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# THERMAL CONTACT FORMULATION BASED ON THE MORTAR METHOD

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**Abstract**

A new approach to model heat transfer between two bodies in mechanical contact is presented. The proposed method is inspired on the "mortar method", more frequently used for mechanical contact, and its development was triggered by the necessity of correctly modelling the heat transfer between glass and moulds in glass forming processes due to the large dependence of glass viscosity on temperature. Typically, when modelling these processes with the finite element method a moving mesh, attached to the deforming glass, deals with the mechanical and thermal problems in the glass. In the moulds due to the low pressures involved only the heat transfer problem is usually addressed and consequently the same mesh is kept throughout the modelling process. In the proposed method a virtual interface, the "mortar", is established between the two bodies to deal with the heat transfer between them. A master/slave strategy, combined with a penalty formulation, is used. Interface elements are established in the discretisation of the "mortar" surface, in which the nodes are projection of the interface nodes of the two bodies. The heat flux between the two bodies is obtained from the interpolation of the temperatures of the two bodies at the interface and the heat transfer coefficient may be evaluated from the contact pressure and viscosity on the slave body. As a result a more effective thermal contact solution is obtained and dependence on the chosen meshes and spurious oscillations, which are typical in standard penalty formulations, are avoided.

**Key words**: thermal contact, heat resistance, mortar method, glass forming

## 1. INTRODUCTION

The study presented here was triggered by the necessity of correctly modelling the heat transfer between glass and moulds in forming processes due to the large dependence of glass viscosity on temperature (César de Sá et al, 1986, 1999, 2002). Forming processes of glass containers are complex coupled thermal/mechanical problem with interaction between the heat transfer analysis and the viscous flow of molten glass. Typically, in these processes, initially a lump of molten glass, at temperatures close to 1200ºC is fed from a furnace to a mould where by means of a plunger or air pressure the blank is formed. Subsequently the blank is inserted in a final mould in which by a blow pressure the final product is obtained with a temperature distribution that may range between 500ºC to 700ºC depending on the final shape or the type or colour of the glass in use. These main stages are schematically represented in figure 1.

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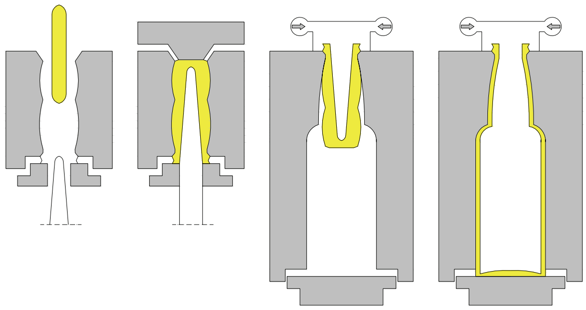

*Figure 1. Schematic representation of forming of a glass container*

Although this process is very fast, taking only a few seconds, the transfer of heat and change of viscosity play a fundamental role. The changes in temperature influence the very process of heat transfer since the thermal properties of glass change with temperature. On the other hand the great dependence of glass viscosity with the temperature (ranging from $10^2$Pa·s to $106^2$ Pa·s in the interval of forming temperatures) influences dramatically the flow of the material and therefore the final product.

The control of the temperature distribution in blank and blow moulds used in the hollow glass industry is therefore a key factor in the making of high quality containers and once mastered is a decisive aspect in the innovation and economy production. In fact any disturbance in the "ideal" temperature pattern may result in a defective product with, for example, an uneven glass distribution, figure 2, or an unfavourable residual stress distribution.

