

## ON PARTIAL HYDRODYNAMIC LUBRICATION EFFECTS IN THE JOURNAL BEARINGS

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### Abstract

The objective of this research is to include mixed friction effects into existing models of hydrodynamic journal bearings. The considerations in this paper are devoted to partial hydrodynamic lubrication effects in typical cylindrical transverse journal bearings applied for supporting of heavy rotor-shafts of the rotating machines. For this purpose there is used an average flow theory for an oil film interaction with the rough journals and bushings. In order to carry out a qualitative analysis of the problem a simple mechanical model of the partially lubricated “super-bearing” is assumed. These viscoelastic properties of the oil film in such bearing are determined by numerical solving of the average Reynolds’ equation describing partial oil flow between the rough journal and bushing. Here, the pressure flow factors and the shear flow factors standing in the average Reynold’s equations are determined analytically using proper empirical formulae obtained by numerical simulations of the model flows between partially lubricated surfaces for various roughness geometry and average oil film thicknesses. Solving in this way the average Reynolds equation for the entire oil gap by means of the finite difference method enables us to determine resultant viscous friction retarding force as well as transverse hydrodynamic load of the bearing. In the numerical examples numerous roughness amplitudes of the journal and the bushing surface as well as various directional orientations of roughness asperities, which follow from final machining of the journal and the bushing, are taken into consideration in order to investigate their influence on kineto-static and dynamic behaviour of the transverse oil-film bearing.

**Key words:** journal bearing, partial and mixed lubrication, average flow model, friction

### 1. INTRODUCTION

The problem of partial hydrodynamic lubrication caused by roughness of sliding surfaces has been investigated for many years by numerous researchers. An extensive survey of various theoretical and experimental methods of research of mixed lubrication problems has been described in the state-of-the-art paper by Spikes and Olver (2002). These methods were used in several fields of technological applications, such as rolling mills, journal bearings, gear teeth, piston engines and others. The crucial aspect of this problem from the viewpoint of considerations in this paper is the effect of transverse roughness of mutually sliding surfaces on a distribu-

tion of pressure and stresses in the elastohydrodynamic lubrication contact zones. This problem is discussed e.g. by Li and Hooke (2002). But majority of theoretical studies in this field were focused on very fundamental aspects taking into consideration general principles of bearing operation from the viewpoint of flow determination between sliding surfaces of various roughness descriptions. Nevertheless, the most recent research of mixed lubrication in journal bearings are devoted to phenomena observed in specialized bearing designs, such as the spiral grooved journal bearings, in which the cavitation effects are investigated by Hirayama et. al. (2002).

The objective of this research is to include mixed friction effects into existing models of hydrodynamic journal bearings described e.g. by Kiciński (1996, 2005). For this purpose a statistical approach has been employed to modify Reynolds equation to take into account the influence of statistical roughness on fluid flow in the bearing contact zones. This aspect of the mixed lubrication has been investigated by Patir and Cheng (1978, 1979). The theoretical method applied in these works as well as the proposed computational algorithm seem to be the most convenient to satisfy the postulated purpose of this paper. Thus, the results obtained by Patir and Cheng (1978, 1979) are regarded here as very fundamental for modelling of the mixed lubrication in the transverse journal bearings interacting with the rotating shaft.

## 2. FUNDAMENTALS OF THE AVERAGE FLOW MODEL TO MIXED LUBRICATION BETWEEN ROUGH SLIDING SURFACES

The considerations in this paper are devoted to partial hydrodynamic lubrication effects in typical cylindrical transverse journal bearings applied for supporting of heavy rotor-shafts of the rotating machines. For this purpose there is used the average flow theory for an oil film interaction with rough journals and bushings. This theory is based on the classical Reynolds equation commonly applied as equation of motion of the oil film interaction in journal bearings, e.g. by Kiciński (1996, 2005). For the transverse journal bearing and for an isothermal, incompressible lubricant, the pressure in elasto-hydrodynamic contact is governed by the Reynolds equation in the following form

