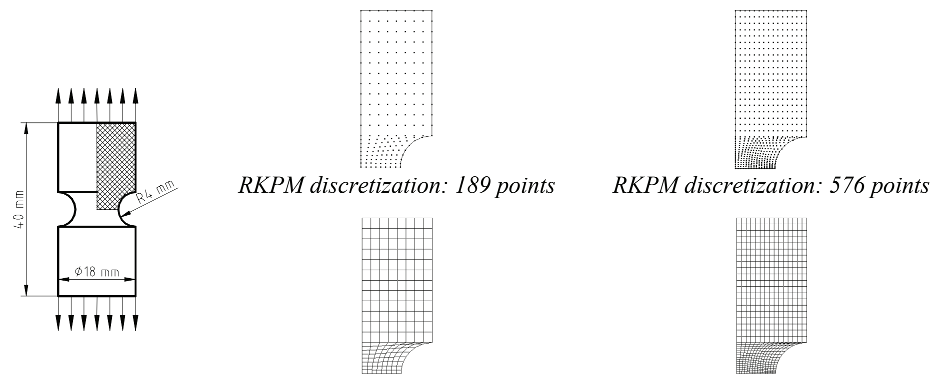


Fig. 1*. Specimen geometry. FEM and RKPM discretization.*

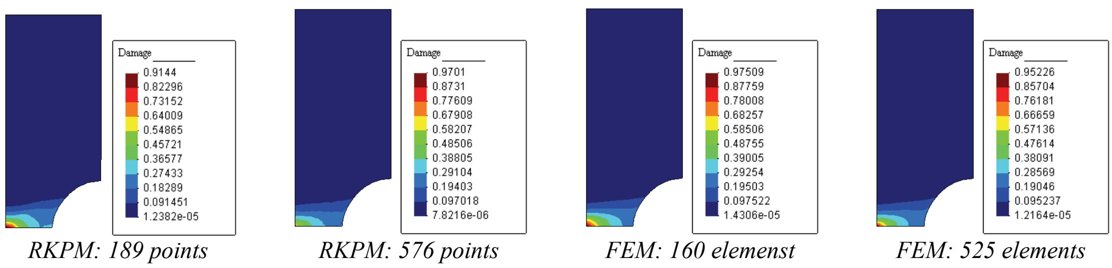
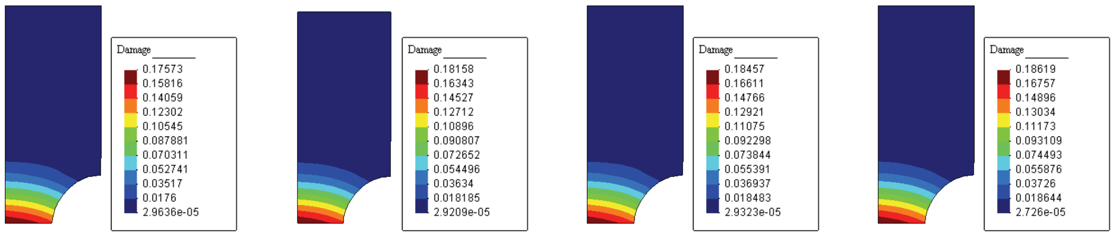


Fig. 2. *Local model. Discretization dependence in the RKPM and FEM.*

A velocity of $0.05 mm/s$ was imposed at the top edge of the specimen. The simulation was stopped for a maximum vertical displacement of 0.6 *mm* or whenever the damage reached the unity value. The length scale parameter was taken as $l = 2\ mm$.

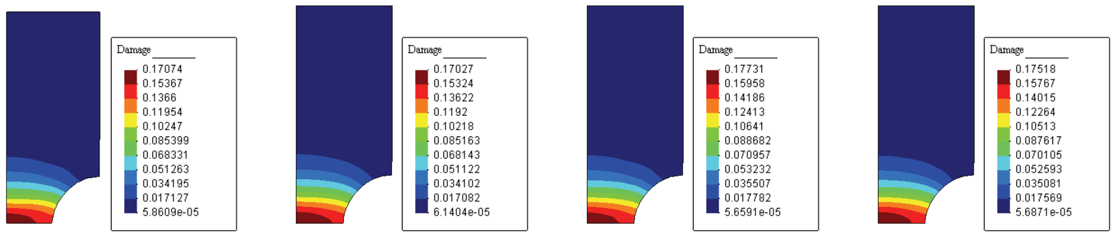
Two different discretizations, a coarser and a finer one, were tested. For the RKPM the first one

Finally it also clear how the FEM and RKPM solutions are very similar for the corresponding discretizations.