*Figure 2. Different glass distributions due to different process conditions*

One of the key aspects affecting the temperature distribution is the control of the heat flux from the glass to the blank and blow moulds (Pchelyakov & Guloyan, 1985; Storck et al., 1998, Keijts & van der Werff, 2001). Locally this heat flux is a function of the heat contact resistance resulting from the existence of a thin gap between the glass and the moulds. This gap consists of air and combustion products from the mould lubricant whose effective conductivity depends on the local pressure and viscosity. In fact the higher the pressure the lesser is the heat contact resistance. Also as the viscosity increases, with the cooling of the mould, the contact becomes less perfect and the contact resistance increases. It is remarkable that only approximately 60% of the theoretical heat flux, if no contact resistance existed, enters, for example, cast iron moulds. In industry this problem is generally not addressed as it is difficult to handle and therefore the possibility of numerically being able to model it would be an important step forward.

Recently, (Grégoire et al., 2006), an interface finite element was developed in order to deal with the thermal contact between glass and moulds in glass forming processes and implemented in a commercial code (Abaqus). This element had however some limitations due to the fact that not all the desired features could be included as some restrictions are posed by the commercial code handling. In order to solve those limitations a new interface element is here presented. The element is inspired in the mortar method developed for mechanical contact (El-Abbasi & Bathe, 2001; Wriggers, 2002), and is addressed for the coupled problem described and with application on press and blow production of hollow glass containers. In this work some tests are performed to evaluate its robustness.

## 2. MORTAR METHOD FOR HEAT RESISTANCE MODELLING

There are two main problems when dealing with the heat transfer between two bodies that come into contact. The first one is the fact that the heat flux between the two bodies may depend on the contact pressure. That is clearly the case of the thermal contact between glass and moulds in glass forming processes in which, as referred before, other phenomena like the local glass viscosity, and therefore local glass temperature, play also an important role. The second one is the fact that, to enforce thermal contact restraints in the discretised form of the governing equations that describe the phenomenon, problems associated to the use of non-matching meshes for the two bodies may result in poor and/or unstable solutions. Also an over constrained pathological numerical behaviour, which is typical when constraints are imposed using weak formulations, may occur when certain conditions, like the Babuska–Brezzi or the *inf-sup* conditions are not satisfied, (El-Abbasi & Bathe, 2001; Wriggers, 2002). This problem has been circumvented, for mechanical contact, by means of the master/slave formulations or the so-called Mortar Method (McDevitt & Laursen, 2000; Wohlmuth, 2000). In this last method, between the master and the slave bodies, an intermediate surface is considered in which the constraint is imposed. Inspired in this method a solution for the heat thermal contact is sought in the next paragraphs.

Suppose two bodies $\Omega_1$ and $\Omega_2$ moving towards each other, in which heat transfer phenomena are taking place, as schematically represented in figure 3. Let us assume that $k_{i=1,2}$ and $\Omega_{i=1,2}$ are the corresponding conductivities and heat source terms in the two bodies, that $T_a$ is the ambient temperature, that in part of the boundaries, $\Gamma_{\tau,i=1,2}$, essential boundary conditions (temperatures) are prescribed and in part of the boundaries, $\Gamma_{\alpha,i=1,2}$, natural boundary conditions (heat flux to the surrounding air) occur, with corresponding heat transfer coefficients $h_{\alpha,i=1,2}$.

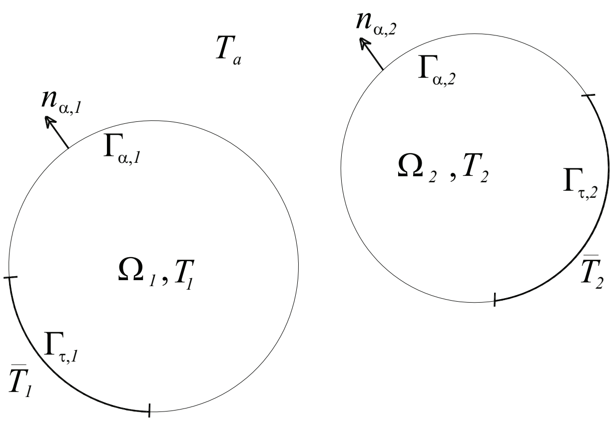


*Figure 3. Two bodies before contact.*

If radiation effects are neglected and a Lagrangian discretisation (mesh) is adopted to model the two bodies the convective terms may be neglected and the heat diffusion may be obtained, for a set of essential boundary conditions and for each time step, from the variational principle associated with the following functional, (Grégoire et al., 2006):