$$\frac{\partial}{\partial \xi} \left( \frac{h_T^3}{12\mu} \frac{\partial p(\xi, \eta)}{\partial \xi} \right) + \frac{\partial}{\partial \eta} \left( \frac{h_T^3}{12\mu} \frac{\partial p(\xi, \eta)}{\partial \eta} \right) = \frac{U_1 + U_2}{2} \frac{\partial h_T}{\partial \xi} + \frac{\partial h_T}{\partial t}, \quad (1)$$

where  $p(\xi, \eta)$  denotes the oil film hydrodynamic pressure expressed as a function of the bearing circumference co-ordinate  $\xi$  and the bearing length co-ordinate  $\eta$ ,  $\mu$  is the oil viscosity,  $U_1$ ,  $U_2$  are respectively the journal and bushing circumferential velocities (usually  $U_2=0$ ) and  $t$  denotes time. In this equation there is applied that  $h_T = h + \delta_1 + \delta_2$ , where  $h$  denotes the nominal film thickness (compliance) defined as the distance between the mean levels of

the journal and bushing sliding surfaces,  $\delta_1$  and  $\delta_2$  are the random roughness amplitudes of the two surfaces measured from their mean levels. The amplitudes  $\delta_1$  and  $\delta_2$  have a Gaussian distribution of heights with zero mean and standard deviations  $\sigma_1$  and  $\sigma_2$ , respectively. Then, the combined roughness  $\delta = \delta_1 + \delta_2$  has a variance  $\sigma^2 = \sigma_1^2 + \sigma_2^2$ . This equation is usually solved with the Reynolds-Swift-Stieber boundary conditions, (Kiciński, 1996 and 2005), which for the transverse journal bearing seem to be very convenient:

$$\frac{\partial p(\xi, \eta)}{\partial \xi} = 0 \quad \text{for } \xi = \xi_e, \quad p\left(\xi, \pm \frac{L}{2}\right) = 0, \quad (2)$$

$$p(\xi_e, \eta) = 0, \quad p(\xi_b, \eta) = p_0,$$

where  $L$  denotes the bearing length,  $\xi_b$ ,  $\xi_e$  are the co-ordinates of the beginning and end of the positive oil pressure zone, respectively, and  $p_0$  denotes the oil supply pressure of the journal bearing. Equation (1) with the boundary conditions (2) can be applied for small enough journal to bushing relative displacements, for which the smooth oil flow only can be assumed. However, according to Patir and Cheng (1978, 1979), for greater journal-to-bushing relative displacements, where the roughness effects of the sliding surfaces start to play more and more significant role, the average Reynolds equation is proposed in the following form:

$$\frac{\partial}{\partial \xi} \left( \phi_n(H, \gamma) \frac{h^3}{12\mu} \frac{\partial \bar{p}(\xi, \eta)}{\partial \xi} \right) + \frac{\partial}{\partial \eta} \left( \phi_n(H, 1/\gamma) \frac{h^3}{12\mu} \frac{\partial \bar{p}(\xi, \eta)}{\partial \eta} \right) = \frac{U_1 + U_2}{2} \frac{\partial \bar{h}_T}{\partial \xi} + \frac{U_1 - U_2}{2} \sigma \frac{\partial \phi_s}{\partial \xi} + \frac{\partial \bar{h}_T}{\partial t}. \quad (3)$$

In this equation  $\bar{p}(\xi, \eta)$  denotes the mean hydrodynamic pressure and  $\bar{h}_T$  is the journal-bushing average gap defined as

$$\bar{h}_T = \int_{-h}^{\infty} (h + \delta) f(\delta) d\delta,$$

where  $f(\delta)$  is the probability density function of  $\delta$ . Equation (3) in comparison with (1) is modified by the pressure flow factors  $\phi_n$  and by the shear flow factor  $\phi_s$ , which are expressed as functions of the ratio  $H=h/\sigma$  and the ratio  $\gamma$  of the surface asperity correlation lengths. The ratio  $H$  describes a temporary mutual distance between the journal and bushing surfaces in comparison with the combined roughness variance and the ratio  $\gamma$  defines directional properties of the roughness asperities. According to Peklenik (1967-68),  $\gamma$  is defined as the length



ratio of one-halves of the autocorrelation functions of asperity profiles. Thus,  $\gamma$  can be visualized as the length-to-width ratio of a representative asperity, where purely transverse, isotropic and longitudinal roughness patterns correspond to  $\gamma=0,1,\infty$ , respectively.