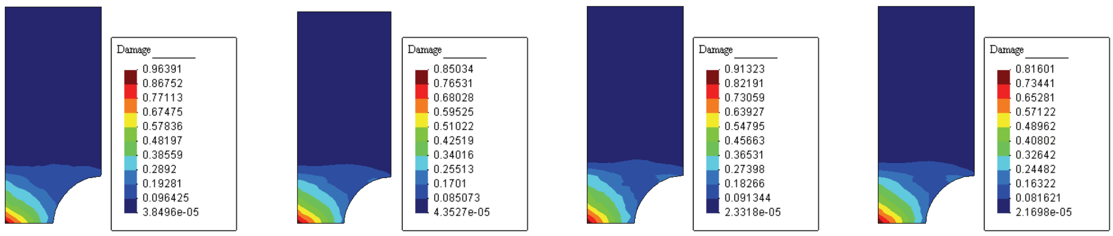
RKPM: 189 points *RKPM: 576 points* *FEM: 160 elements* *FEM: 525 elements*

Fig. 3. *Non-local integral model.*



RKPM: 189 points *RKPM: 576 points* *FEM: 160 elemenst* *FEM: 525 elements*

Fig. 4. *Non-local implicit gradient model.*



RKPM: 189 points *RKPM: 576 points* *FEM: 160 elemenst* *FEM: 525 elements*

Fig. 5. *Non-local explicit gradient model.*

4. CONCLUSIONS

The use of non-local damage models in the modelling of large deformation processes is an important feature in order to avoid discretization dependence that characterise the standard local models. The integral and implicit gradient models show a typical non-local character by providing a spatial interaction in the constitutive response, as suggested in Peerlings et al. (2001). The explicit gradient model, that Peerlings et al. (2001) suggest that has not a truly non-local character, avoids discretization dependence but localizes the damage in a zone of small width, advising its use in processes where localization is within very narrow zones.

The solutions obtained the FEM and RKPM were very similar with the disadvantage of the latter of being much more costly in computational time and not confirming, as somehow expected, some statements that meshless methods would be a solution for the numerical pathologies described.

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REFERENCES

Andrade Pires, F. M., César de Sá, J. M. A., Costa Sousa, L., Natal Jorge, R. M., 2003, Numerical modelling of ductile plastic damage in bulk metal forming, *Int. J. of Mech. Sci.*, 45, 273-294.

César de Sá, J. M A., Areias, P. M. A., Zheng, C., 2006, Damage modelling in metal forming problems using an implicit non-local gradient model, *Comput. Methods Appl. Mech.* Engrg., 195, 6646-6660.

Lemaitre, J., *A course on damage mechanics*, 1996, Berlin, Heidelberg, New York: Springer.

Liu, W. K., Jun, S., Zhang, Y. F., 1995, Reproducing kernel particle methods, *Int. J. Numer. Meth. Fluids*, 20, 1081-1106.

Peerlings, R.H.J., de Borst, R., Brekelmans, W.A.M., de Vree, J.H.P., 1996, Gradient enhanced damage for quasi-brittle materials, *Int. J. Num. Meths in Engng*, 39**,** 3391-3403.

Peerlings, R.H.J., Geers, M.G.D., de Borst, R., Brekelmans, W.A.M., 2001, A critical comparison of nonlocal and gradient-enhanced softening continua, *Int. J. Solids and Structures*, 38**,** 7732-7746.

Pijaudier-Cabot, G., Bazant, Z.P., 1987, Nonlocal damage theory, Journal of Engineering Mechanics, 113, 1512-1533.

Pijaudier-Cabot, G. , Bazant, Z.P., Tabbara, M., 1988, Comparison of various models for strain-softening, *Engineering Computations*, **5**, 141-150.

Zienkiewicz, O. C., Godbole, P. N., 1974, Flow of plastic and viscoplastic solids with special reference to extrusion and forming processes, *Int. J. Numer. Meth. Eng*., 8, 15-38.

Vaz Jr., M., de Santi Jr., N., Verran, G.O., de Souza Neto, E.A., 2005, Further studies on assessing ductile fracture using continuous damage coupled to an elasto-plastic material model. In D.R.J. Owen, E. Oñate, B. Suárez, eds, *Computational Plasticity VIII - Fundamentals and Applications*, 355-359. CIMNE.

PORÓWNANIE METOD BEZSIATKOWEJ I ELEMENTÓW SKOŃCZONYCH W ANALIZIE PLASTYCZNEGO PĘKANIA W PROCESACH PRZERÓBKI PLASTYCZNEJ

Streszczenie

Możliwość wykorzystania metody bez siatkowej Reproducing Kernel Particle Method (RKPM) do symulacji plastycznego pękania w procesach przeróbki plastycznej jest tematem niniejszej pracy. Zalety metody RKPM są porównane z konwencjonalnymi modelami MES, szczególnie pod względem problemów z dyskretyzacją badanego obszaru. Zastosowany model pękania bazuje na podejściu Lemaitre z uwzględnieniem rozgraniczenia pękania dla lokalnych obszarów rozciąganych i spęczanych. Zaimplementowane lokalne i globalne modele w formie jawnej i niejawnej są porównane i omówione w niniejszej pracy.

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