$$\Pi_i = \int_{\Omega_i} \left( \tfrac{1}{2} k_i \ \left\| \nabla T_i \right\|^2 - Q_i \ T_i \right) \ d\Omega_i + \int_{\Gamma_{\alpha,i}} \left( \tfrac{1}{2} h_{\alpha,i} \left( T_i - T_a \right)^2 \right) d\Gamma, \qquad i=1,2 \qquad (1)$$

When contact is made, as in figure 4, the heat transfer in the two bodies is constrained by the thermal conditions in the contact zone.

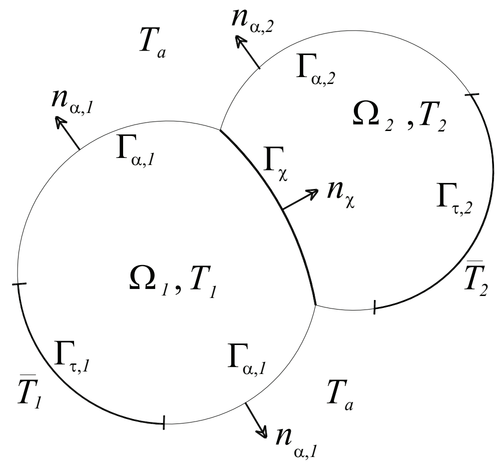


*Figure 4. Two bodies after contact.*

Inspired in the mortar method we may assume that the system is equivalent to two virtual non-contacting bodies, as in figure 5, in which in the surfaces $\Gamma_{\chi,i=1,2}$ corresponding to the original contact surface the two bodies are insulated, plus a virtual surface $\Gamma_\chi$, corresponding to the original contact zone, in which the heat transfer between the two bodies is taking place.

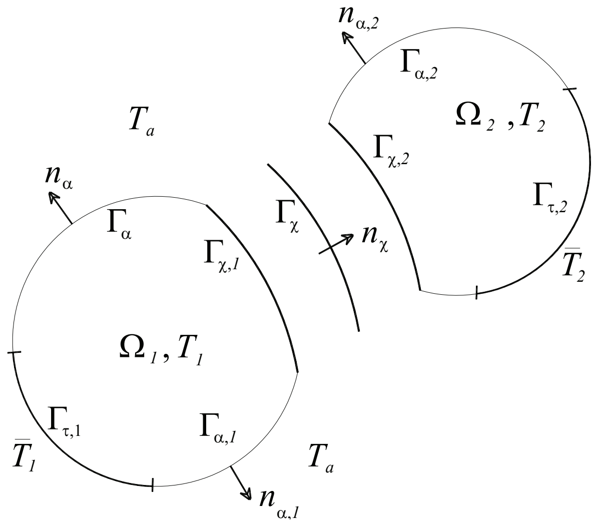


*Figure 5. "Mortar" surface after contact of the two bodies.*

The variational principle may now be obtained from original functional including the restraint in a penalised form:

$$\Pi_p = \Pi_1 + \Pi_2 + \tfrac{1}{2} \int_{\Gamma_\chi} h_\chi \left( T_1 - T_2 \right)^2 \ d\Gamma \qquad (2)$$

The penalty term $h_\chi$ may be identified with the heat transfer coefficient between the surfaces in contact, (Grégoire et al., 2006), and may be set, locally, as a function of pressure and/or viscosity and allows for the possibility of having a discontinuity between the temperatures in the glass and the mould at the interface.