The mentioned above flow factors are obtained through flow simulation using model problem formulated for the assumed character of sliding surface roughness. According to Patir and Cheng (1978, 1979), after generating  $\delta_1$  and  $\delta_2$  randomly with known statistical properties over the area of the bearing (or using the measured real surfaces), this task is many times solved for pressure by means of the finite difference method. The results of simulations for the model problems obtained for various ratios  $H$  and  $\gamma$  can be approximated by analytical functions describing the flow factors standing in the average Reynolds equation (3). Then, the pressure flow factor can be expressed in the following form

$$\phi_n(H, \gamma) = 1 - Ce^{-rH} \quad \text{for } \gamma \leq 1 \quad \text{and}$$

$$\phi_n(H, \gamma) = 1 + CH^{-r} \quad \text{for } \gamma > 1, \quad (4)$$

where  $C$  and  $r$  are the constant coefficients tabularized by Patir and Cheng (1978, 1979) for various ratios  $H$  and  $\gamma$ . The shear flow factor  $\phi_s$  is approximated by the following analytical function

$$\phi_s(H, \gamma_1, \gamma_2) = \left(\frac{\sigma_1}{\sigma}\right)^2 \Phi_s(H, \gamma_1) - \left(\frac{\sigma_2}{\sigma}\right)^2 \Phi_s(H, \gamma_2), \quad (5)$$

where

$$\Phi_s(H, \gamma) = A_1 H^{\alpha_1} e^{-\alpha_2 H + \alpha_3 H^2} \quad \text{for } 0.5 < H \leq 5 \quad \text{or}$$

$$\Phi_s(H, \gamma) = A_2 e^{-0.25H} \quad \text{for } H > 5 \quad \text{and } A_1, A_2, \alpha_1,$$

$\alpha_2, \alpha_3$  are the constant coefficients tabularized by Patir and Cheng (1979) for various ratios  $\gamma$  regarded here separately for each sliding surface "1" and "2", i.e. denoted by  $\gamma_1$  and  $\gamma_2$ .

The approach described above is valid for  $H > 0.5$ , because for smaller  $H = h/\sigma$  ratios the metal-to-metal contact between the journal and bushing surfaces takes place. Then the Coulomb-type friction effects should have been taken into consideration. However, if  $H$  tends to infinity, i.e. when a distance between the journal and bushing surfaces is much greater than heights of the roughness asperities, from formulae (4) and (5) it follows that the pressure flow factors  $\phi_n$  tend to 1 and the shear flow factor  $\phi_s$  tends to 0. Thus, the average Reynolds equation (3) becomes the standard Reynolds equation in the form (1).

### 3. ASSUMPTIONS AND FORMULATION OF THE PROBLEM

The approach proposed by Patir and Cheng (1978, 1979) and summarized above is very convenient for dynamic analysis of rotating shafts supported in journal bearings, where the mixed lubrication between journals and bushings are taken into consideration. Then, for each current temporary position of the journal with respect to the bushing it is necessary to solve the average Reynolds equation (3) with boundary conditions (2) using formulae (4) and (5). Thus, according to Kiciński (1996, 2005), by means of proper partial differentiation of the vertical and horizontal components of the bearing hydrodynamic reaction force with respect of journal-to-bushing mutual displacements or displacement velocities it is possible to determine temporary values of the oil film stiffness and damping coefficients as well as to calculate the frictional power of the bearing. In order to carry out a qualitative analysis of the problem a simple mechanical model of the partially lubricated "super-bearing" is assumed. This model shown in figure 1 consists of the rigid mass  $m_R$  rotating with constant speed  $\Omega$  and performing translational in-plane motion within the non-rotating rigid ring which is elastically suspended in this plane. The rigid mass  $m_R$  represents the entire inertia of the rotating parts of the rotor machine and the rigid ring of mass  $m_B$  corresponds to the housing of such "super bearing". Translational in-plane motion of the rigid mass  $m_R$  is coupled with translational motion of the rigid ring by means of mass-less springs and dampers representing respectively visco-elastic properties of the oil film in the "super-bearing" with rough sliding surfaces. Motion of this model is governed by the following system of four ordinary differential equations, which correspond to the number of degrees of freedom of the assumed object