## 3. MORTAR INTERFACE ELEMENT

The thermal contact is associated with the mechanical contact which is typically dealt with the Lagrange multiplier or penalty methods. To avoid over constrained solutions in the discretised problem usually a master/slave strategy is used in which the mesh of the slave body is more refined to prevent penetration from the master body (Wriggers, 2002). In the case of glass forming problems, in principle, the obvious solution for the slave side is the glass, where the deformation occurs and consequently requiring a finer mesh and remeshing operations. In the mould, as the forming pressures are very low, only the heat transfer problem is important and the initial mesh remains the same during the simulation of the process, making it the obvious choice for the master side.

Nevertheless, in many applications the existence of curved zones with small radius of curvature imposes, locally, fine meshes in the moulds. Also, during the forming simulation, the elements on the glass may be highly stretched, meaning that at the contact zones the slave mesh may be coarser then the master one. Then, at those locations the heat transfer will not be well captured which is an important weakness due to the high dependence of glass viscosity on the temperature.

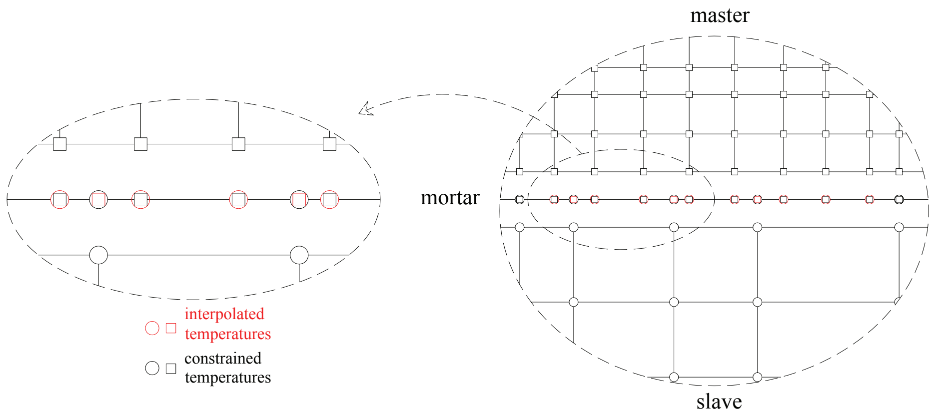


*Figure 6. Intermediate surface (mortar) mesh between slave and master meshes.*

When discretising the variational principle associated to equation (2), within a finite element framework, the meshes representing the two bodies usually do not match at the interface. According to what was previously assumed in section 2, no boundary conditions are imposed at the contact areas for the two meshes, i.e., they are assumed to be insulated. The heat transfer between the two bodies, represented in equation (2) by the third term of the right hand side, is dealt with interface elements that are defined in the virtual mortar interface, as in figure 6. They are therefore formulated from the discretisation of the following terms obtained from the first variation on equation (2):

$$\ldots \int_{\Gamma_\chi} h_\chi \left(T_1 - T_2\right) \delta T_1 \, d\Gamma - \int_{\Gamma_\chi} h_\chi \left(T_1 - T_2\right) \delta T_2 \, d\Gamma \ldots \tag{3}$$

using the standard finite element procedures.

The nodes of these "mortar" elements are obtained from the projection of the interface nodes of the meshes representing the two bodies. In the definition of the element temperatures and properties the following strategy is followed:

*i)* in the nodes obtained from the projection of the nodes of the mesh of the slave body ($\Omega_1$):

- temperatures ($T_1$) are constrained to have the same temperatures as the corresponding ones in the mesh of the slave body ($\Omega_1$);
- pressure values and viscosity used to evaluate $h_\chi$ are prescribed to be those obtained in the corresponding nodes of the slave body ($\Omega_1$);
- temperatures ($T_2$) are interpolated from the mesh of the master body ($\Omega_2$);

*ii)* in the nodes obtained from the projection of the nodes of the mesh of the master body($\Omega_2$):

- temperatures ($T_1$) are interpolated from the mesh of the slave body ($\Omega_1$);
- pressure values and viscosity used to evaluate $h_\chi$ are interpolated from the mesh of the slave body ($\Omega_1$);
- temperatures ($T_2$) are constrained to have the same temperatures as the corresponding ones in the mesh of the master body ($\Omega_2$);

The described procedure results in a more effective thermal contact solution avoiding "double pass strategies" (Wriggers, 2002) and consequent over constrained problems.

## 4. ASSESSMENT OF THE PROPOSED METHOD

A simple example, in which a cylinder of crystal glass at the temperature of 1150ºC is put into contact with a steel (XC38) cylinder at the temperature of 485ºC, see figure 7, was used to test the proposed method in modelling thermal contact. The cylinders are insulated to the surrounding ambient. The thermal properties of the glass and steel are described in table 1.

The "mortar" element was implemented in a commercial code (Abaqus), by means of a user subroutine facility, in order to better assess its performance.

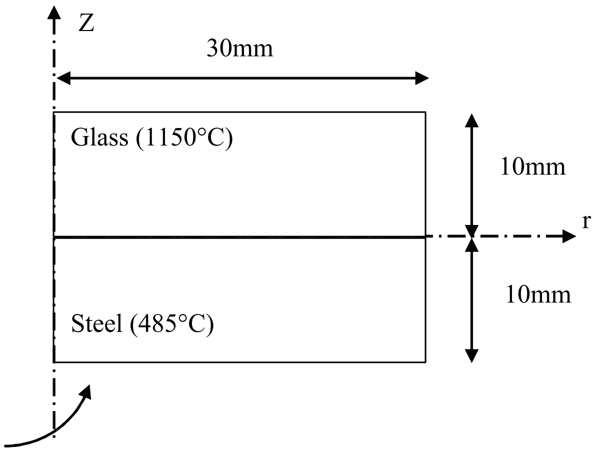


*Figure 7. Geometry of glass and steel cylinders (axi-symmetry relative to Z- axis).*

| | Glass (crystal) at 1150°C | Steel (XC38) at 485°C |
|---|---|---|
| Thermal conductivity (W/m/°C) | 2.77 | 37 |
| Specific heat (J/kg/°C) | 1282 | 620 |
| Density (kg/m$^3$) | 2500 | 7800 |

*Table 1. Glass and steel thermal properties.*

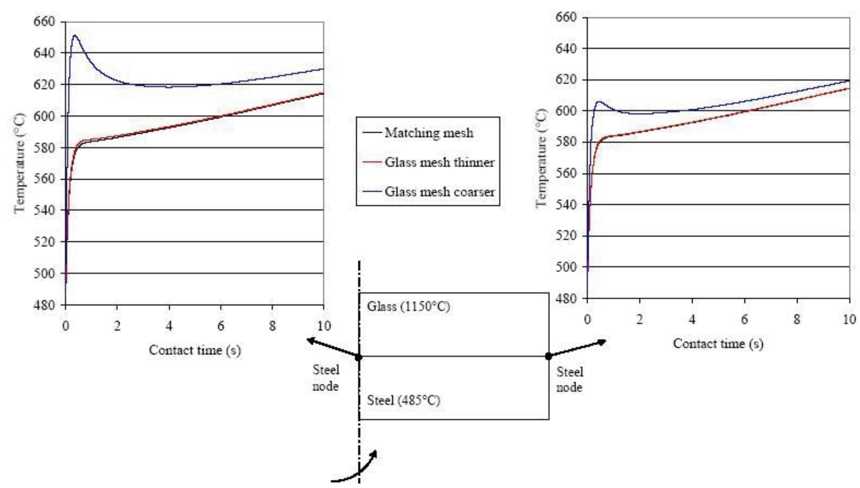


*Figure 9. Solution with Abaqus. Temperature evolution for two points in the steel cylinder.*

A set of different situations was used, including simulations with coarse and fine meshes for situations with conforming and non-conforming meshes at the interface, and, in particular, the critical situation where a coarser mesh is at the slave side. The reference cases for comparison are chosen to be the solutions with conforming meshes. Three types of meshes were used and are represented in figure 8.