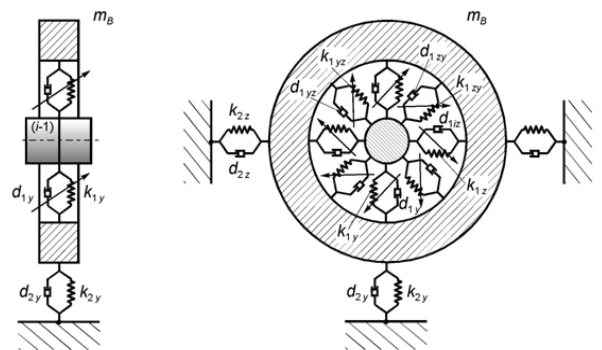


Fig. 1. Model of the super-rotor supported by the super-bearing.



$$\begin{aligned}
 & m_R \frac{d^2 u(t)}{dt^2} + d_{1y} \left[ \frac{du(t)}{dt} - \frac{dy(t)}{dt} \right] + k_{1y} [u(t) - y(t)] + \\
 & k_{1y} [u(t) - y(t)] + d_{1yz} \left[ \frac{dw(t)}{dt} - \frac{dz(t)}{dt} \right] + \\
 & \quad + k_{1yz} [w(t) - z(t)] = m_R g, \\
 & m_R \frac{d^2 w(t)}{dt^2} + d_{1z} \left[ \frac{dw(t)}{dt} - \frac{dz(t)}{dt} \right] + k_{1z} [w(t) - z(t)] \\
 & + d_{1zy} \left[ \frac{du(t)}{dt} - \frac{dy(t)}{dt} \right] + \\
 & \quad + k_{1zy} [u(t) - y(t)] = 0, \\
 & m_B \frac{d^2 y(t)}{dt^2} + d_{2y} \frac{dy(t)}{dt} + k_{2y} y(t) + \\
 & d_{1y} \left[ \frac{dy(t)}{dt} - \frac{du(t)}{dt} \right] + k_{1y} [y(t) - u(t)] + \\
 & \quad + d_{1yz} \left[ \frac{dz(t)}{dt} - \frac{dw(t)}{dt} \right] + k_{1yz} [z(t) - w(t)] = 0,
 \end{aligned} \tag{6}$$

$$\begin{aligned}
 & m_B \frac{d^2 z(t)}{dt^2} + d_{2z} \frac{dz(t)}{dt} + k_{2z} z(t) + \\
 & d_{1z} \left[ \frac{dz(t)}{dt} - \frac{dw(t)}{dt} \right] + k_{1z} [z(t) - w(t)] + \\
 & \quad + d_{1zy} \left[ \frac{dy(t)}{dt} - \frac{du(t)}{dt} \right] + k_{1zy} [y(t) - u(t)] = 0.
 \end{aligned}$$

In these equations  $u(t)$ ,  $w(t)$  denote respectively the vertical and horizontal displacement of mass  $m_R$ ,  $y(t)$ ,  $z(t)$  are the vertical and horizontal displacement of mass  $m_B$ , respectively,  $k_{1y}$ ,  $k_{1z}$ ,  $k_{1yz}$ ,  $k_{1zy}$  denote the stiffness coefficients of the oil film,  $d_{1y}$ ,  $d_{1z}$ ,  $d_{1yz}$ ,  $d_{1zy}$  are the oil film damping coefficients,  $k_{2y}$ ,  $k_{2z}$ ,  $d_{2y}$ ,  $d_{2z}$  denote respectively the stiffness and damping coefficients of the super-bearing housing and  $g$  is the gravitational acceleration which multiplied by mass  $m_R$  yields a static loading of the considered system.