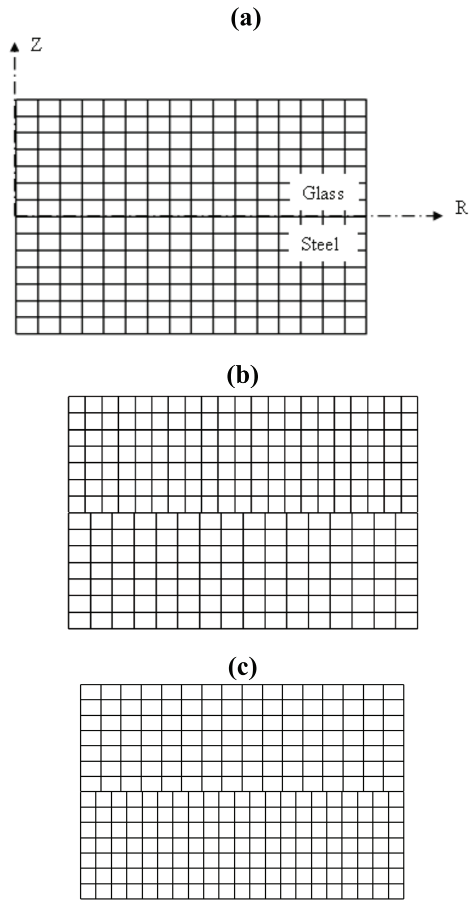


*Figure 8. Meshes: (a) Conforming meshes – (b) and (c) Non-conforming meshes.*

In order to compare solutions with and without the interface element a constant heat transfer coefficient of 5000 W/m²/°C is adopted at the interface, as it was not possible to deal with a pressure and temperature dependence in the commercial code without including the interface element.

### 4.1. Solution without the "mortar" element

Using only the commercial code the dependence on the choice of the master and slave surfaces is evident as shown in

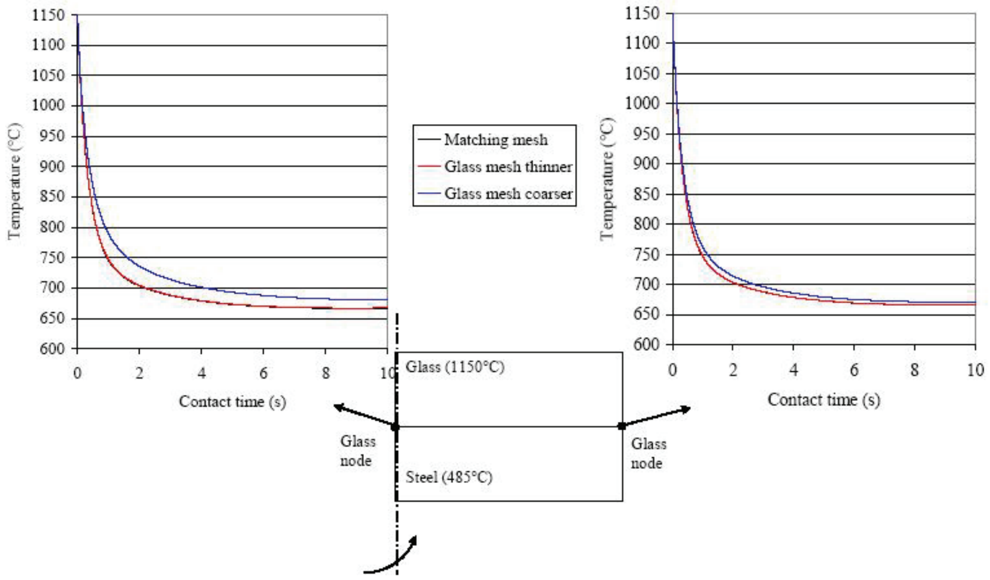


*Figure 10. Solution with Abaqus. Temperature evolution for two points in the glass cylinder.*

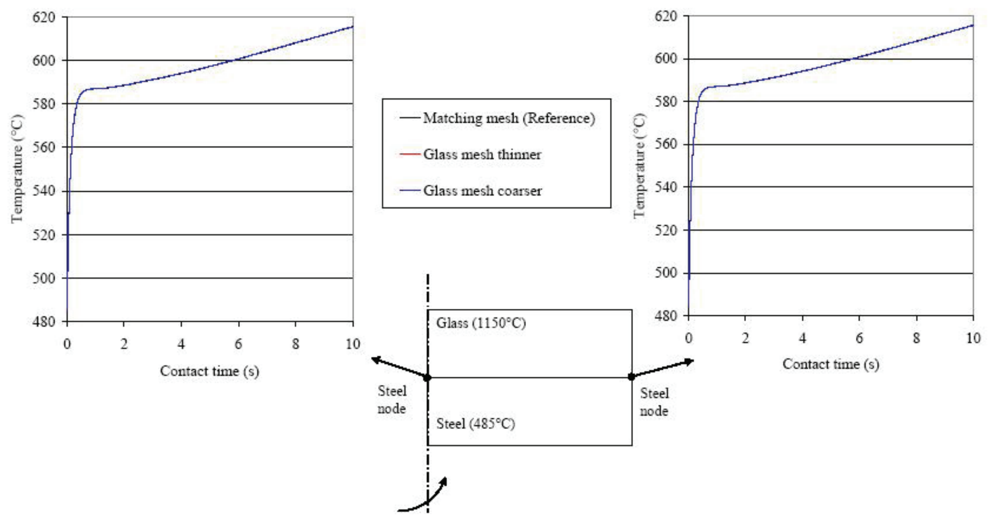


*Figure 11. Solution with the mortar element. Temperature evolution for two points in the steel cylinder.*

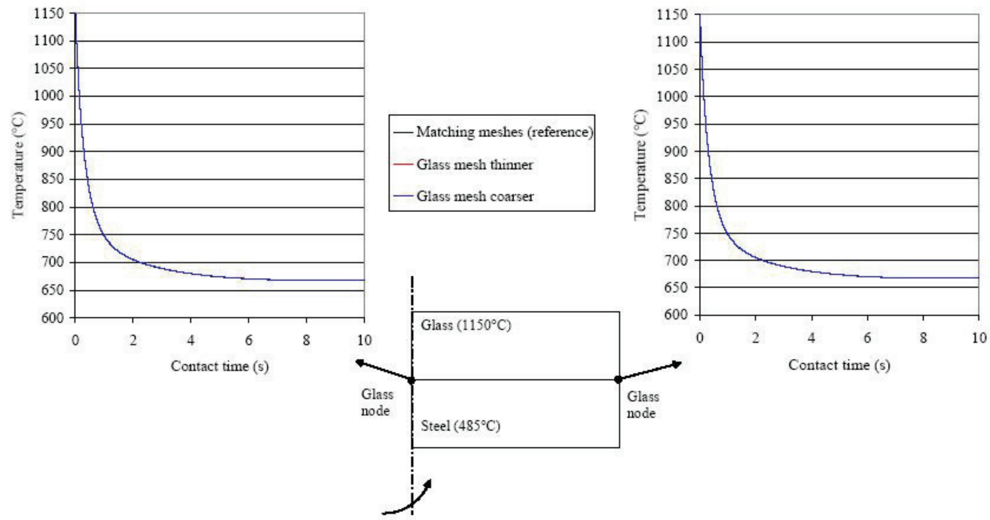


*Figure 12. Solution with the mortar element. Temperature evolution for two points in the glass cylinder.*

figures 9 and 10 where the temperatures for two points at the interface in the first *10 s*, both on the glass and steel side, are represented. It is clear that when the coarse mesh is used on the glass side (the slave body) the temperature evolution departs from the reference solution (with conforming meshes) both on the glass and steel sides at the interface.