The visco-elastic properties of the oil film in such bearing are determined by the stiffness and damping coefficients  $k_{1y}$ ,  $k_{1z}$ ,  $k_{1yz}$ ,  $k_{1zy}$ ,  $d_{1y}$ ,  $d_{1z}$ ,  $d_{1yz}$ ,  $d_{1zy}$ , temporary values of which are response dependent in (6). These numerical values are computed in each direct integration step of (6) by means of solving of the average Reynolds equation (3) describing partial oil flow between the rough journal and bushing. Here, according to Patir and Cheng (1978, 1979), the pressure flow factors and the shear

flow factors standing in the average Reynolds equations are determined analytically using empirical formulae (4) and (5) obtained by numerical simulations of the model flows between partially lubricated surfaces for various roughness geometry and average oil film thicknesses. Solving in this way the average Reynolds equation for the entire oil gap by means of the finite difference method enables us to determine resultant viscous friction retarding force as well as transverse hydrodynamic load of the bearing. For various roughness geometry the former enables us to determine the frictional power yielded by the bearing and the latter makes possible calculation of stiffness and damping coefficients of the mass-less springs representing visco-elastic properties of the oil film in the assumed model of the “super-bearing”.

#### 4. COMPUTATIONAL EXAMPLES

In the numerical examples numerous roughness amplitudes of the journal and the bushing surface as well as various directional orientations of roughness asperities, which follow from final machining of the journal and the bushing, will be taken into consideration in order to investigate their influence on kineto-static and dynamic behaviour of the transverse oil-film bearing. In the first step a kineto-static problem is investigated, i.e. for the shaft rotating with the constant speed  $\Omega$  loaded by the gravitational force only. For this purpose all inertial and damping terms in equations (6) have been neglected. Thus, the non-linear static problem is described by the following system of algebraic equations:

$$\begin{aligned}
 & k_{1y} [u_0 - y_0] + k_{1yz} [w_0 - z_0] = m_R g, \\
 & k_{1z} [w_0 - z_0] + k_{1zy} [u_0 - y_0] = 0, \\
 & k_{2y} y_0 + k_{1y} [y_0 - u_0] + k_{1yz} [z_0 - w_0] = 0, \\
 & k_{2z} z_0 + k_{1z} [z_0 - w_0] + k_{1zy} [y_0 - u_0] = 0,
 \end{aligned} \tag{7}$$

where  $u_0$ ,  $w_0$ ,  $y_0$ ,  $z_0$  denote static displacement components of the respective degrees of freedom of the model shown in figure 1 and defined in (6). If, additionally, the rigid bearing housing is assumed, then  $y_0 = z_0 = 0$  and equations (7) reduce to

$$\begin{aligned}
 & k_{1y} u_0 + k_{1yz} w_0 = m_R g, \\
 & k_{1z} w_0 + k_{1zy} u_0 = 0.
 \end{aligned} \tag{8}$$



Moreover, if the same statistical roughness level on both mutually sliding surfaces is assumed for the journal and the bushing, then  $\sigma_1 = \sigma_2$  in (5) which results in  $\phi_s = 0$ . For isotropic roughness on both surfaces, i.e. for  $\gamma=1$ , the pressure flow factors are approximated by the formula (4) with the following coefficients, (Patir and Cheng, 1978 and 1979):

$$\phi_n(H, \gamma) = 1 - 0.90e^{-0.56H}. \quad (9)$$

According to the above, in order to solve the nonlinear kineto-static problem of mixed lubrication in the journal bearing by solving (8) and calculating the frictional power in the oil film it is necessary to integrate the average Reynolds equation (3) for  $U_2=0$  with the right hand side the same as in (1), where the pressure flow factors are determined using formula (9).

As the first example 'TEST3000.DAN described in CD disc attached to the monography by Kiciński (2005) was modified and used for calculations. Then, the following parameters of the journal bearing have been assumed: several bearing static loads PST, the relative journal-to-bushing eccentricity defined as

$$\varepsilon_0 = \left| \frac{O_p - O_c}{\Delta R} \right| = \frac{e}{\Delta R} = 0.001,$$