### 4.2. Solution including the "mortar" element

By including the "mortar" element in the commercial code it is possible to have a coarser mesh on the slave side. This is shown in figures 11 and 12 where the temperature evolutions for the same two points at the interface as in the previous test, are represented for the first 10 s. In all cases the solutions totally agree with the reference solution (with conforming meshes).

The inclusion of the "mortar" element also eliminates spurious oscillations of the temperatures at the interface that are present in the standard solution of the commercial code even if a fine mesh is used on the slave side. In the solution with the "mortar" element, as shown in figure 13 the temperature is constant along the contact surface at each instant, as expected, while using only the commercial code, even with a finer mesh in the slave side, spurious temperature oscillations are present for both a thermomecanical solution, i.e., not neglecting the self weight, or a pure heat transfer analysis.

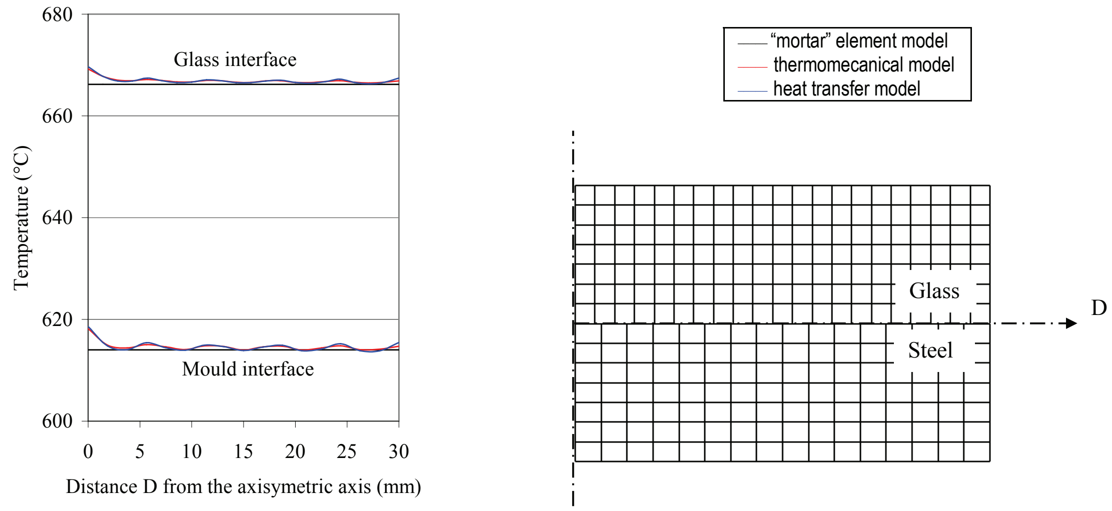


*Figure 13. Glass and steel temperature distribution at the glass/steel interface after 10s.*

## 5. CONCLUSIONS

A new approach to heat contact between two bodies was developed based on the Mortar Method usually associated with the mechanical contact. The method is particularly useful when the discretisations of the bodies do not fit at the contact interface. The proposed solution proved to be very efficient in dealing with non matching meshes even in cases where the finite element mesh on the slave side is coarser than the mesh in the master side eliminating dependence on the chosen meshes and avoiding oscillations which are typical in standard penalty formulations.

This study was triggered by the necessity of correctly modelling the heat transfer between glass and moulds in forming processes due to the large dependence of glass viscosity on temperature. Nevertheless its range of application is general, i.e., it may be utilised in any type of thermal contact problem and it will be particularly useful whenever thermal contact is a decisive process parameter.

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