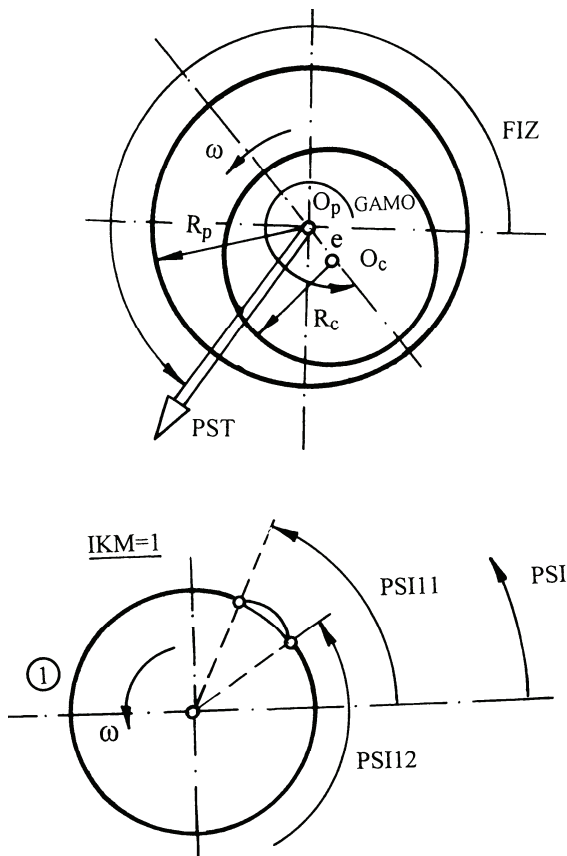


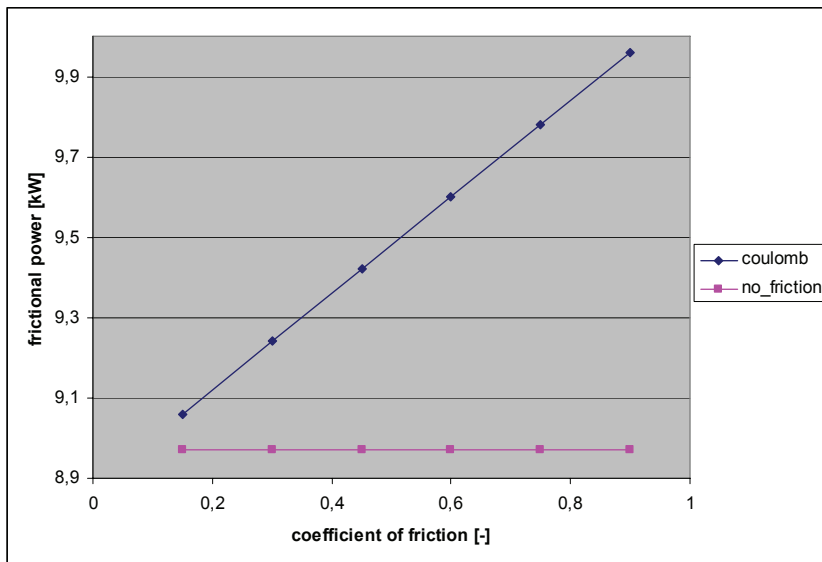
Fig. 2. Scheme of the journal bearing geometry, according to Kiciński (1996, 2005)

where  $O_p, O_c$  are the geometrical centres of the bushing and journal, respectively, as shown in figure 1, and  $\Delta R$  is the bearing nominal clearance equal to  $0.8 \cdot 10^{-4}$  m. The incoming oil temperature has been assumed  $40^\circ\text{C}$ , the oil dynamic viscosity  $0.05 \text{ Ns/m}^2$ , the constant journal rotational speed 3000 rpm, the journal diameter 0.1 m, the bearing length 0.2 m, the oil pocket width 0.15 m, the oil supply pressure  $10 \cdot 10^5 \text{ N/m}^2$  and for the oil density  $870 \text{ kg/m}^3$ . For the assumed journal bearing with single oil pocket (IKM=1) angle PSI11 of the oil-supply input edge is equal to 15 deg and the oil output edge angle PSI12=345 deg.

At current stage of the project following algorithm modifications are proposed:

1. Calculate from the roughness data the initial minimal oil film thickness  $H_0$  for which the mixed-friction conditions are assumed.
2. Search for the circumferential journal sectors I,  $I=1, M1$ , where the oil film thickness  $H$  is smaller than the limit value  $H_0$  assumed from the roughness data. Denote boundaries of the mixed-friction area as "b1" and "b2".
3. Calculate the area of all those sectors  $A(J) = (b2(J) - b1(J)) * LL$ , where LL corresponds to the bearing width.
4. Calculate from the roughness data the probable area of metal-to-metal contact.
5. Calculate the normal force acting in the metal-to-metal friction area as a part of the total shaft gravitational force.
6. Calculate the additional mixed friction force assuming simple the Coulomb law for this initial model.
7. Calculate friction power by means of simple multiplication of the friction force times velocity.
8. Calculate the new values of the oil-film thicknesses, oil pressure, new mixed friction area  $A(J+1)$  (according to points 1÷3).
9. If the difference  $A(J+1) - A(J)$  is smaller than the assumed tolerance, finish the calculations.
10. Else, go to point 1.





**Fig. 3.** Friction power dependence on Coulomb friction coefficient for the assumed metal-to-metal friction zone in sectors 10 to 15 (of the total number 36 sectors along the bushing circumference).

The nonlinear scheme described above was included into the existing code ISOTER developed by Kiciński (1996, 2005). As it was expected for such simple dependencies of friction power on Coulomb friction coefficient nearly linear results were obtained, as it shown in figure 3.

Solving the average Reynolds equation (3), beyond the frictional power, for each value of friction coefficient the oil film stiffness coefficients  $k_{1y}$ ,  $k_{1z}$ ,  $k_{1yz}$ ,  $k_{1zy}$  have been determined. Then, using formulae (7) or (8) the kinetostatic equilibrium position of the journal and bushing of the super-bearing shown in Fig. 1 can be calculated.

## 5. FINAL REMARKS AND PREVIEW

In the paper the problem of mixed-lubrication in the transverse journal bearing has been investigated. For this task, when the journal-to-bushing centres mutual distance is smaller than the bearing clearance and the partial lubrication process can be assumed, the average flow model for the oil film is proposed. This model is based on the average Reynolds equation modified by the pressure and shear factors determined by proper analytical functions of the sliding surface statistical roughness properties and of the journal-to-bushing mutual distance. For greater journal-to-bushing centres mutual distances, i.e. when the metal-to-metal contact between the sliding rough surfaces of the journal and bushing can be observed, the Coulomb model of friction has been applied. Because of natural difficulties connected with determination of reliable values of friction co-

efficients for the partially lubricated rough surfaces, various realistic numerical values of this coefficient were taken into consideration in the numerical example. In the next steps of research in this field, beyond the partial lubrication, also the metal-to-metal friction effects are still going to be investigated, i.e. for so high loadings of the journal bearing, when the hydrodynamic lubrication theory for the oil film stops to be in force. Then, various roughness parameters of the sliding surfaces as well as several types of bearing loadings will be investigated.

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**NUMERYCZNA ANALIZA TARCIA MIESZANEGO  
W ŁOŻYSKACH ŚLIZGOWYCH**

## Streszczenie

Celem pracy jest przedstawienie algorytmu, który umożliwi uwzględnienie tarcia mieszane w łożyskach ślizgowych. Zaproponowano takie modyfikacje równania Reynoldsa, które pozwolą uwzględnić wysokość nierówności na powierzchniach wału i panwi. Następnie założono możliwość wystąpienia warunków tarcia mieszane, do którego może dojść, gdy najmniejsza odległość wału i panewki zmniejszy się osiągając wartość

przy której może dojść do kontaktu obydwu współpracujących części. Przyjęto, że w takim przypadku tarcie mieszane zostanie opisane z wykorzystaniem klasycznego modelu Coulomba. Podstawą obliczeń jest prosty algorytm numeryczny wbudowany w program IZOTER prof. Kicińskiego (1996, 2005) oparty na Metodzie Różnic Skończonych i przeznaczony do symulacji zjawisk towarzyszących pracy łożysk ślizgowych. Zaproponowany algorytm przetestowano na przykładzie łożyska stosowanego w turbinie siłowni energetycznej.

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